Subdivision Surfaces

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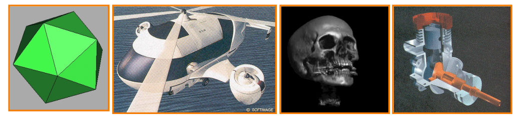
Modeling

- How do we ...
  - Represent 3D objects in a computer?
  - Construct 3D representations quickly/easily?
  - Manipulate 3D representations efficiently?



Different representations for different types of objects

3D Object Representations

- Raw data
  - Voxels
  - Point cloud
  - Range image
  - Polygons
- Solids
  - Octree
  - BSP tree
  - CSG
  - Sweep
- Surfaces
  - Mesh
  - Subdivision
  - Parametric
  - Implicit
- High-level structures
  - Scene graph
  - Skeleton
  - Application specific

3D Object Representations

- Raw data
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- **Surfaces**
  - **Mesh**
  - **Subdivision**
  - Parametric
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  - Scene graph
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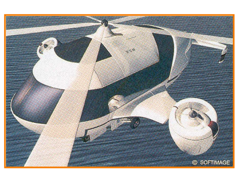
  - Application specific

Surfaces

- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



H&B Figure 10.46

Surfaces

- What makes a good surface representation?
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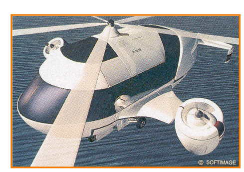

H&B Figure 10.46

Subdivision

- How do you make a smooth curve?

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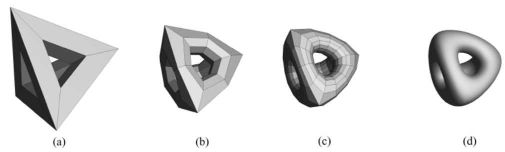
Subdivision Surfaces

- Coarse mesh & subdivision rule
  - Define smooth surface as limit of sequence of refinements



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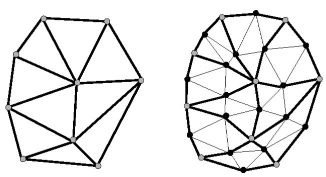
Key Questions

- How refine mesh?
  - Aim for properties like smoothness
- How store mesh?
  - Aim for efficiency for implementing subdivision rules



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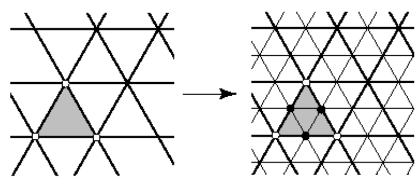
Loop Subdivision Scheme

- How refine mesh?
  - Refine each triangle into 4 triangles by splitting each edge and connecting new vertices



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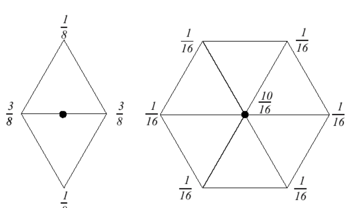
Loop Subdivision Scheme

- How position new vertices?
  - Choose locations for new vertices as weighted average of original vertices in local neighborhood



What if vertex does not have degree 6?

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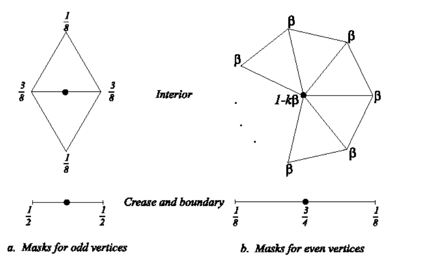
Loop Subdivision Scheme

- Rules for *extraordinary vertices* and *boundaries*:



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Loop

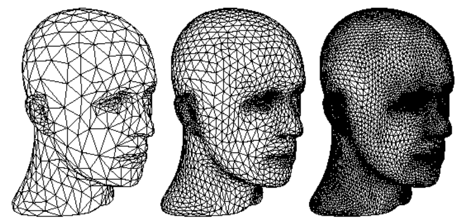
- How to choose β?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness
  - Involves calculating eigenvalues of matrices
    - » Original Loop

$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

    - » Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision Scheme



Limit surface has provable smoothness properties!

