

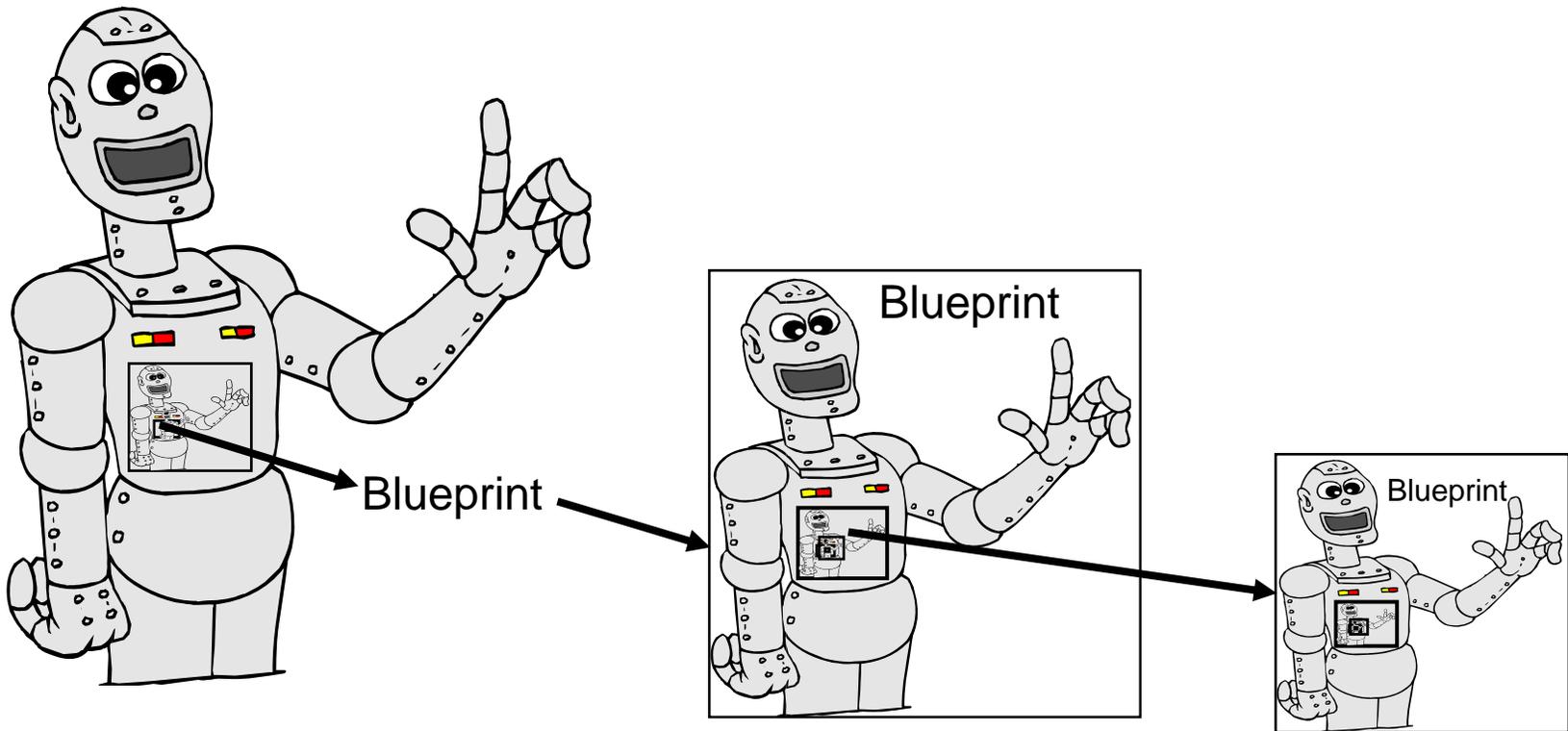


# Discussion

- Did the “Theory of Everything” article make you look at something in a new way?
- What is the Church-Turing thesis and how convincing is it to you?

# Self-Reproduction

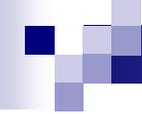
Fallacious argument for impossibility:





