3D Polygon Rendering Pipeline

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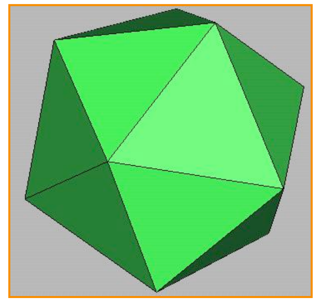
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination



3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

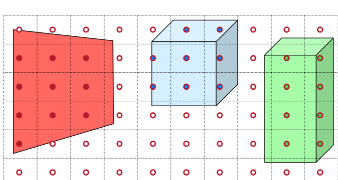
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

Quake II
(Id Software)

Ray Casting Revisited

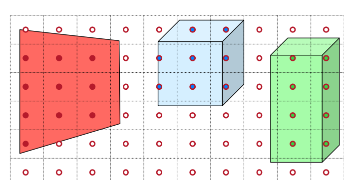
- For each sample …
  - o Construct ray from eye position through view plane
  - o Find first surface intersected by ray through pixel
  - o Compute color of sample based on surface radiance



More efficient algorithms utilize spatial coherence!

3D Polygon Rendering

- What steps are necessary to utilize spatial coherence while drawing these polygons into a 2D image?



3D Rendering Pipeline (for direct illumination)

3D Primitives
→ Modeling Transformation
→ Lighting
→ Viewing Transformation
→ Projection Transformation
→ Clipping
→ Viewport Transformation
→ Scan Conversion
→ Image

This is a pipelined sequence of operations to draw a 3D primitive into a 2D image

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→ Image

```
glBegin(GL_POLYGON);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(1.0, 0.0, 0.0);
glVertex3f(1.0, 1.0, 1.0);
glVertex3f(0.0, 1.0, 1.0);
glEnd();
```

OpenGL executes steps of 3D rendering pipeline for each polygon

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3D Rendering Pipeline (for direct illumination)

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3D Rendering Pipeline (for direct illumination)

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↓
- Modeling Transformation — Transform into 3D world coordinate system
- Lighting — Illuminate according to lighting and reflectance
- Viewing Transformation — Transform into 3D camera coordinate system
- Projection Transformation — Transform into 2D camera coordinate system
- **Clipping — Clip primitives outside camera's view**
- Viewport Transformation
- Scan Conversion

↓
Image

3D Rendering Pipeline (for direct illumination)

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- Clipping — Clip primitives outside camera's view
- **Viewport Transformation — Transform into image coordinate system**
- Scan Conversion

↓
Image

3D Rendering Pipeline (for direct illumination)

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- Clipping — Clip primitives outside camera's view
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- **Scan Conversion — Draw pixels (includes texturing, hidden surface, ...)**

↓
Image

3D Rendering Pipeline (for direct illumination)

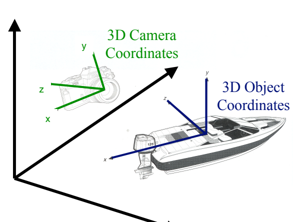
3D Primitives
↓
- **Modeling Transformation** — **Transform** into 3D world coordinate system
- Lighting — Illuminate according to lighting and reflectance
- **Viewing Transformation** — **Transform** into 3D camera coordinate system
- **Projection Transformation** — **Transform** into 2D camera coordinate system
- Clipping — Clip primitives outside camera's view
- **Viewport Transformation** — **Transform** into image coordinate system
- Scan Conversion — Draw pixels (includes texturing, hidden surface, ...)

↓
Image

Transformations

p(x,y,z)
↓ 3D Object Coordinates
Modeling Transformation
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
p'(x',y')

Transformations map points from one coordinate system to another



3D World Coordinates

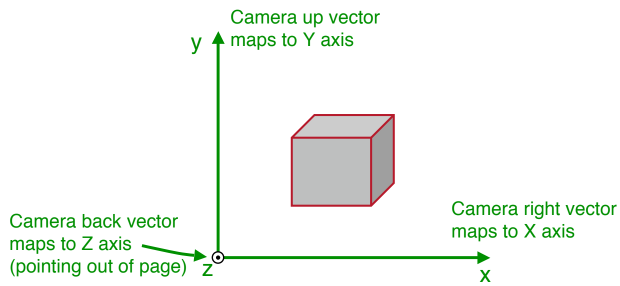
Viewing Transformations

p(x,y,z)
↓ 3D Object Coordinates
Modeling Transformation
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
p'(x',y')

} Viewing Transformations

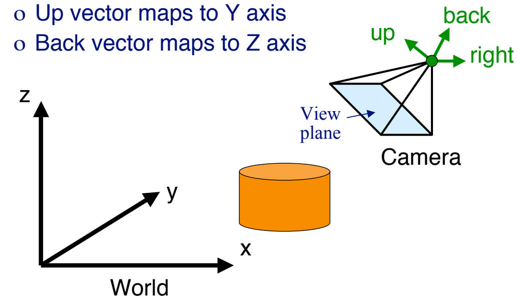
Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis



Finding the viewing transformation

- We have the camera (in world coordinates)
- We want T taking objects from world to camera

$$p^C = T\, p^W$$

- Trick: find T^{-1} taking objects in camera to world

$$p^W = T^{-1} p^C$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

?