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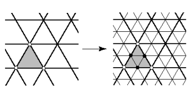
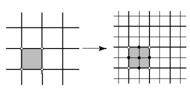
Subdivision Schemes

- There are different subdivision schemes
  - Different methods for refining topology
  - Different rules for positioning vertices
    - » Interpolating versus approximating



Face split for triangles



Face split for quads

Face split		
	Triangular meshes	*Quad. meshes*
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

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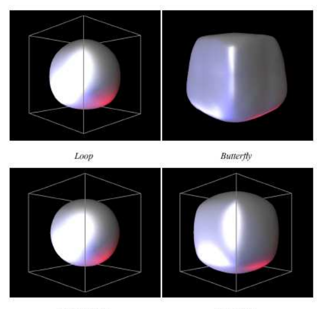
Subdivision Schemes



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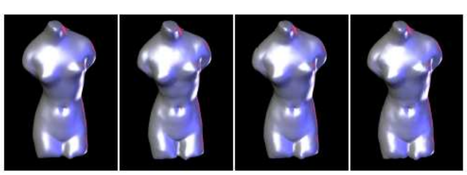
Subdivision Schemes



Loop *Butterfly* *Catmull-Clark* *Doo-Sabin*

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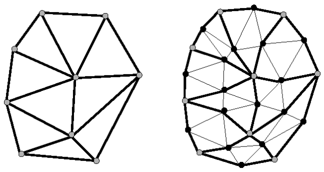
Subdivision Surfaces

- Properties:
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections

Pixar

Key Questions

- How refine mesh?
  - Aim for properties like smoothness
- **How store mesh?**
  - Aim for efficiency for implementing subdivision rules



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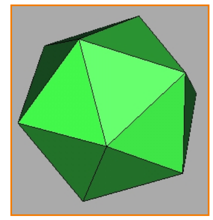
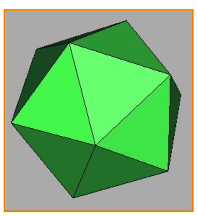
Polygon Meshes

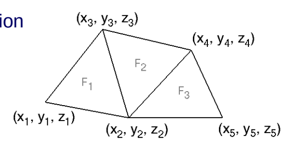
- Mesh Representations
  - Independent faces
  - Vertex and face tables
  - Adjacency lists
  - Winged-Edge



Independent Faces

- Each face lists vertex coordinates
  - Redundant vertices
  - No topology information

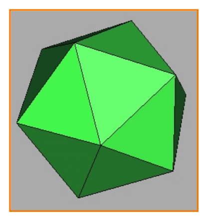


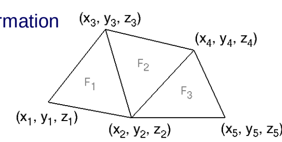


FACE TABLE	
F_1	(x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3)
F_2	(x_2, y_2, z_2) (x_4, y_4, z_4) (x_3, y_3, z_3)
F_3	(x_2, y_2, z_2) (x_5, y_5, z_5) (x_4, y_4, z_4)

Vertex and Face Tables

- Each face lists vertex references
  - Shared vertices
  - Still no topology information



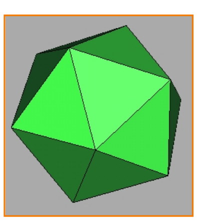


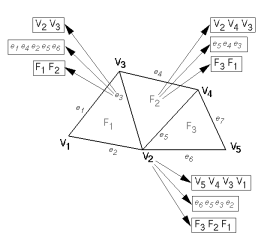
VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

Adjacency Lists

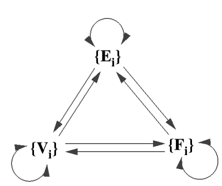
- Store all vertex, edge, and face adjacencies
  - Efficient topology traversal
  - Extra storage

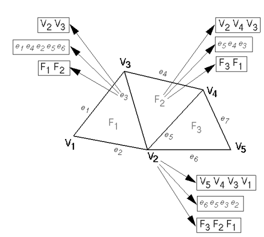




Partial Adjacency Lists

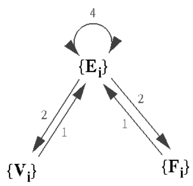
- Can we store only some adjacency relationships and derive others?