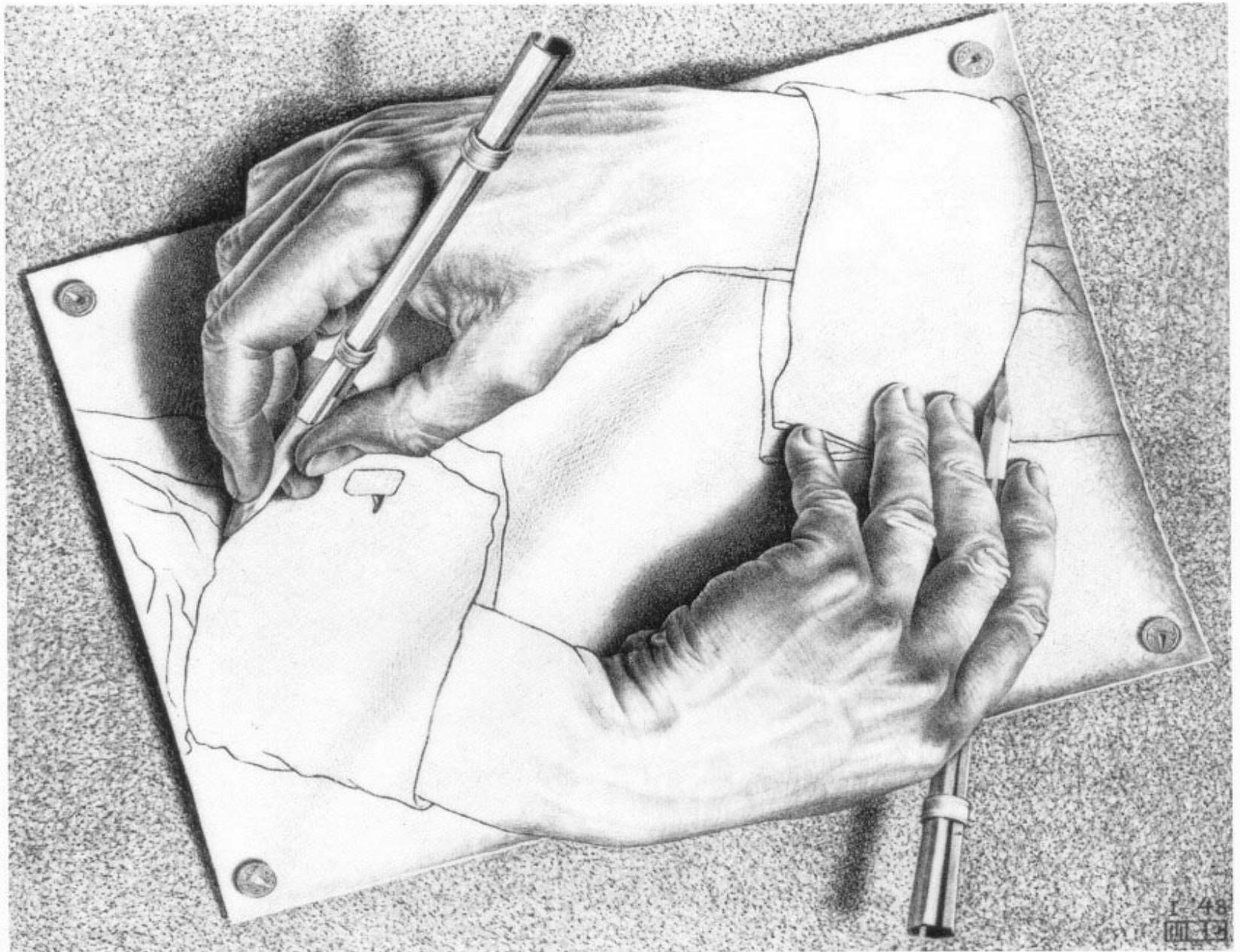
M.C. Escher

*Print Gallery*

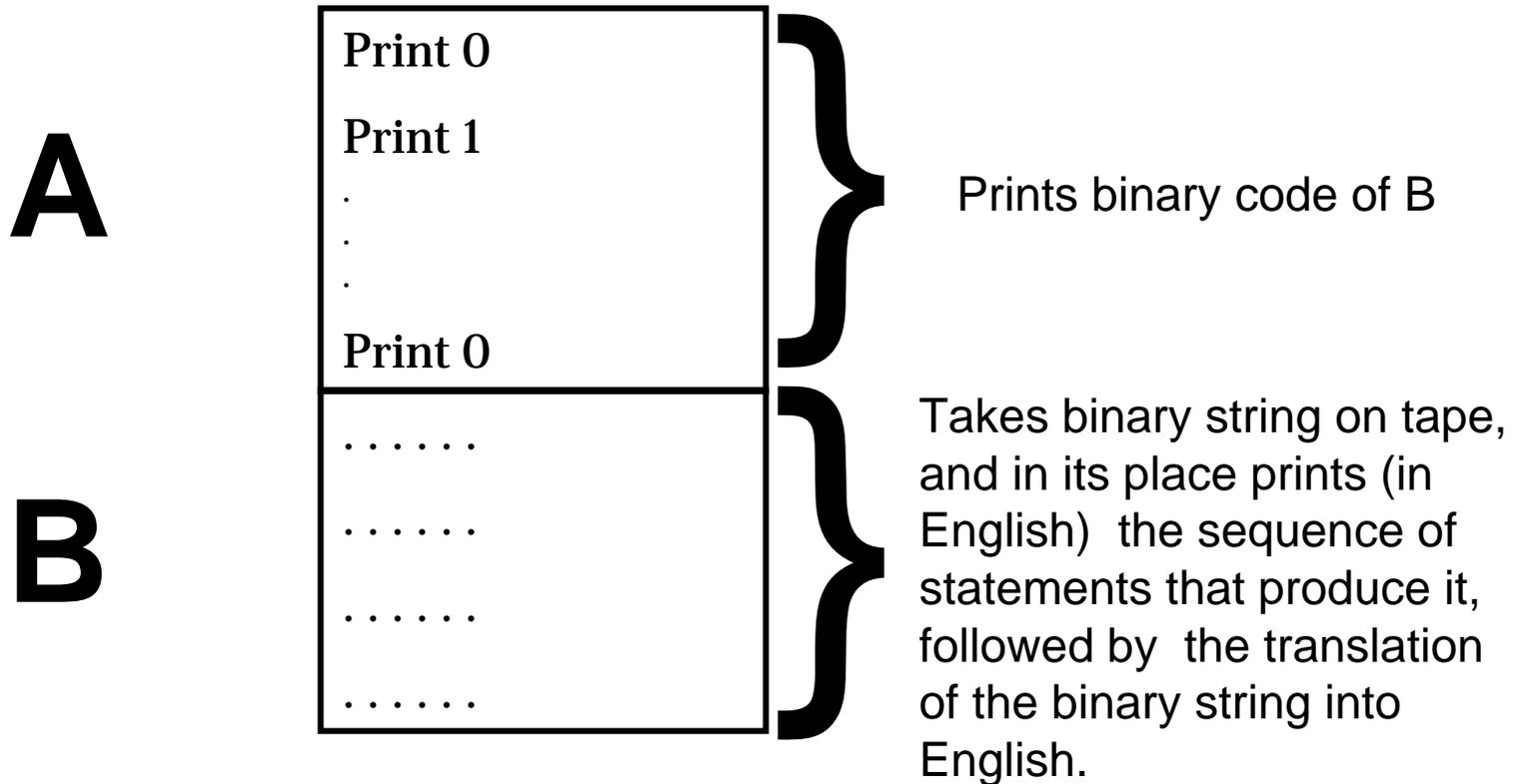


Fallacy Resolved: “Blueprint” can involve some computation; need not be an exact copy!

Print this sentence twice, the second time in quotes. “Print this sentence twice, the second time in quotes.”



# High-level description of program that self-reproduces





# Next several lectures: Computational Hardware

- Boolean logic and Boolean circuits
- Sequential circuits (circuits with memory)
- Clocked circuits and Finite State Machines
- CPUs
- Operating System
- Networks, Internet

# Logical Reasoning

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late to class (he really hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

It is raining today, Ben overslept, and there's an exam. Will Ben bike today?

*“Propositional logic.”*

# Propositional Logic: History

- Aristotle – Law of excluded middle and Law of contradiction.
- Stoic Philosophers (3<sup>rd</sup> century BC) – Basic inference rules (*modus ponens* etc.)
- Some work by medieval philosophers
- De Morgan and Boole (19<sup>th</sup> century): symbolic logic – “automated”, “mechanical”
- C. Shannon (1930s) – proposal to use digital hardware

# Example

Ed goes to the party if Dan does not and Stella does.

Associate Boolean variables with 3 events

$\left\{ \begin{array}{l} \mathbf{E: Ed goes to party} \\ \mathbf{D: Dan goes to party} \\ \mathbf{S: Stella goes to party} \end{array} \right\}$  Each is either TRUE or FALSE

$$\mathbf{E = S \text{ AND } (\text{NOT } D)}$$

