7. Theory of Computation

Introduction to Theoretical CS

Two fundamental questions.

- What can a computer do?
- What can a computer do with limited resources?

General approach.

- Don't talk about specific machines or problems. ← e.g., Pentium M running Linux kernel 2.6.15
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory?

In theory …

- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice …

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

"In theory there is no difference between theory and practice. In practice there is." -Yogi Berra

Regular Expressions and DFAs

Pattern Matching Applications

Test if a string matches some pattern.

- Process natural language.
- Scan for virus signatures.
- Search for information using Google.
- Access information in digital libraries.
- Retrieve information from Lexis/Nexis.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

Describing a Pattern

PROSITE. Huge database of protein families and domains.

Q. How to describe a protein motif?

Ex. [signature of the C2H2-type zinc finger domain]

- C
- Between 2 and 4 amino acids.
- C
- 3 more amino acids.
- One of the following amino acids: **LIVMFYWCX**.
- 8 more amino acids.
- H
- Between 3 and 5 more amino acids.
- H

CAASCGGPYACGGWAGYHAGWH

Regular Expressions: Basic Operations

Regular expression. Notation to specify a set of strings.

Operation	Regular Expression	Yes	No
Concatenation	`aabaab`	`aabaab`	every other string
Wildcard	`.u.u.u.`	`cumulus` `jugulum`	`succubus` `tumultuous`
Union	`aa | baab`	`aa` `baab`	every other string
Closure	`ab*a`	`aa` `abbba`	`ab` `ababa`
Parentheses	`a(a|b)aab`	`aaaab` `abaab`	every other string
Parentheses	`(ab)*a`	`a` `ababababa`	`aa` `abbba`

Regular Expressions: Examples

Regular expression. Notation is surprisingly expressive.

Regular Expression	Yes	No
`.*spb.*` contains the trigraph `spb`	`raspberry` `crispbread`	`subspace` `subspecies`
`a* | (a*ba*ba*ba*)*` multiple of three `b`'s	`bbb` `aaa` `bbbaababbaa`	`b` `bb` `baabbbaa`
`.*0....` fifth to last digit is `0`	`1000234` `98701234`	`111111111` `403982772`
`gcg(cgg|agg)*ctg` fragile X syndrome indicator	`gcgctg` `gcgcggctg` `gcgcggaggctg`	`gcgcgg` `cggcggcggctg` `gcgcaggctg`

Generalized Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ….
- Additional operations typically added for convenience.
- Ex: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.

Operation	Regular Expression	Yes	No
One or more	`a(bc)+de`	abcde abcbcde	ade bcde
Character classes	`[A-Za-z][a-z]*`	word Capitalized	camelCase 4illegal
Exactly k	`[0-9]{5}-[0-9]{4}`	08540-1321 19072-5541	111111111 166-54-1111
Negations	`[^aeiou]{6}`	rhythm	decade

Regular Expressions in Java

Validity checking. Is `input` in the set described by the `re`?

```
public class Validate {
   public static void main(String[] args) {
      String re    = args[0];
      String input = args[1];
      StdOut.println(input.matches(re));
   }                                ↖
}                          powerful string library method
```

```
                          C2H2 type zinc finger domain
% java Validate "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H" CAASCGGPYACGGAAGYHAGAH
true
                          legal Java identifier
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true
                          valid email address (simplified)
% java Validate "[a-z]+@([a-z]+\.)+(edu|com)" wayne@cs.princeton.edu
true
                                 need quotes to "escape" the shell
```

PROSITE Patterns

PROSITE. Huge database of protein families and domains.
PROSITE patterns. Special notation to describe protein motif.

Ex. The signature of the C2H2-type zinc finger domain is:

```
C-x(2,4)-C-x(3)-[LIVMFYWC]-x(8)-H-x(3,5)-H
```

Cys wildcard concatenation any one of these amino acids 8 copies His

Regular expressions. Easy to translate PROSITE patterns to regular expressions and use rich RE libraries.

More String Library Functions

String searching methods.

