Lecture 1: Introduction

Algorithms and Data Structures
Princeton University
Spring 2004

Kevin Wayne

Overview

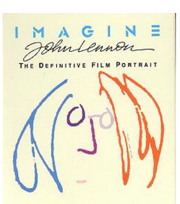
What is COS 226?

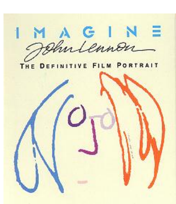
- Intermediate-level survey course.
- Programming and problem solving with applications.
- Algorithms: method for solving a problem.
- Data structures: method to store information.

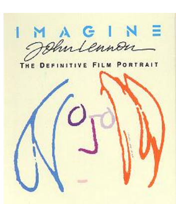
Data Structure	Algorithms
union-find	weighted quick union with path compression
sorting	quicksort, mergesort, heapsort. radix sorts
priority queue	binary heap
symbol table	BST, red-black tree, hash table, TST, k-d tree
string	KMP, Rabin-Karp, Huffman, LZW, Burrows-Wheeler
graph	Prim, Kruskal, Dijkstra, Bellman-Ford, Ford-Fulkerson

Imagine a World With No Good Algorithms

Multimedia. CD player, DVD, MP3, JPG, DivX, HDTV.
Internet. Packet routing, Google, Akamai.
Secure communications. Cell phones, e-commerce.
Information processing. Database search, data compression.
Computers. Circuit layout, file system, compilers.
Computer graphics. Hollywood movies, video games.
Biology. Human genome project, protein folding.
Astrophysics. N-body simulation.
Transportation. Airline crew scheduling, map routing.
. . .







Why Study Algorithms

Using a computer?

- Want it to go faster? Process more data?
- Want it to do something that would otherwise be impossible?

Technology improves things by a constant factor.

- But might be costly.
- Good algorithmic design can do much better and might be cheap.
- Supercomputers cannot rescue a bad algorithm.

Algorithms as a field of study.

- Old enough that basics are known.
- New enough that new discoveries arise.
- Burgeoning application areas.
- Philosophical implications.

The Usual Suspects

Lectures: Kevin Wayne (Kevin)
- MW 11-12:20, CS 105.

Precepts: Nir Ailon (Nir), Miro Dudik (Miro)
- T 12:30, Friend 005.
- T 1:30, Friend 005.
- T 3:30, Friend 005.
- Clarify programming assignments, exercises, lecture material.
- First precept meets 2/10.

Coursework and Grading

Weekly programming assignments: 45%
- Due Thursdays 11:59pm, starting 2/12.

Weekly written exercises: 15%
- Due at beginning of Monday lecture, starting 2/9.

Exams:
- Closed book with cheatsheet.
- Midterm. 15%
- Final. 25%

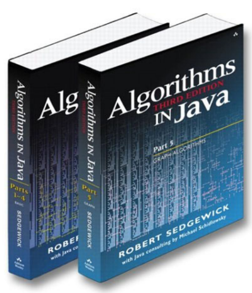
Staff discretion. Adjust borderline cases.

Course Materials

Course web page. `http://www.princeton.edu/~cos226`

note change

- Syllabus.
- Programming assignments.
- Exercises.
- Lecture notes.
- Old exams.



Algorithms in Java, 3rd edition.
- Parts 1-4 (COS 126 text).
- Part 5 (graph algorithms).

Algorithms in C, 2nd edition.
- Strings and geometry handouts.

Union Find

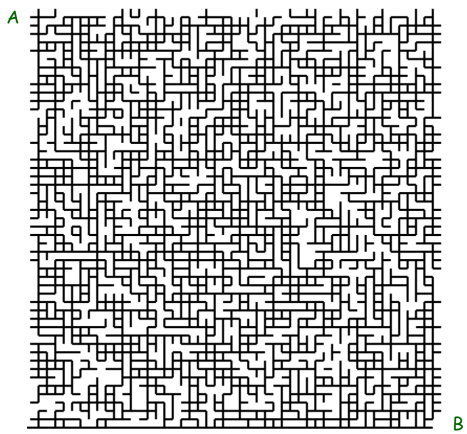
Quick find
Quick union
Weighted quick union
Path compression

Reference: Chapter 1, Algorithms in Java, 3rd Edition, Robert Sedgewick.

