3D Polygon Rendering Pipeline

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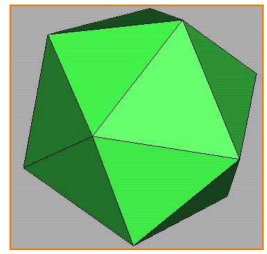
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination



3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

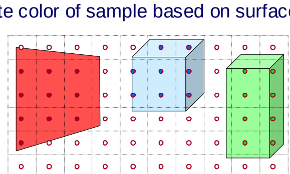
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

Quake II
(Id Software)

Ray Casting Revisited

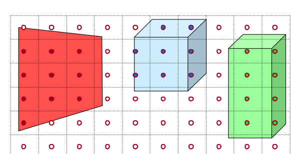
- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on surface radiance



More efficient algorithms utilize spatial coherence!

3D Polygon Rendering

- What steps are necessary to utilize spatial coherence while drawing these polygons into a 2D image?



3D Rendering Pipeline (for direct illumination)

3D Primitives
Modeling Transformation
Lighting
Viewing Transformation
Projection Transformation
Clipping
Viewport Transformation
Scan Conversion
Image

This is a pipelined sequence of operations to draw a 3D primitive into a 2D image

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```
glBegin(GL_POLYGON);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(1.0, 0.0, 0.0);
glVertex3f(1.0, 1.0, 1.0);
glVertex3f(0.0, 1.0, 1.0);
glEnd();
```

OpenGL executes steps of 3D rendering pipeline for each polygon

3D Rendering Pipeline (for direct illumination)

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Image

3D Rendering Pipeline (for direct illumination)

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3D Rendering Pipeline (for direct illumination)

3D Primitives

Modeling Transformation	Transform into 3D world coordinate system
Lighting	Illuminate according to lighting and reflectance
Viewing Transformation	Transform into 3D camera coordinate system
Projection Transformation	Transform into 2D camera coordinate system
Clipping	Clip primitives outside camera's view
Viewport Transformation	
Scan Conversion	

Image

3D Rendering Pipeline (for direct illumination)

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Image

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Scan Conversion	Draw pixels (includes texturing, hidden surface, ...)

Image

3D Rendering Pipeline (for direct illumination)

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Image

Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

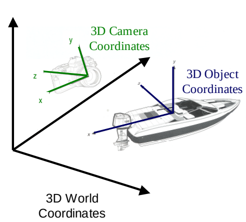