Finding the Viewing Transformation

- Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- This matrix is T^{-1} so we invert it to get T … easy!

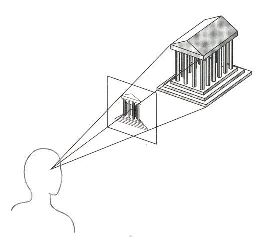
Viewing Transformations

p(x,y,z)
↓ 3D Object Coordinates
Modeling Transformation
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
p'(x',y')

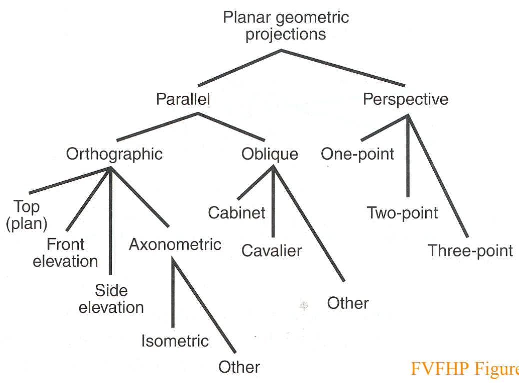
} Viewing Transformations

Projection

- General definition:
  - Transform points in *n*-space to *m*-space ($m<n$)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

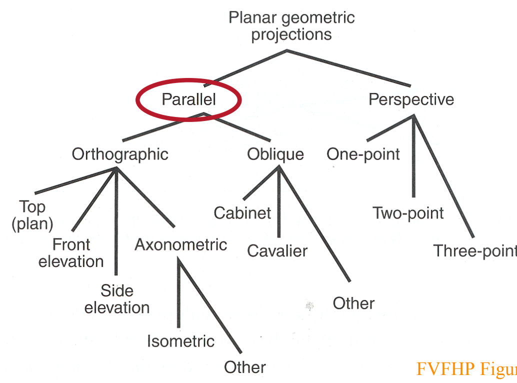


Taxonomy of Projections



FVFHP Figure 6.10

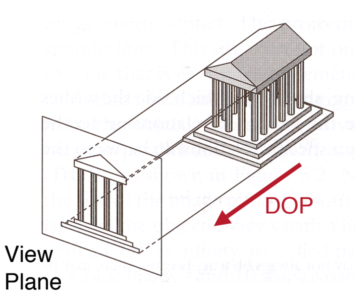
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

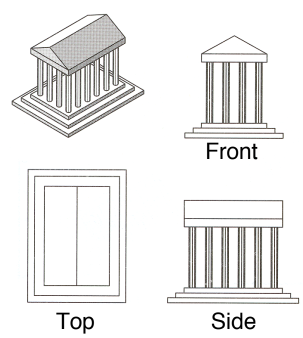
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Orthographic Projections

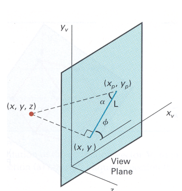
- DOP perpendicular to view plane

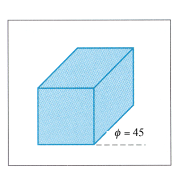


Angel Figure 5.5

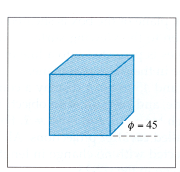
Oblique Projections

- DOP not perpendicular to view plane





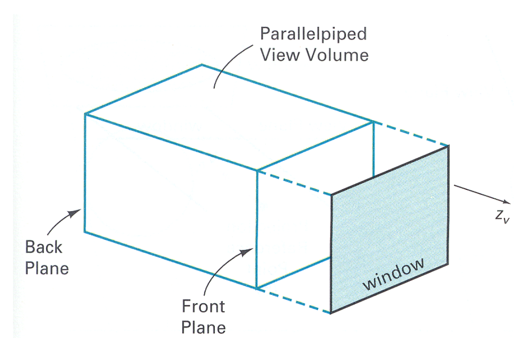
Cavalier
(DOP $\alpha = 45^{\circ}$)



Cabinet
(DOP $\alpha = 63.4^{\circ}$)

H&B Figure 12.24

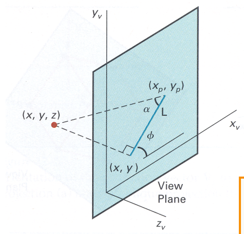
Parallel Projection View Volume



H&B Figure 12.30

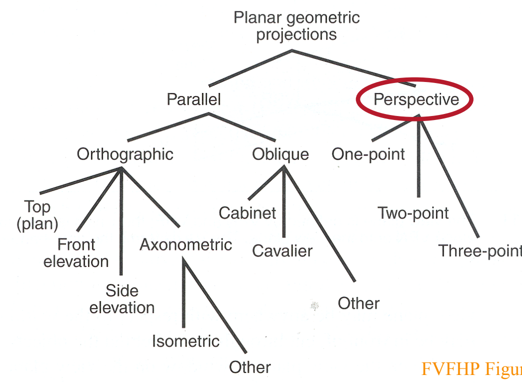
Parallel Projection Matrix

- General parallel projection transformation:



$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L\cos\phi & 0 \\ 0 & 1 & L\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