An Example Problem: Network Connectivity

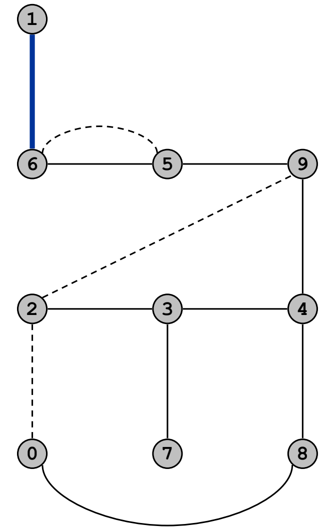
Network connectivity.

- Nodes at grid points.
- Add connections between pairs of nodes.
- Is there a path from node A to node B?



Network Connectivity

in	out	evidence
3 4	3 4	
4 9	4 9	
8 0	8 0	
2 3	2 3	
5 6	5 6	
2 9		(2–3–4–9)
5 9	5 9	
7 3	7 3	
4 8	4 8	
5 6		(5–6)
0 2		(2–3–4–8–0)
6 1	6 1	



Union-Find Abstraction

What are critical operations we need to support?

- N objects.
  - grid points
- FIND: test whether two objects are in same set.
  - is there a connection between A and B?
- UNION: merge two sets.
  - add a connection

Design efficient data structure to store connectivity information and algorithms for UNION and FIND.

- Number of operations M can be huge.
- Number of objects N can be huge.

Other Applications

More union-find applications.

- Hex.
- Percolation.
- Image processing.
- Minimum spanning tree.
- Least common ancestor.
- Equivalence of finite state automata.
- Compiling EQUIVALENCE statements in FORTRAN.
- Micali-Vazarani algorithm for nonbipartite matching.
- Weihe's algorithm for edge-disjoint s-t paths in planar graphs.
- Scheduling unit-time tasks to P processors so that each job finishes between its release time and deadline.
- Scheduling unit-time tasks with a partial order to two processors in order to minimize last completion time.

References.

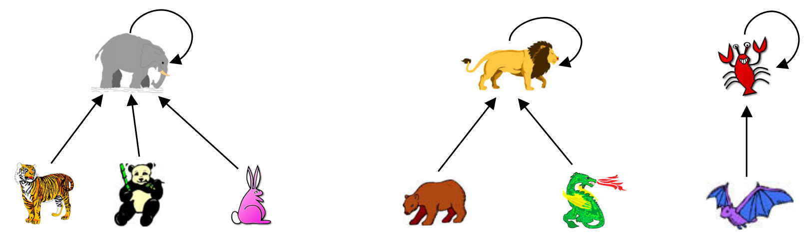
- *A Linear Time Algorithm for a Special Case of Disjoint Set Union,* Gabow and Tarjan.
- *The Design and Analysis of Computer Algorithms,* Aho, Hopcroft, and Ullman.

Objects

Elements are arbitrary objects in a network.

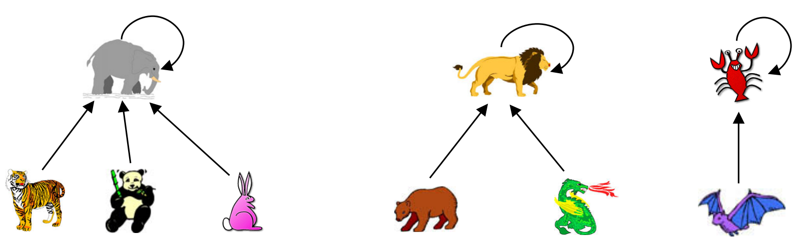
- Pixels in a digital photo.
- Computers in a network.
- Transistors in a computer chip.
- Web pages on the Internet.
- Metallic sites in a composite system.
- When programming, convenient to name them 0 to N-1.
- When drawing, fun to use animals!