↓ 2D Screen Coordinates

Viewport Transformation

↓ 2D Image Coordinates

p'(x',y')

Transformations map points from one coordinate system to another



Viewing Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

↓ 2D Screen Coordinates

Viewport Transformation

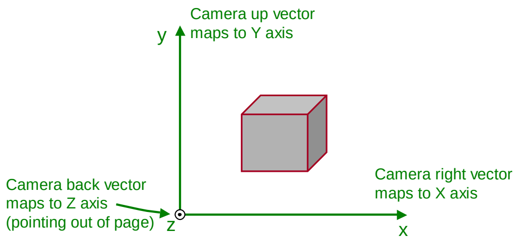
↓ 2D Image Coordinates

p'(x',y')

Viewing Transformations (Viewing Transformation, Projection Transformation)

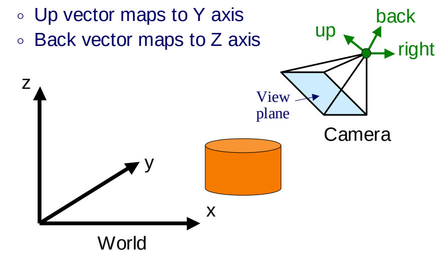
Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis



Finding the viewing transformation

- We have the camera (in world coordinates)
- We want T taking objects from world to camera

$$p^C = T\ p^W$$

- Trick: find T^{-1} taking objects in camera to world

$$p^W = T^{-1} p^C$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

?

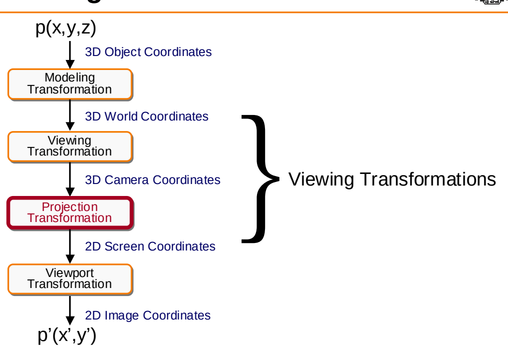
Finding the Viewing Transformation

- Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

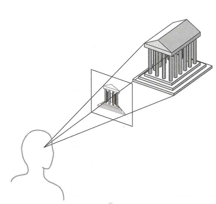
- This matrix is T^{-1} so we invert it to get T … easy!

Viewing Transformations

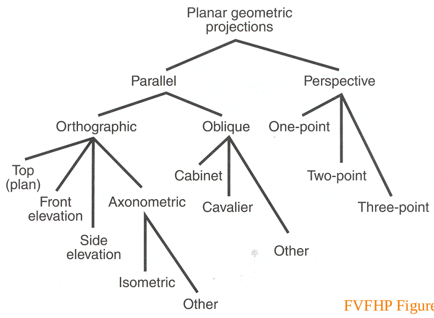


Projection

- General definition:
  - Transform points in *n*-space to *m*-space (*m*<*n*)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

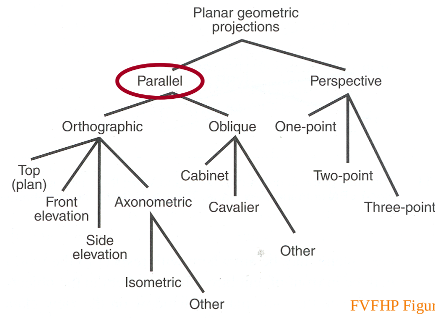


Taxonomy of Projections



FVFHP Figure 6.10

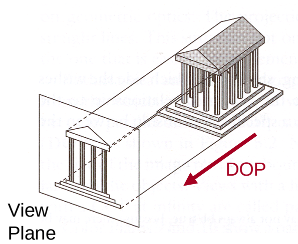
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

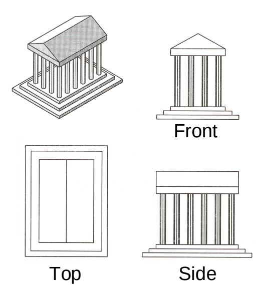
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Orthographic Projections

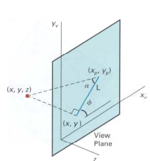
- DOP perpendicular to view plane

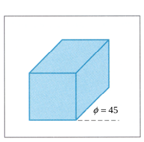


Angel Figure 5.5

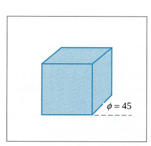
Oblique Projections

- DOP not perpendicular to view plane





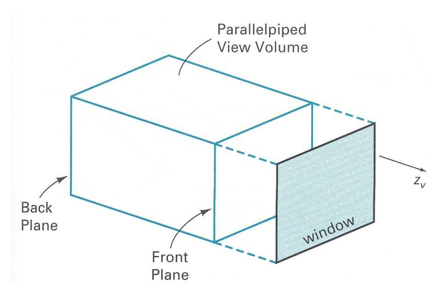
Cavalier
(DOP $\alpha = 45^{o}$)



Cabinet
(DOP $\alpha = 63.4^{o}$)

H&B Figure 12.24

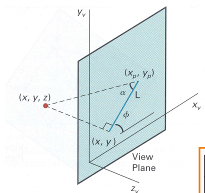
Parallel Projection View Volume



H&B Figure 12.30

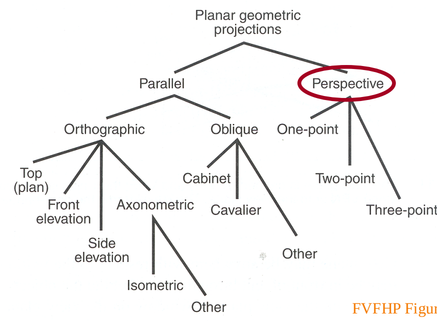
Parallel Projection Matrix

- General parallel projection transformation:



$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

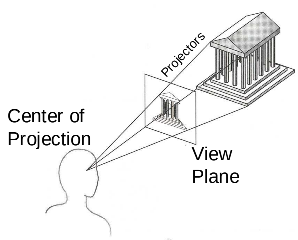
Taxonomy of Projections



FVFHP Figure 6.10

Perspective Projection