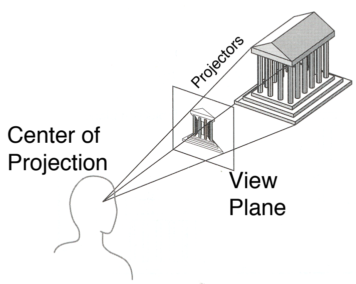
Taxonomy of Projections



FVFHP Figure 6.10

Perspective Projection

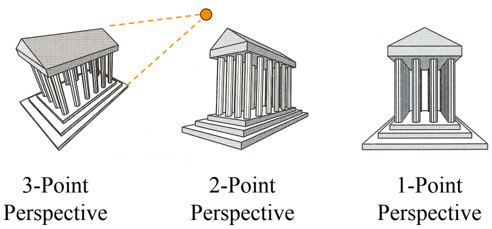
- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)



Angel Figure 5.9

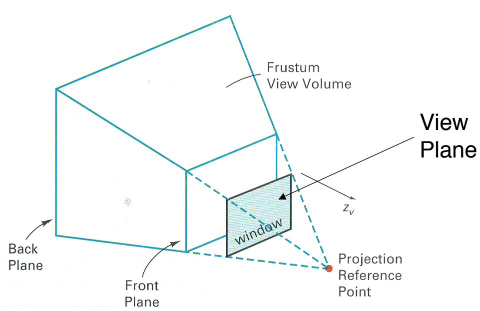
Perspective Projection

- How many vanishing points?



Angel Figure 5.10

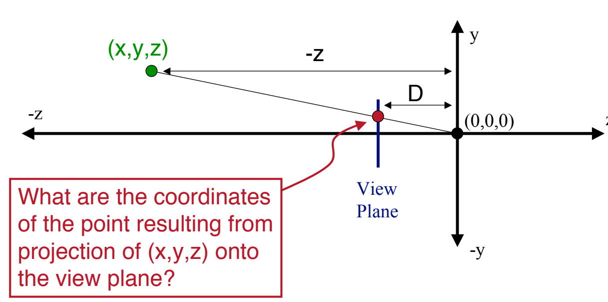
Perspective Projection View Volume



H&B Figure 12.30

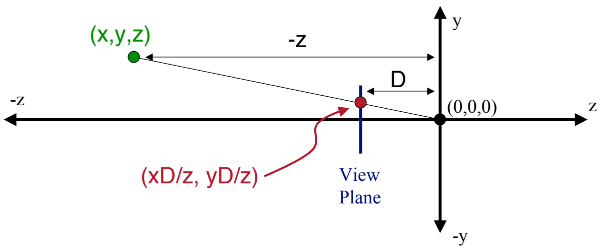
Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

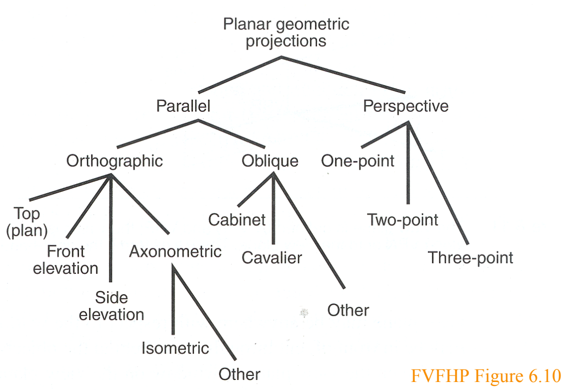
- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

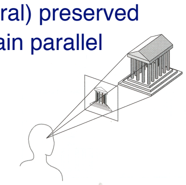
Taxonomy of Projections



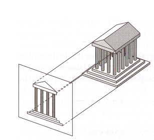
FVFHP Figure 6.10

Perspective vs. Parallel

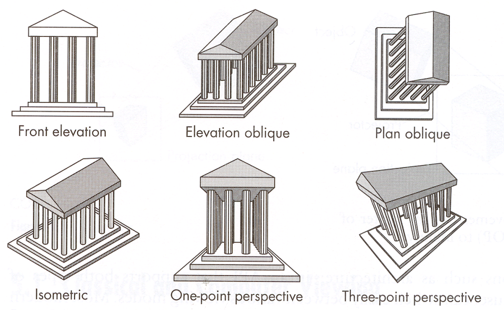
- Perspective projection
  - + Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - + Good for exact measurements
  - + Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



Classical Projections



Angel Figure 5.3

Viewing Transformations Summary

- Camera transformation
  - Map 3D world coordinates to 3D camera coordinates
  - Matrix has camera vectors as rows
- Projection transformation
  - Map 3D camera coordinates to 2D screen coordinates
  - Two types of projections:
    - » Parallel
    - » Perspective

3D Rendering Pipeline (for direct illumination)

3D Primitives
↓ 3D Modeling Coordinates
Modeling Transformation
↓ 3D World Coordinates
Lighting
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Clipping
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
Scan Conversion
↓ 2D Image Coordinates
Image

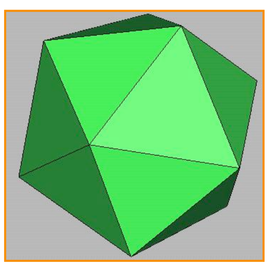


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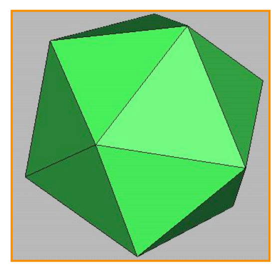
Image