`public class String`	*(Java's String library)*
`boolean matches(String re)`	*does this string match the given regular expression*
`String replaceAll(String re, String str)`	*replace all occurrences of regular expression with the replacement string*
`int indexOf(String r, int from)`	*return the index of the first occurrence of the string r after the index from*
`String[] split(String re)`	*split the string around matches of the given regular expression*

```
String s = StdIn.readAll();
s = s.replaceAll("\\s+", " ");
```

replace all sequences of whitespace characters with a single space

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```
String s = StdIn.readAll();
String[] words = s.split("\\s+");
```

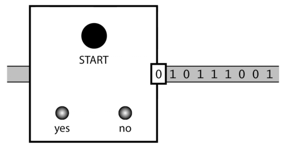
create array of words in document

regular expression that matches any whitespace character

Solving the Pattern Match Problem

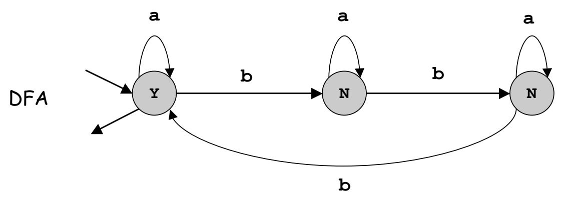
Regular expressions are a concise way to describe patterns.
- How would you implement the method `matches()` ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

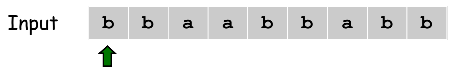
DFA: simple machine that solves a pattern match problem.
- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on `true` or `false` questions for simplicity.



Deterministic Finite State Automaton (DFA)

Simple machine with N states.
- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept input string if last state is labeled Y.

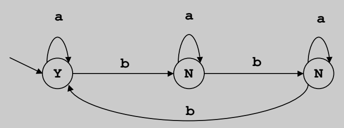




DFA and RE Duality

RE. Concise way to describe a set of strings.
DFA. Machine to recognize whether a given string is in a given set.

Duality. For any DFA, there exists a RE that describes the same set of strings; for any RE, there exists a DFA that recognizes the same set.



multiple of 3 b's

`a* | (a*ba*ba*ba*)*`

multiple of 3 b's

Practical consequence of duality proof: to match RE, (i) build DFA and (ii) simulate DFA on input string.

Implementing a Pattern Matcher

Problem. Given a RE, create program that tests whether given input is in set of strings described.

Step 1. Build the DFA.

- A compiler!
- See COS 226 or COS 320.

Step 2. Simulate it with given input.

```
State state = start;
while (!StdIn.isEmpty()) {
   char c = StdIn.readChar();
   state = state.next(c);
}
StdOut.println(state.accept());
```

Application: Harvester

Harvest information from input stream.

- Harvest patterns from DNA.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

- Harvest email addresses from web for spam campaign.

```
% java Harvester "[a-z]+@([a-z]+\\.)+(edu|com)" http://www.princeton.edu/~cos126
rs@cs.princeton.edu
maia@cs.princeton.edu
doug@cs.princeton.edu
wayne@cs.princeton.edu
```

Application: Harvester

equivalent, but more efficient representation of a DFA

Harvest information from input stream.

- Use `Pattern` data type to compile regular expression to NFA.
- Use `Matcher` data type to simulate NFA.

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester {
   public static void main(String[] args) {
      String re        = args[0];
      In in            = new In(args[1]);
      String input     = in.readAll();
      Pattern pattern  = Pattern.compile(re);
      Matcher matcher  = pattern.matcher(input);
                                    find next substring matching pattern
      while (matcher.find()) {
         Stdout.println(matcher.group());
      }
   }                                the substring that matches the pattern
}
```

Application: Parsing a Data File

Ex: parsing an NCBI genome data file.

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus (platypus)
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

```
String re = "[ ]*[0-9]+([actg ]*).*";
Pattern pattern = Pattern.compile(re);
In in = new In(filename);
while (!in.isEmpty()) {
   String line = in.readLine();
   Matcher matcher = pattern.matcher(line);
   if (matcher.find()) {      extract the RE part in parentheses
      String s = matcher.group(1).replaceAll(" ", "");
      // do something with s
   }
}
```

Summary

Programmer.

- Regular are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.

- RE is a compact description of a set of strings.
- DFA is an abstract machine that solves RE pattern match problem.

You. Practical application of core CS principles.

Variations.

- Terminology: DFA, FSA, FSM.
- DFAs with output: Moore machines, Mealy machines.

Fundamental Questions

Q. Are there patterns that cannot be described by any RE/DFA?
A. Yes.

- Bit strings with equal number of 0s and 1s.
- Decimal strings that represent prime numbers.
- DNA strings that are Watson-Crick complemented palindromes.

Q. Can we extend RE/DFA to describe richer patterns?
A. Yes.

- Context free grammar (e.g., Java).
- Turing machines.