

COS 451 – Assignment (due April 4, 2006)

1 Decomposing Polygons

Let P be a simple polygon with n vertices, v_1, \dots, v_n in clockwise order. An optimal convex decomposition of P (abbreviated *OCD*) is any partition of P into a minimum number of convex polygons (called *parts*).

1. Prove that in some cases it might be necessary to introduce new vertices in the interior of P in order to obtain an *OCD*.
2. Prove that there always exists an *OCD* of P with no interior part (a part is called interior if it contains no point on the boundary of P).

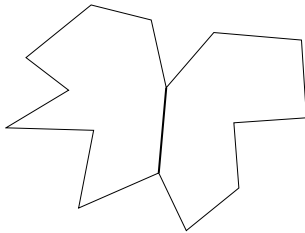


Figure 1: A *link*

Let a *link* be a segment connecting two reflex vertices v_i, v_j such that $v_i v_j$ lies completely inside P and resolves both reflex angles (Fig.1). A convex decomposition of P is called *regular* if it can be obtained by drawing segments in this manner: first draw a collection of non-intersecting links, then iterate on the following process. As long as there is still some unresolved reflex vertex, pick one of them and draw a segment from it along a *good* direction until you hit a point previously drawn or a point on the boundary of the polygon (Fig.2). This point is *not* allowed to be a reflex vertex. A direction is *good* if it resolves the reflex angle of the vertex in question.

3. Prove that if it is possible to draw k non-intersecting links, then a convex decomposition can be obtained with at most $c - k + 1$ parts (c is the number of reflex angles in P).
4. Describe an $O(n^3)$ algorithm for computing a regular convex decomposition with a minimum number of parts (hint: think of dynamic programming).

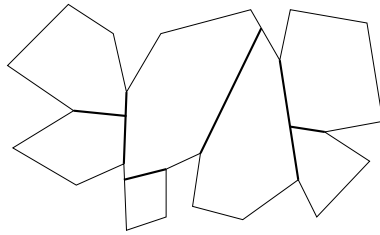


Figure 2: A regular decomposition with two links

2 Furthest Neighbors

Given n points in the plane sorted by x -coordinates, find an $O(n)$ time algorithm for computing the two points whose interdistance is maximum. Establish the correctness of your algorithm.

3 Triangulation

A polygon is called hollow if it has polygonal holes (Fig.3). Prove that to compute a triangulation of a hollow n -gon requires $\Omega(n \log n)$ time in the worst case; hint: assume that the number h of holes is $\Omega(n)$. For the general case where h is arbitrary (it could be zero), can you establish a worst-case lower bound on the time for triangulation as a function of both n and h .

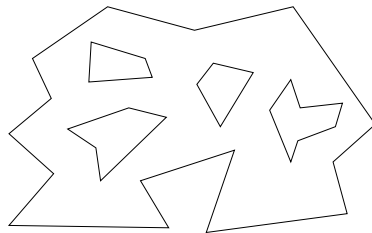


Figure 3: A hollow polygon with $n = 35$ and $h = 3$

4 Intersecting Convex Polygons

Design and analyze a linear time algorithm for computing the intersection of two convex n -gons. Extra credit: Implement it and test it thoroughly.

5 A Containment Problem

Let P and Q be two convex polygons with respectively p and q vertices. Find an $O(p + q)$ time algorithm for deciding whether P can be made to fit inside Q , allowing only translations.