2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

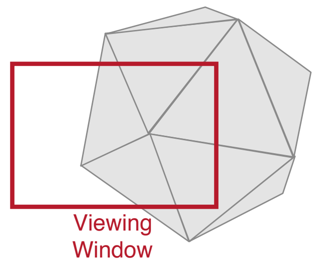
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window

Screen Coordinates

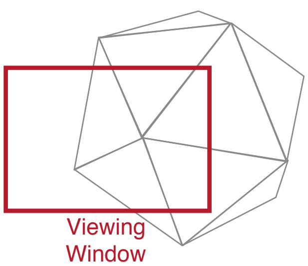
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window



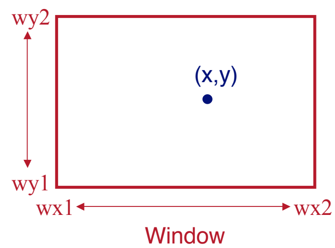
Clipping

- Avoid drawing parts of primitives outside window
  - Points
  - Lines
  - Polygons
  - Circles
  - etc.



Point Clipping

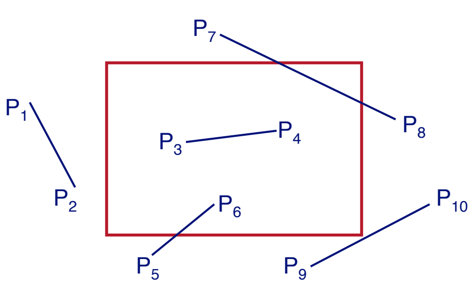
- Is point (x,y) inside the clip window?




```
inside =
   (x >= wx1)  &&
   (x <= wx2)  &&
   (y >= wy1)  &&
   (y <= wy2);
```

Line Clipping

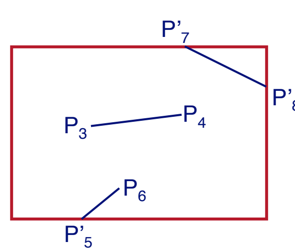
- Find the part of a line inside the clip window



Before Clipping

Line Clipping

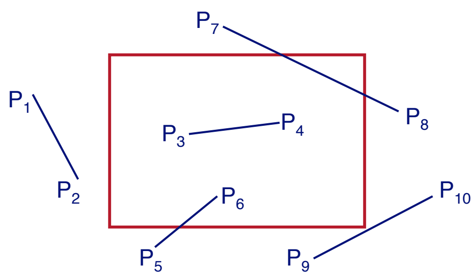
- Find the part of a line inside the clip window



After Clipping

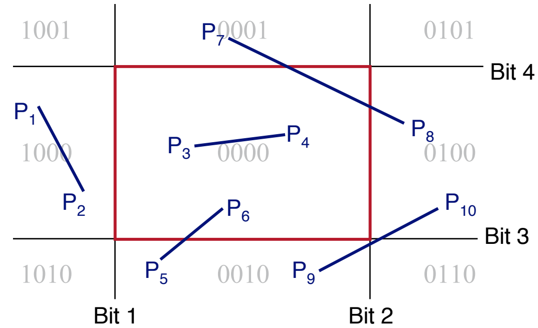
Cohen Sutherland Line Clipping

- Use simple tests to classify easy cases first



Cohen Sutherland Line Clipping

- Classify some lines quickly by AND of bit codes representing regions of two endpoints (must be 0)



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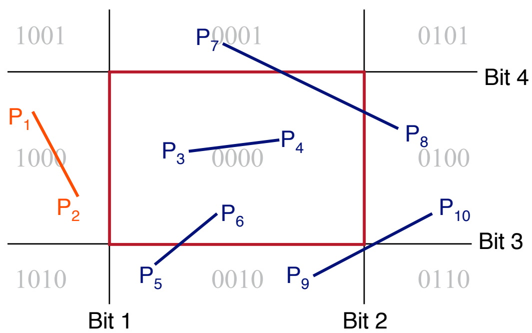

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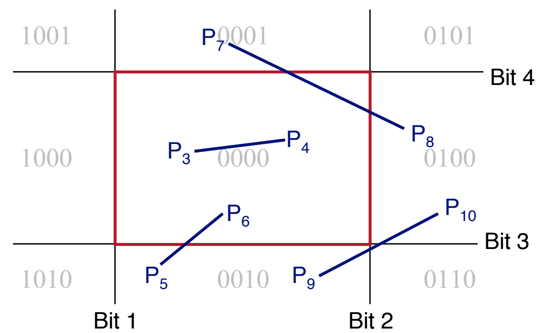