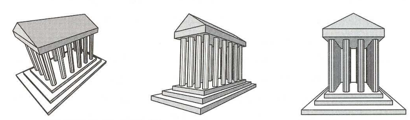
- Map points onto "view plane" along "projectors" emanating from "center of projection" (COP)



Angel Figure 5.9

Perspective Projection

- How many vanishing points?



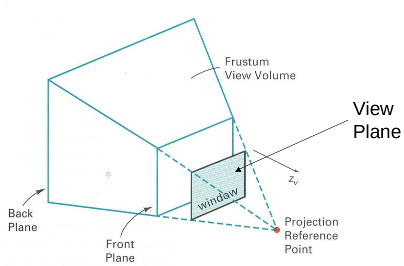
3-Point Perspective

2-Point Perspective

1-Point Perspective

Angel Figure 5.10

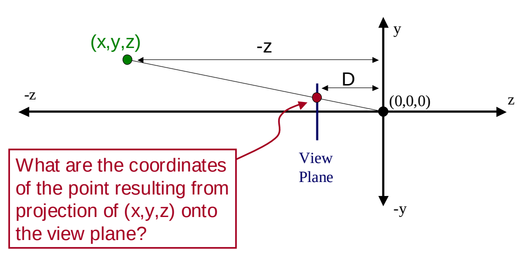
Perspective Projection View Volume



H&B Figure 12.30

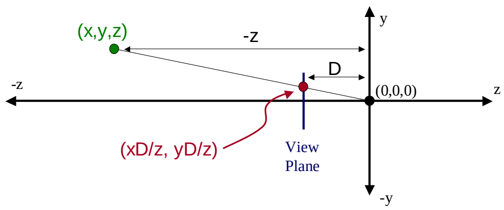
Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

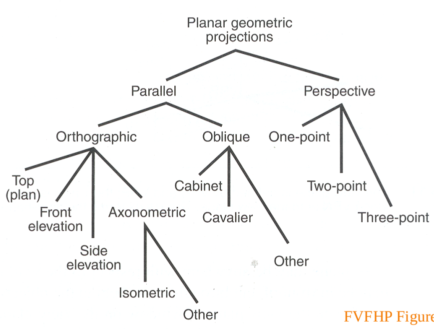
- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

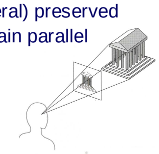
Taxonomy of Projections



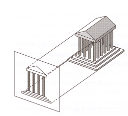
FVFHP Figure 6.10

Perspective vs. Parallel

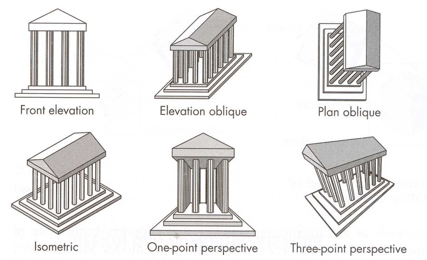
- Perspective projection
  - + Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - + Good for exact measurements
  - + Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



Classical Projections



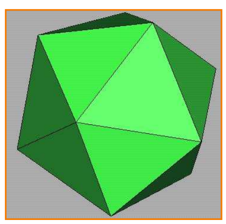
Angel Figure 5.3

Viewing Transformations Summary

- Camera transformation
  - Map 3D world coordinates to 3D camera coordinates
  - Matrix has camera vectors as rows
- Projection transformation
  - Map 3D camera coordinates to 2D screen coordinates
  - Two types of projections:
    - » Parallel
    - » Perspective

3D Rendering Pipeline (for direct illumination)

3D Primitives
↓ 3D Modeling Coordinates
Modeling Transformation
↓ 3D World Coordinates
Lighting
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Clipping
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
Scan Conversion
↓ 2D Image Coordinates
Image



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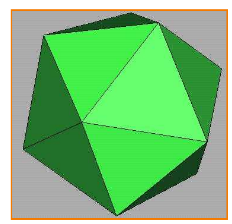


2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

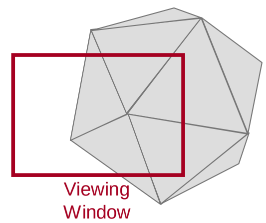
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window

Screen Coordinates

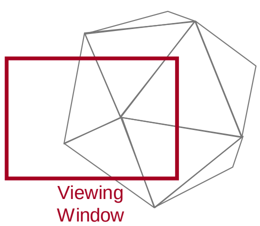
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window



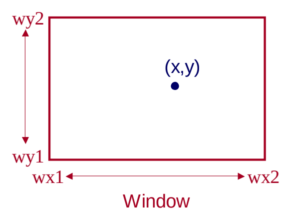
Clipping

- Avoid drawing parts of primitives outside window
  - Points
  - Lines
  - Polygons
  - Circles
  - etc.



Point Clipping

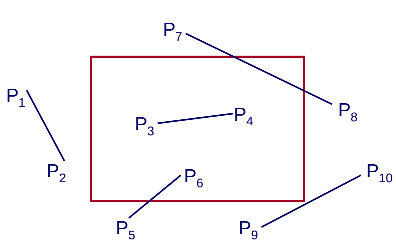
- Is point (x,y) inside the clip window?