Quick-Find



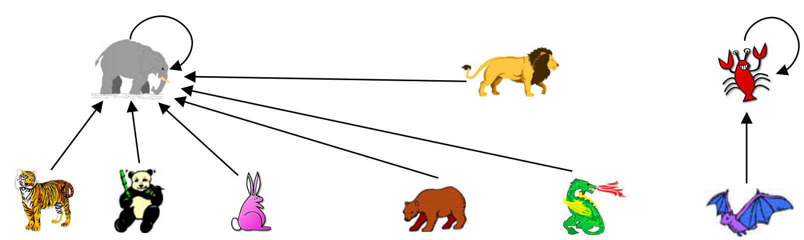
id[tiger] = id[panda] = id[bunny] = id[elephant] = elephant
id[bear] = id[dragon] = id[lion] = lion
id[bat] = id[lobster] = lobster

Quick-Find



Union(tiger, bear)

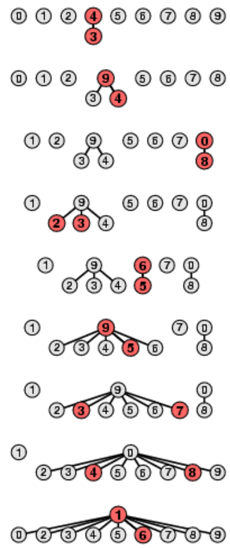
Quick-Find



Union(tiger, bear)

Quick-Find

3-4	0	1	2	4	4	5	6	7	8	9
4-9	0	1	2	9	9	5	6	7	8	9
8-0	0	1	2	9	9	5	6	7	0	9
2-3	0	1	9	9	9	5	6	7	0	9
5-6	0	1	9	9	9	6	6	7	0	9
5-9	0	1	9	9	9	9	9	7	0	9
7-3	0	1	9	9	9	9	9	9	0	9
4-8	0	1	0	0	0	0	0	0	0	0
6-1	1	1	1	1	1	1	1	1	1	1



Quick-Find Algorithm

Data structure. ← integer between 0 and N-1

- Maintain array `id[]` with name for each of N elements.
- If p and q are connected, then they have the same id.
- Initially, set id of each element to itself.

```
for (int i = 0; i < N; i++)
   id[i] = i;
```

N operations

Find. To check if p and q are connected, see if they have same id.

```
return (id[p] == id[q]);
```

1 operations

Union. To merge components containing p and q, change all entries with `id[p]` to `id[q]`.

```
int pid = id[p];
for (int i = 0; i < N; i++)
   if (id[i] == pid) id[i] = id[q];
```

N operations

Problem Size and Computation Time

Rough standard for 2000.

- 10^9 operations per second.
- 10^9 words of main memory.
- Touch all words in approximately 1 second. (unchanged since 1950!)

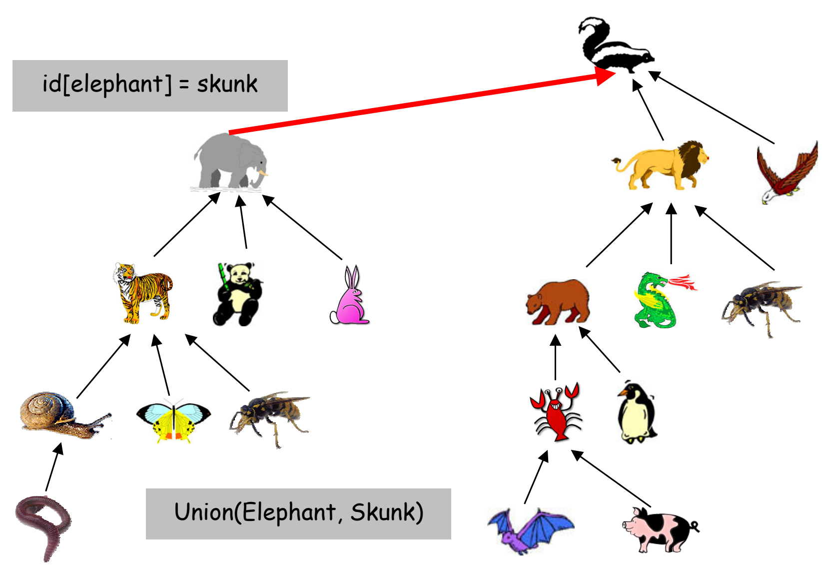
Ex. Huge problem for quick find.