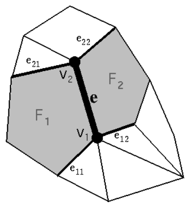




Winged Edge

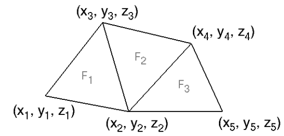
- Adjacency encoded in edges
  - All adjacencies in O(1) time
  - Little extra storage (fixed records)
  - Arbitrary polygons





Winged Edge

- Example:



VERTEX TABLE

V_1	X_1	Y_1	Z_1	e_1
V_2	X_2	Y_2	Z_2	e_6
V_3	X_3	Y_3	Z_3	e_3
V_4	X_4	Y_4	Z_4	e_5
V_5	X_5	Y_5	Z_5	e_6

EDGE TABLE

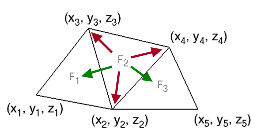
					11	12	21	22
e_1	V_1	V_3		F_1	e_2	e_2	e_4	e_3
e_2	V_1	V_2	F_1		e_1	e_1	e_3	e_6
e_3	V_2	V_3	F_1	F_2	e_2	e_5	e_1	e_4
e_4	V3	V_4		F_2	e_1	e_3	e_7	e_5
e_5	V_2	V_4	F_2	F_3	e_3	e_6	e_4	e_7
e_6	V_2	V_5	F_3		e_5	e_2	e_7	e_7
e_7	V_4	V_5		F_3	e_4	e_5	e_6	e_6

FACE TABLE

F_1	e_1
F_2	e_3
F_3	e_5

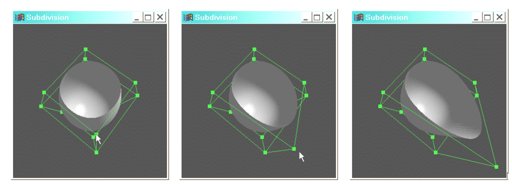
Triangle Meshes

- Relevant properties:
  - Exactly 3 vertices per face
  - Any number of faces per vertex
- Useful adjacency structure for Loop subdivision:
  - Do not represent edges explicitly
  - Faces store refs to vertices and neighboring faces
  - Vertices store refs to adjacent faces and vertices



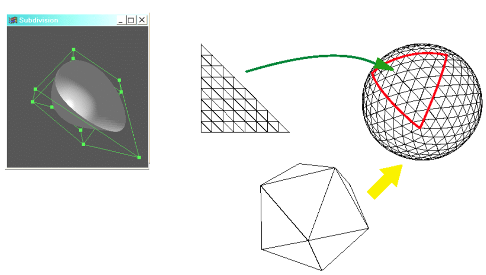
Assignment 3

- Interactive editing of subdivision surfaces
  - Loop subdivision scheme
  - Partial adjacency list mesh representation
  - Interactive vertex dragging



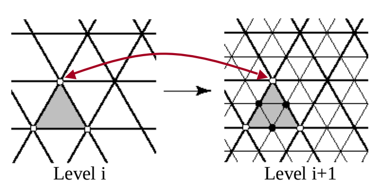
Assignment 3

- Edit coarse mesh while display subdivided mesh



Assignment 3

- Store hierarchy of meshes
  - Full triangle mesh at every level
  - Vertices store references to counterparts one level up and one level down
  - Enables efficient re-positioning of mesh vertices after interactive dragging



Summary

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Local support
  - Guaranteed continuity
  - Multiresolution
- Difficulties:
  - Intuitive specification
  - Parameterization
  - Intersections