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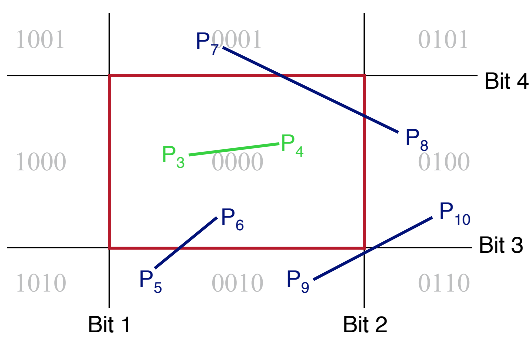
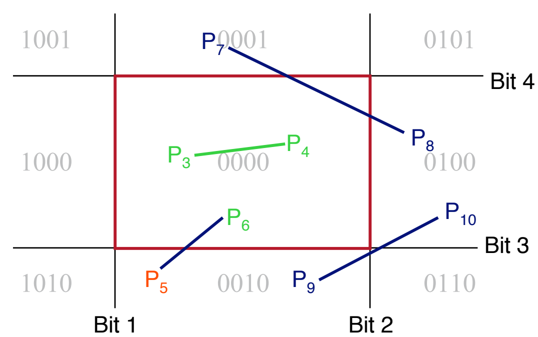


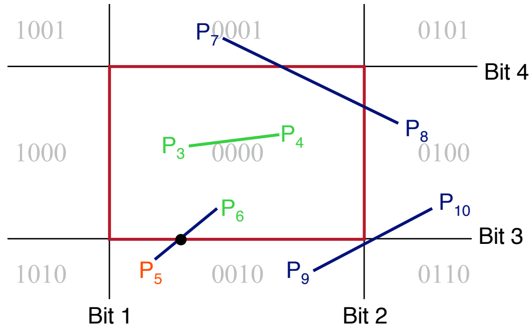
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



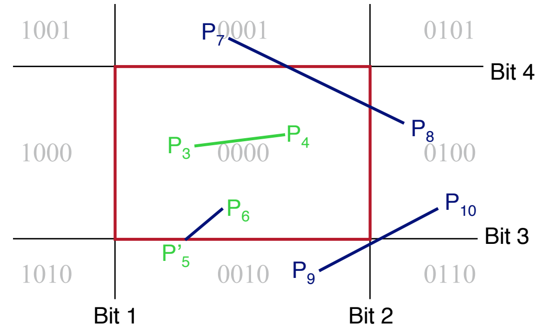
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



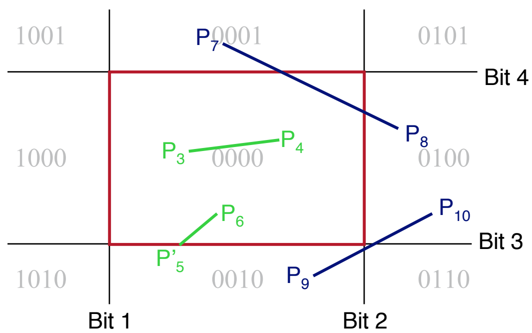
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



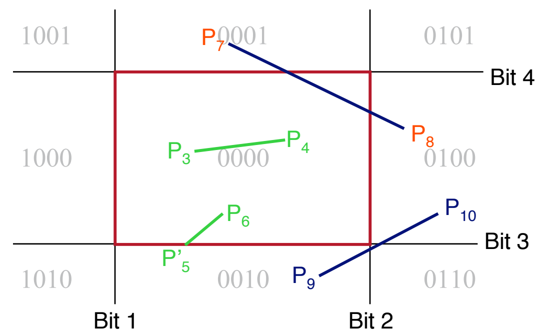
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



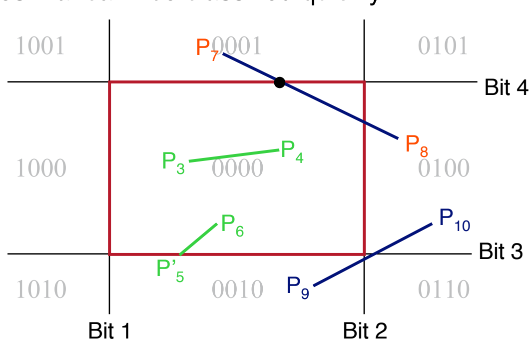
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



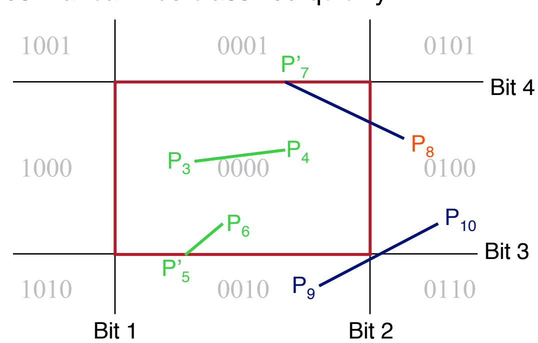
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



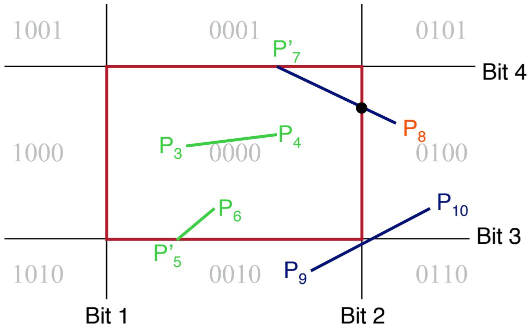
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