```
inside =
  (x >= wx1) &&
  (x <= wx2) &&
  (y >= wy1) &&
  (y <= wy2);
```

Line Clipping

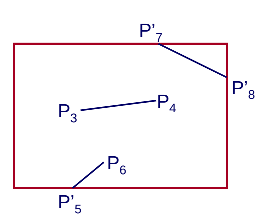
- Find the part of a line inside the clip window



Before Clipping

Line Clipping

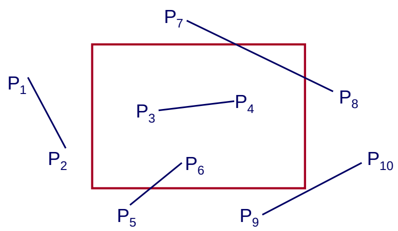
- Find the part of a line inside the clip window



After Clipping

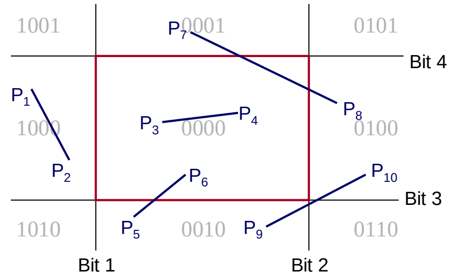
Cohen Sutherland Line Clipping

- Use simple tests to classify easy cases first



Cohen Sutherland Line Clipping

- Classify some lines quickly by AND of bit codes representing regions of two endpoints (must be 0)



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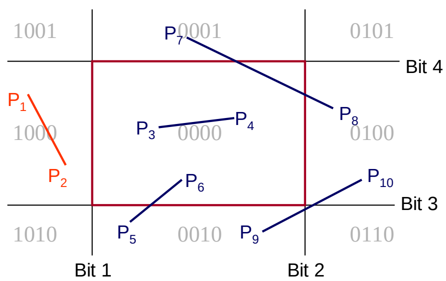

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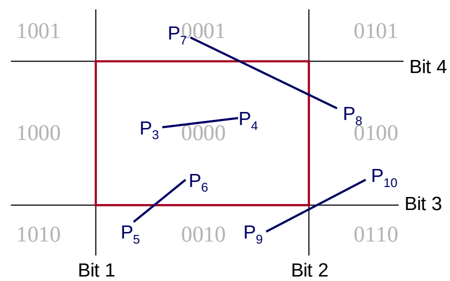

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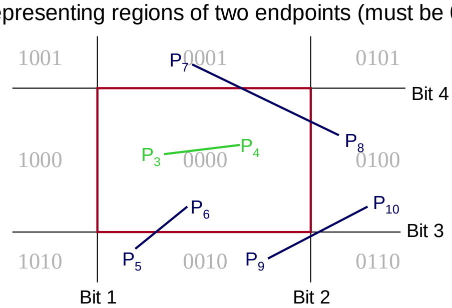
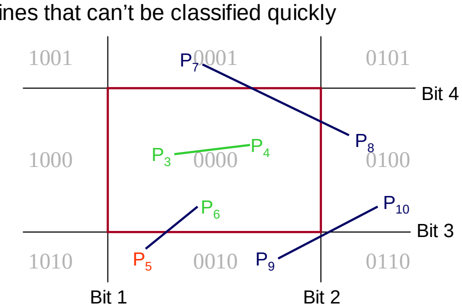


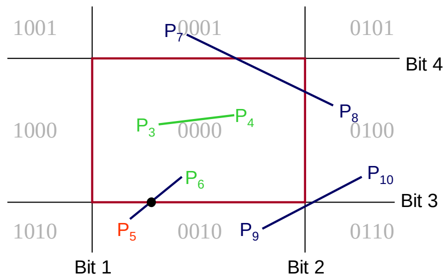
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



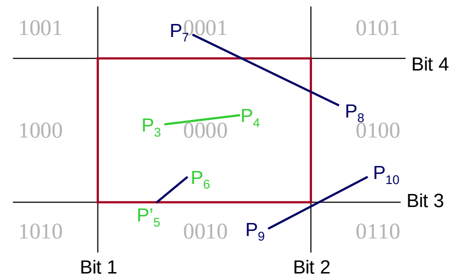
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



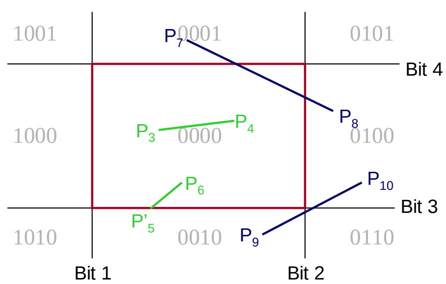
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



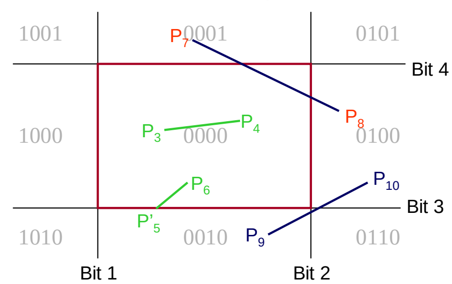