- 10^{10} edges connecting 10^9 nodes.
- Quick-find might take 10^{20} operations. (10 ops per query)
- 3,000 years of computer time!

Paradoxically, quadratic algorithms get worse with newer equipment.

- New computer may be 10x as fast.
- But, has 10x as much memory so problem may be 10x bigger.
- With quadratic algorithm, takes 10x as long!

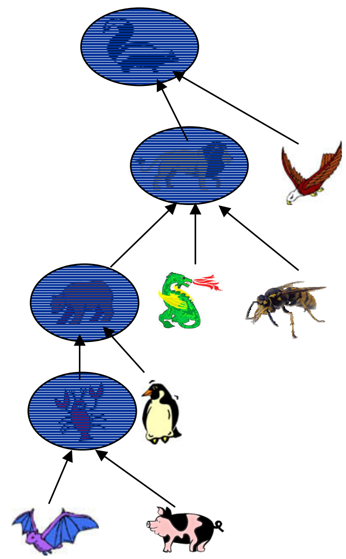
Quick-Union



Find with Quick-Union

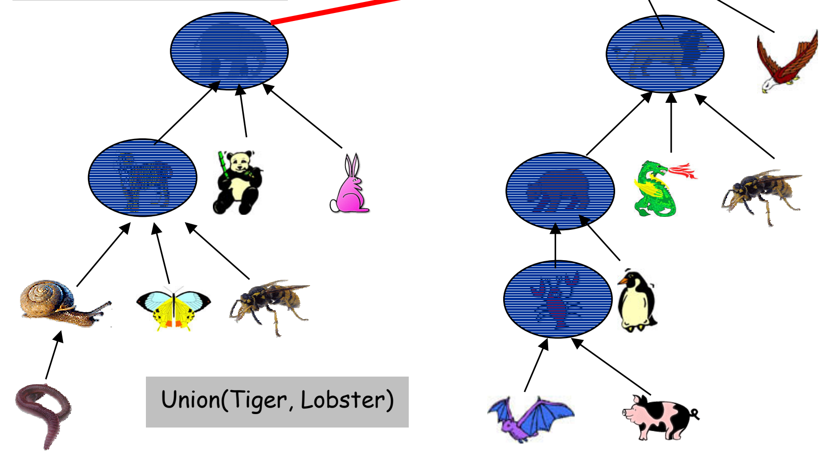
Answer = Skunk

Find(Lobster)



Quick-Union

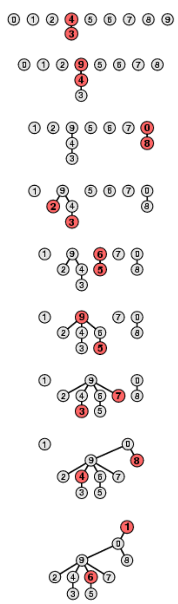
root(Tiger) = Elephant
root(Lobster) = Skunk
id[Elephant] = Skunk



Union(Tiger, Lobster)

Quick-Union

3-4	0	1	2	4	4	5	6	7	8	9
4-9	0	1	2	4	9	5	6	7	8	9
8-0	0	1	2	4	9	5	6	7	0	9
2-3	0	1	9	4	9	5	6	7	0	9
5-6	0	1	9	4	9	6	6	7	0	9
5-9	0	1	9	4	9	6	9	7	0	9
7-3	0	1	9	4	9	6	9	9	0	9
4-8	0	1	9	4	9	6	9	9	0	0
6-1	1	1	9	4	9	6	9	9	0	0



Quick-Union

Data structure: disjoint forests.