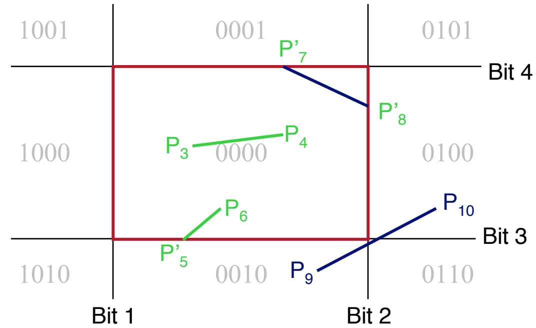
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



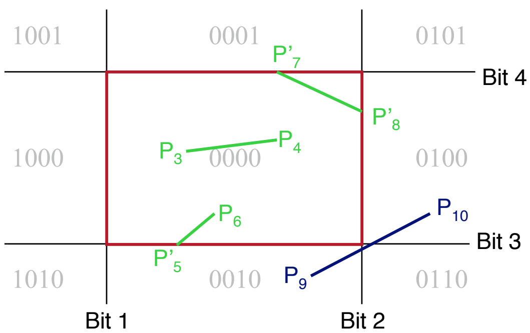
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



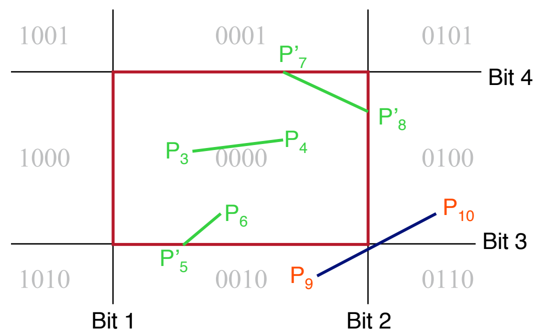
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



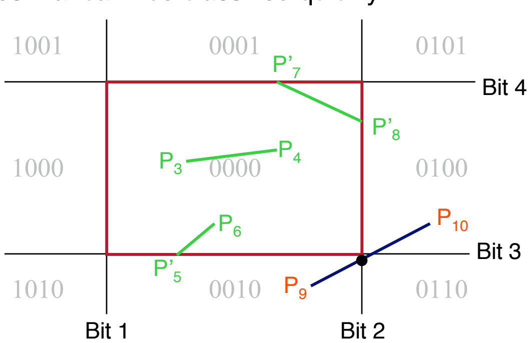
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



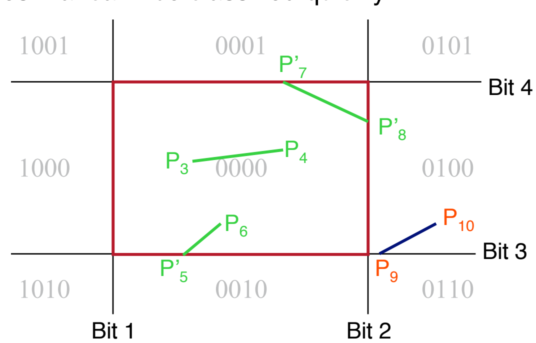
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



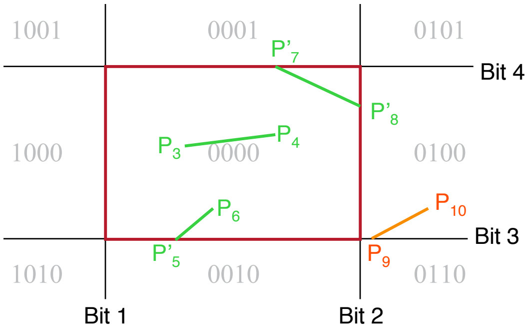
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



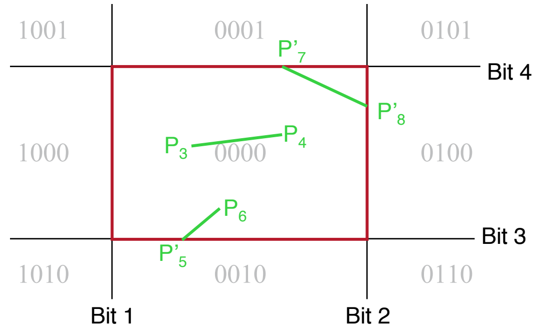
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



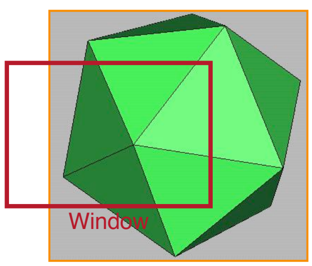
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can’t be classified quickly



Clipping

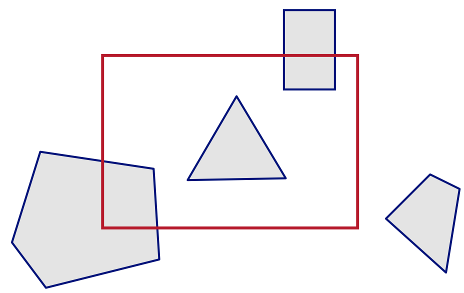
- Avoid drawing parts of primitives outside window
  - Points
  - Lines
  - Polygons
  - Circles
  - etc.