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



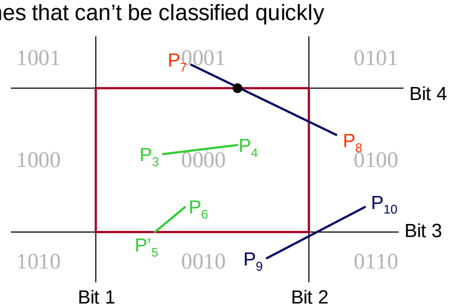
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



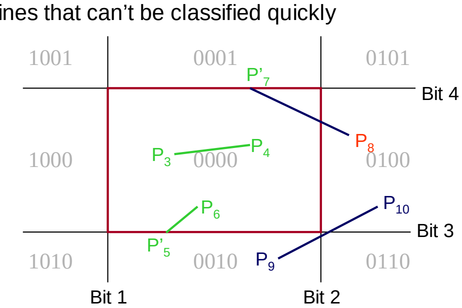
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



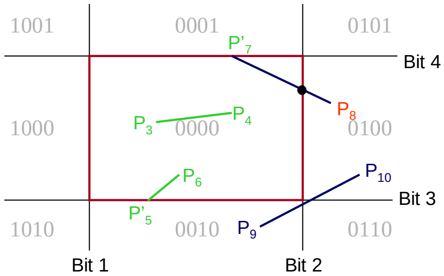
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



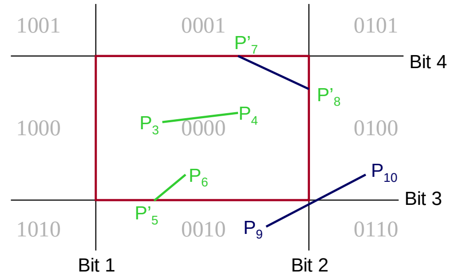
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



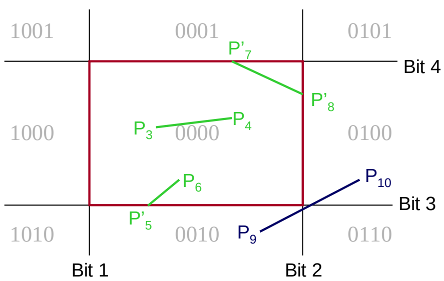
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



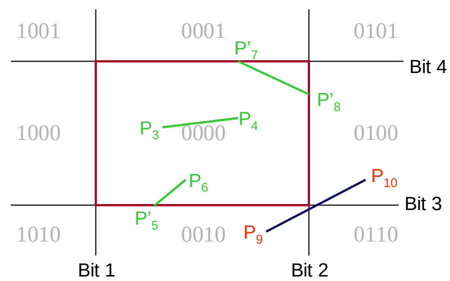
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



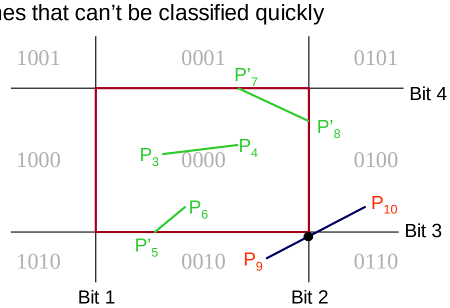
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



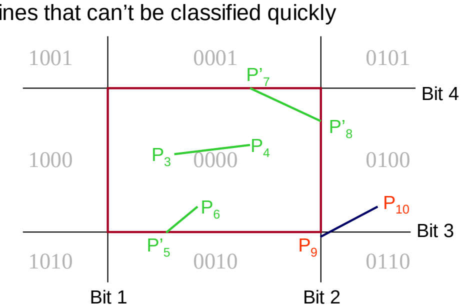
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



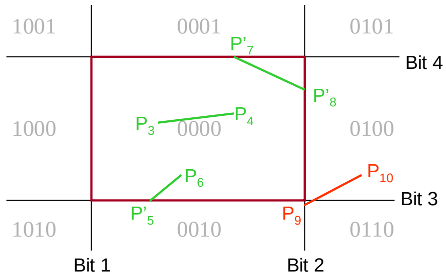
Cohen-Sutherland Line Clipping

- Compute interesections with window boundary for lines that can't be classified quickly



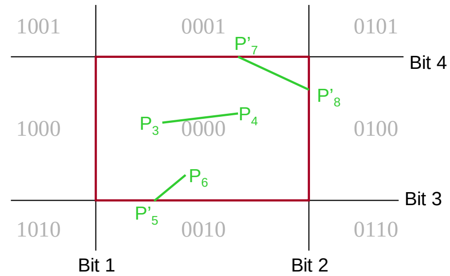
Cohen-Sutherland Line Clipping

- Compute intersections with window boundary for lines that can't be classified quickly



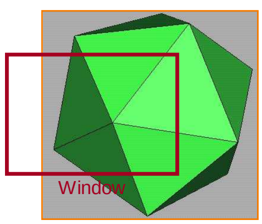
Cohen-Sutherland Line Clipping

