Discussion

- Did the “Theory of Everything” article make you look at something in a new way?

- What is the Church-Turing thesis and how convincing is it to you?
Self-Reproduction

Fallacious argument for impossibility:
M.C. Escher

Print Gallery
Fallacy Resolved: “Blueprint” can involve some computation; need not be an exact copy!

Print this sentence twice, the second time in quotes. “Print this sentence twice, the second time in quotes.”
High-level description of program that self-reproduces

A

\{  \text{Print} \ 0 \\
\text{Print} \ 1 \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{Print} \ 0 \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \}

B

\{ \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \\
\text{.} \}

Prints binary code of B

Takes binary string on tape, and in its place prints (in English) the sequence of statements that produce it, followed by the translation of the binary string into English.
Next several lectures: Computational Hardware

- Boolean logic and Boolean circuits
- Sequential circuits (circuits with memory)
- Clocked circuits and Finite State Machines
- CPUs
- Operating System
- Networks, Internet
Logical Reasoning

Ben only rides to class if he overslept, but even then if it is raining he’ll walk and show up late to class (he really hates to bike in the rain). But if there’s an exam that day he’ll bike if he overslept, even in the rain.

It is raining today, Ben overslept, and there’s an exam. Will Ben bike today?

“Propositional logic.”
Propositional Logic: History

- Stoic Philosophers (3rd century BC) – Basic inference rules (modus ponens etc.)
- Some work by medieval philosophers
- De Morgan and Boole (19th century): symbolic logic – “automated”, “mechanical”
- C. Shannon (1930s) – proposal to use digital hardware
Example

Ed goes to the party if Dan does not and Stella does.

Associate Boolean variables with 3 events

\{ 
\begin{align*}
E: & \text{ Ed goes to party} \\
D: & \text{ Dan goes to party} \\
S: & \text{ Stella goes to party}
\end{align*}
\}

Each is either TRUE or FALSE

\[ E = S \text{ AND } (\text{NOT } D) \]

Alternatively, \( E = S \text{ AND } \overline{D} \)
OR

Ed goes to the party if Dan goes or Stella goes

\[ E = D \text{ OR } S \]

Means E is TRUE if one or both of D and S are TRUE

Note: Different from everyday meaning of OR!

Example: You can eat an orange or an apple
Boolean expressions

Composed of boolean variables, **AND, OR, and NOT**

Examples:

D \( \text{AND} \) ( P \ OR \ (\text{NOT} \ Q) )

C \ OR \ D \ OR \ E
Truth table

Lists the truth value of the boolean expression for all combinations of values for the variables.

Boolean Expression  \[ E = S \text{ AND } \overline{D} \]

<table>
<thead>
<tr>
<th>D</th>
<th>S</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Boolean “algebra”

A \textbf{AND} B written as \( A \cdot B \)

A \textbf{OR} B written as \( A + B \)

\[
\begin{array}{ccc}
0 \cdot 0 &=& 0 \\
0 \cdot 1 &=& 0 \\
1 \cdot 1 &=& 1 \\
0 + 0 &=& 0 \\
1 + 0 &=& 1 \\
1 + 1 &=& 1
\end{array}
\]

Funny arithmetic

See assigned reading. (More next time)
3 equivalent ways of representation

Boolean Expression \[ E = S \text{ AND } \overline{D} \]

Boolean Circuit

Truth table – Gives value of \( E \) for every possible assignment to \( D, S \).
\[
\begin{array}{ccc}
D & S & E \\
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 0 \\
1 & 1 & 0 \\
\end{array}
\]

TRUE=1; FALSE=0.
Ben Revisited

Ben only rides to class if he overslept, but even then if it is raining he’ll walk and show up late to class (he really hates to bike in the rain). But if there’s an exam that day he’ll bike if he overslept, even in the rain.

**B**: Ben Bikes  
**R**: It is raining  
**E**: There is an exam today  
**O**: Ben overslept

Give boolean expression for B in terms of R, E and O