2D Screen Coordinates

Polygon Clipping

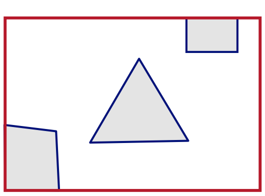
- Find the part of a polygon inside the clip window?



Before Clipping

Polygon Clipping

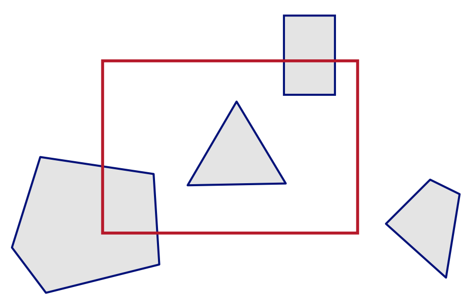
- Find the part of a polygon inside the clip window?



After Clipping

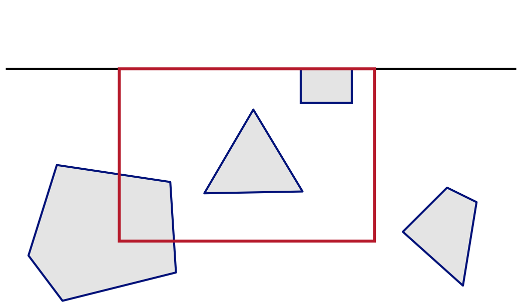
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



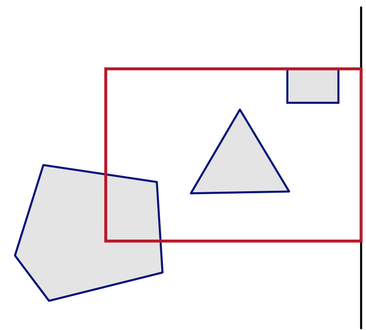
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



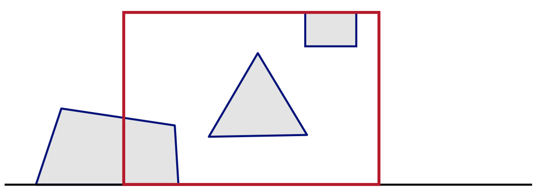
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



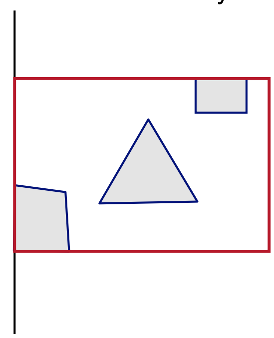
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



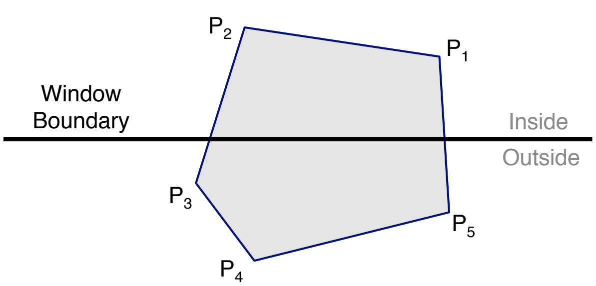
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



Clipping to a Boundary

- Do inside test for each point in sequence, Insert new points when cross window boundary, Remove points outside window boundary



Clipping to a Boundary

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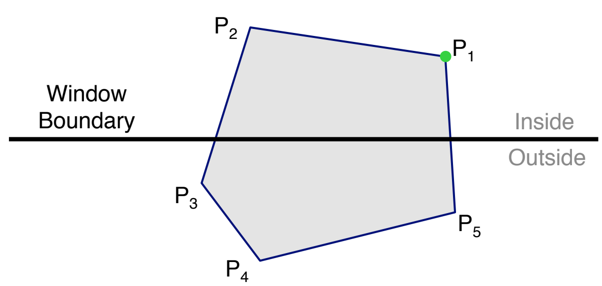
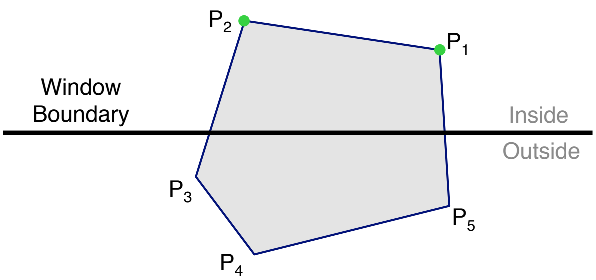


Clipping to a Boundary

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary



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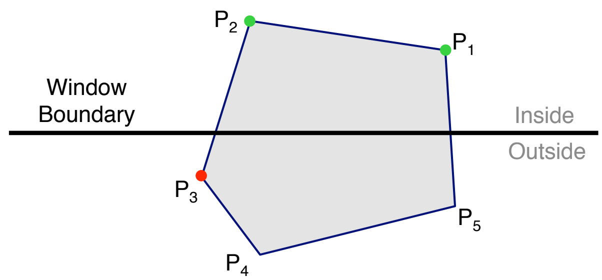