- Compute intersections with window boundary for lines that can't be classified quickly



Clipping

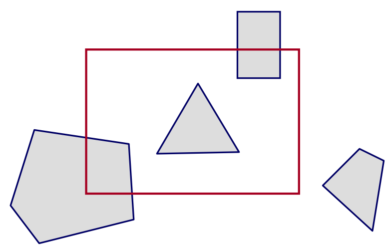
- Avoid drawing parts of primitives outside window
  - Points
  - Lines
  - Polygons
  - Circles
  - etc.



2D Screen Coordinates

Polygon Clipping

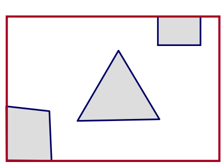
- Find the part of a polygon inside the clip window?



Before Clipping

Polygon Clipping

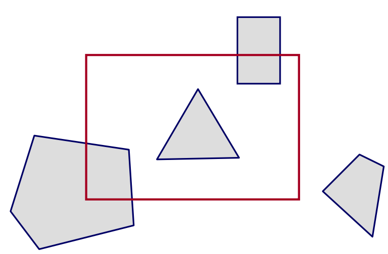
- Find the part of a polygon inside the clip window?



After Clipping

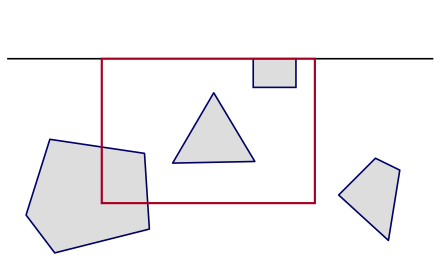
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



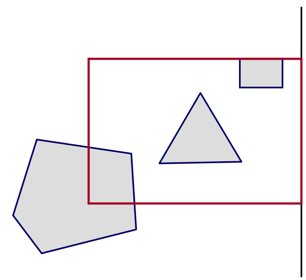
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



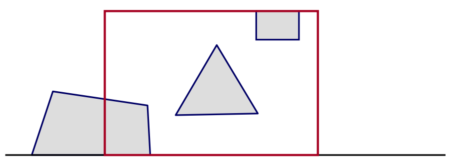
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



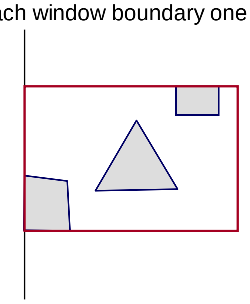
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



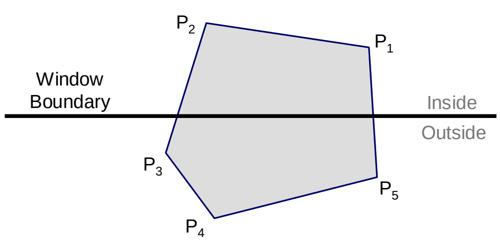
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



Clipping to a Boundary

- Do inside test for each point in sequence, Insert new points when cross window boundary, Remove points outside window boundary



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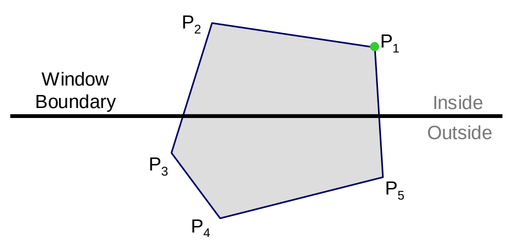