$$\text{Alternatively, } \mathbf{E = S \text{ AND } \bar{D}}$$

# OR

Ed goes to the party if Dan goes **or** Stella goes

$E = D \text{ OR } S$

Means E is TRUE if one or both of D and S are TRUE

Note: Different from everyday meaning of OR!

Example: You can eat an orange or an apple



# Boolean expressions

Composed of boolean variables, **AND**, **OR**, and **NOT**

Examples:

**D AND ( P OR (NOT Q))**

**C OR D OR E**

# Truth table

Lists the truth value of the boolean expression for all combinations of values for the variables.

**Boolean Expression**       $E = S \text{ AND } \bar{D}$

**Truth table**

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

# Boolean “algebra”

A **AND** B written as  $A \cdot B$

A **OR** B written as  $A + B$

$$0 \cdot 0 = 0$$

$$0 + 0 = 0$$

$$0 \cdot 1 = 0$$

$$1 + 0 = 1$$

$$1 \cdot 1 = 1$$

$$1 + 1 = 1$$

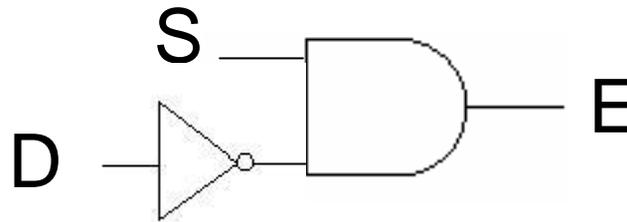
← Funny arithmetic

See assigned reading. (More next time)

# 3 equivalent ways of representation

**Boolean Expression**       $E = S \text{ AND } \bar{D}$

**Boolean Circuit**



**Truth table** – Gives value of E for every possible assignment to D, S.  
TRUE=1; FALSE= 0.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

# Ben Revisited

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late to class (he really hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

**B:** Ben Bikes

**R:** It is raining

**E:** There is an exam today

**O:** Ben overslept

Give boolean expression for B in terms of R, E and O