

COS 451 – Assignment (due April 18, 2006)

1 Point-Set Triangulation

Here is a problem that has eluded satisfactory answers for a long time. Now is your chance to have a crack at it: Given n points in the plane (assumed to be in general position) a triangulation of the points is a triangulation of their convex hull such that a point is a vertex of the triangulation if and only if it is one of the n points. We'd like to find the triangulation that minimizes the added length of the edges. It is tempting to conjecture that the Delaunay triangulation is just what we want. Disprove this conjecture.

2 On a Problem of Area vs. Perimeter

Give a simple, yet rigorous proof that if a convex polygon P lies entirely inside another convex polygon Q , the perimeter of P cannot exceed that of Q .

3 Equidecomposibility

Prove that a disk (i.e. a circle and its interior) cannot be equidecomposed with a square of equal area, using a finite number of pieces. Assume that all cuts are piecewise smooth. You are allowed to make reasonable assumptions to make this problem tractable.

4 4-Way Cut

Given $4n$ points in the plane in general position and a line not passing through any of them that has $2n$ points on each side, prove the existence of another line that partitions the plane into 4 regions, each one with exactly n points.

5 Ant Trails

Write a program that starts with a large polygon (preferably not simple) and connects the midpoints of the edges in sequence, thus producing a new polygon of the same size. Erase the old polygon and display. Iterate at infinitum. (Keep rescaling because it shrinks fast.) What's happening? Can you suggest any mathematical explanation?