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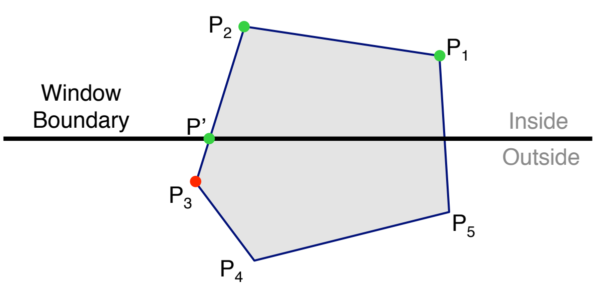

Clipping to a Boundary

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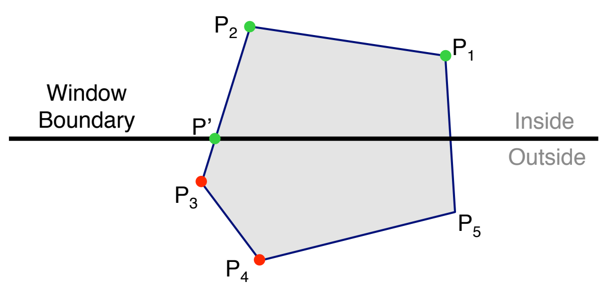
  Remove points outside window boundary



Clipping to a Boundary

- Do inside test for each point in sequence,
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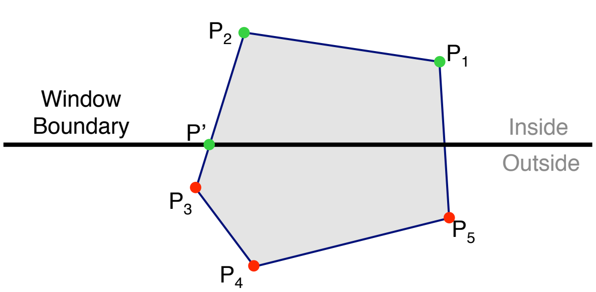
  Remove points outside window boundary



Clipping to a Boundary

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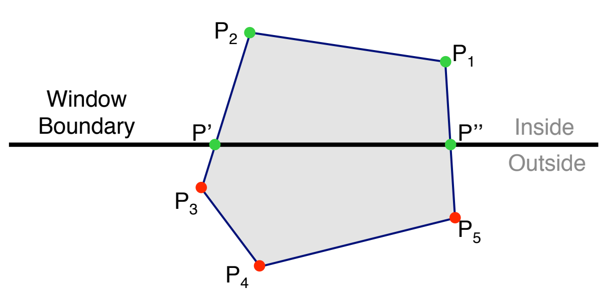

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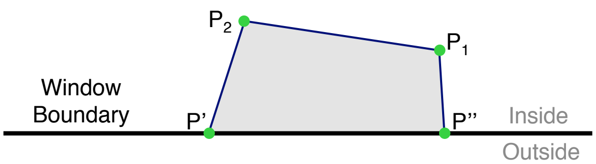
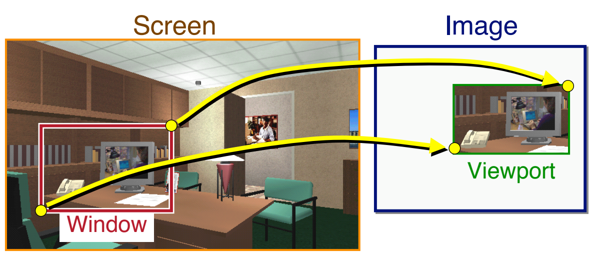


2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

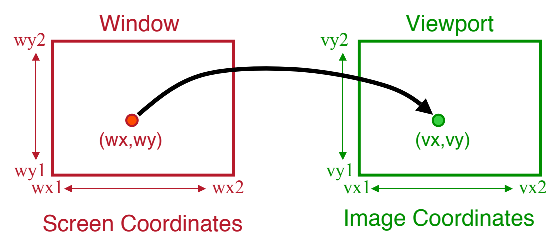
Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



Viewport Transformation

- Window-to-viewport mapping




```
vx = vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1);
vy = vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1);
```

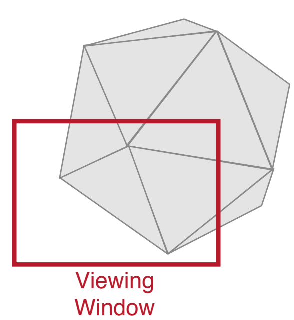
Summary of Transformations

p(x,y,z)
↓ 3D Object Coordinates
Modeling Transformation
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
p'(x',y')

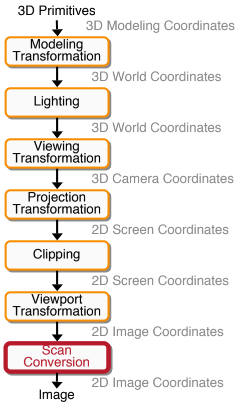
Modeling transformation
Viewing transformations
Viewport transformation

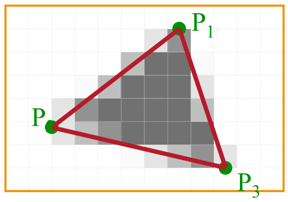
Summary

3D Primitives
↓ 3D Modeling Coordinates
Modeling Transformation
↓ 3D World Coordinates
Lighting
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Clipping
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
Scan Conversion
↓ 2D Image Coordinates
Image



Next Time





Scan Conversion!