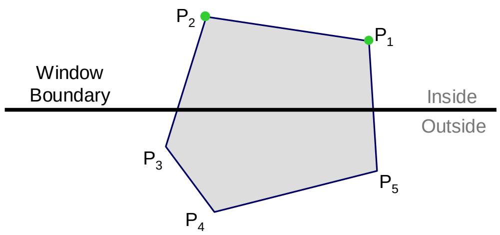


Clipping to a Boundary

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary



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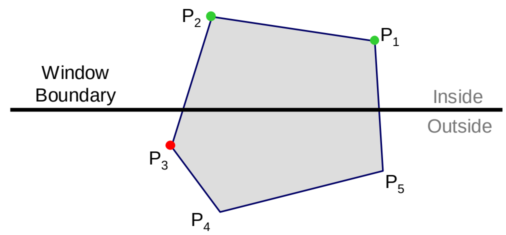

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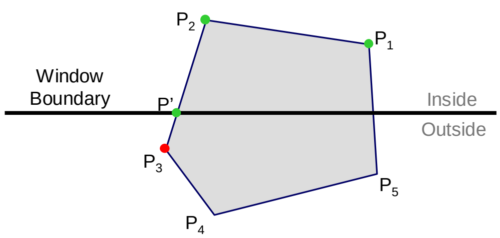

Clipping to a Boundary

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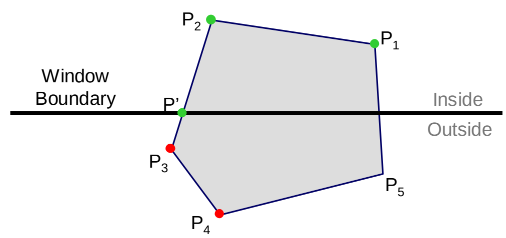

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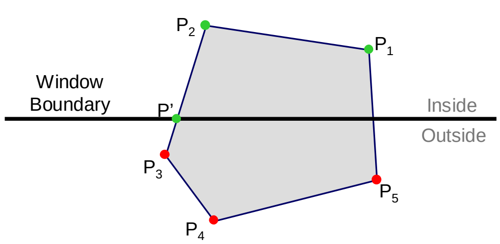
  Remove points outside window boundary



Clipping to a Boundary

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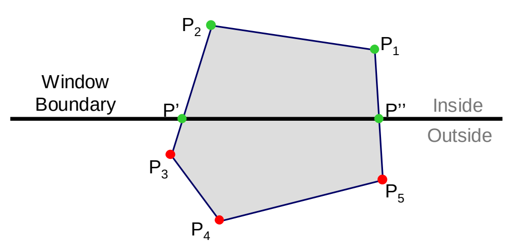
  Remove points outside window boundary



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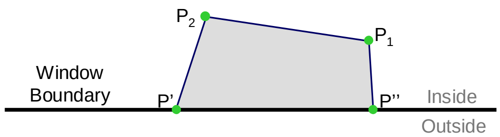
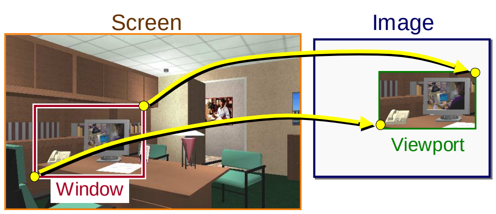


2D Rendering Pipeline

3D Primitives
↓
2D Primitives
↓
Clipping — Clip portions of geometric primitives residing outside the window
↓
Viewport Transformation — Transform the clipped primitives from screen to image coordinates
↓
Scan Conversion — Fill pixels representing primitives in screen coordinates
↓
Image

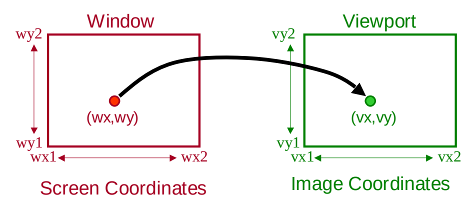
Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



Viewport Transformation

- Window-to-viewport mapping



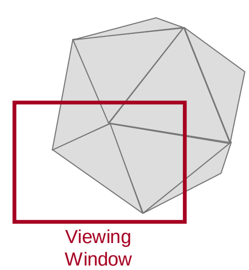

```
vx = vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1);
vy = vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1);
```

Summary of Transformations

p(x,y,z)
↓ 3D Object Coordinates
Modeling Transformation — Modeling transformation
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation — Viewing transformations
↓ 2D Screen Coordinates
Viewport Transformation — Viewport transformation
↓ 2D Image Coordinates
p'(x',y')

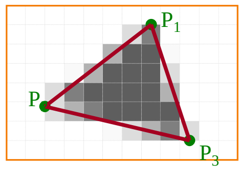
Summary

3D Primitives
↓ 3D Modeling Coordinates
Modeling Transformation
↓ 3D World Coordinates
Lighting
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Clipping
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
Scan Conversion
↓ 2D Image Coordinates
Image



Next Time

3D Primitives
↓ 3D Modeling Coordinates
Modeling Transformation
↓ 3D World Coordinates
Lighting
↓ 3D World Coordinates
Viewing Transformation
↓ 3D Camera Coordinates
Projection Transformation
↓ 2D Screen Coordinates
Clipping
↓ 2D Screen Coordinates
Viewport Transformation
↓ 2D Image Coordinates
Scan Conversion
↓ 2D Image Coordinates
Image



Scan Conversion!