- Maintain array `id[]` for each of N elements.
- Root of element x = `id[id[id[...id[p]...]]]` ← keep going until it doesn't change

```
public int root(int x) {
   while (x != id[x])
      x = id[x];
   return x;
}
```

time proportional to depth of x

Find. Check if p and q have same root.

```
return (root(p) == root(q));
```

time proportional to depth of p and q

Union. Set the id of p's root to q's root.

```
int i = root(p);
int j = root(q);
id[i] = j;
```

time proportional to depth of p and q

Weighted Quick-Union

Quick-find defect.

- UNION too expensive.
- Trees are flat, but too hard to keep them flat.

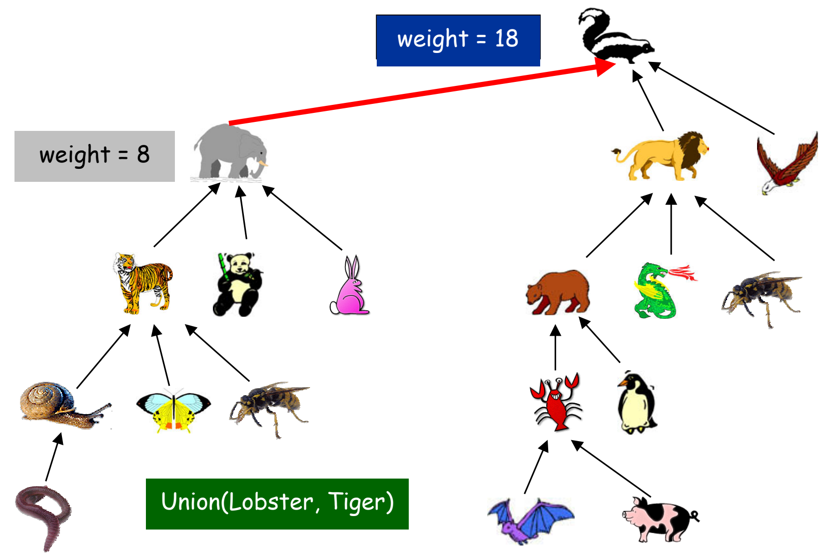
Quick-union defect.

- Finding the root can be expensive.
- Trees could get tall.

Weighted quick-union.

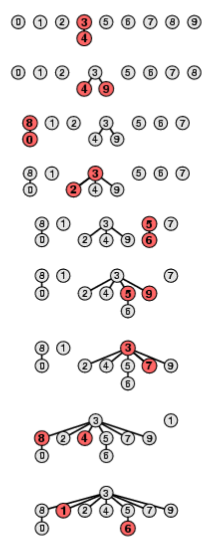
- Modify quick-union to avoid tall trees.
- Keep track of size of each component.
- Balance by linking small tree below large one.

Weighted Quick-Union



Weighted Quick-Union

3-4	0	1	2	3	3	5	6	7	8	9
4-9	0	1	2	3	3	5	6	7	8	3
8-0	8	1	2	3	3	5	6	7	8	3
2-3	8	1	3	3	3	5	6	7	8	3
5-6	8	1	3	3	3	5	5	7	8	3
5-9	8	1	3	3	3	3	5	7	8	3
7-3	8	1	3	3	3	3	5	3	8	3
4-8	8	1	3	3	3	3	5	3	3	3
6-1	8	3	3	3	3	3	5	3	3	3



Weighted Quick-Union

Data structure: disjoint forests.

- Also maintain array `sz[i]` that counts the number of elements in the tree rooted at i.

Find. Same as quick union.

Union. Same as quick union, but merge smaller tree into the larger tree and update the `sz[]` array.

```
if (sz[i] < sz[j]) { id[i] = j; sz[j] += sz[i]; }
else               { id[j] = i; sz[i] += sz[j]; }
```

Analysis.

now, provably at most lg N

- FIND takes time proportional to depth of p and q in tree.
- UNION takes constant time, given roots.

Weighted Quick-Union

Is performance improved?

- Theory: lg N per union or find operation.
- Practice: constant time.

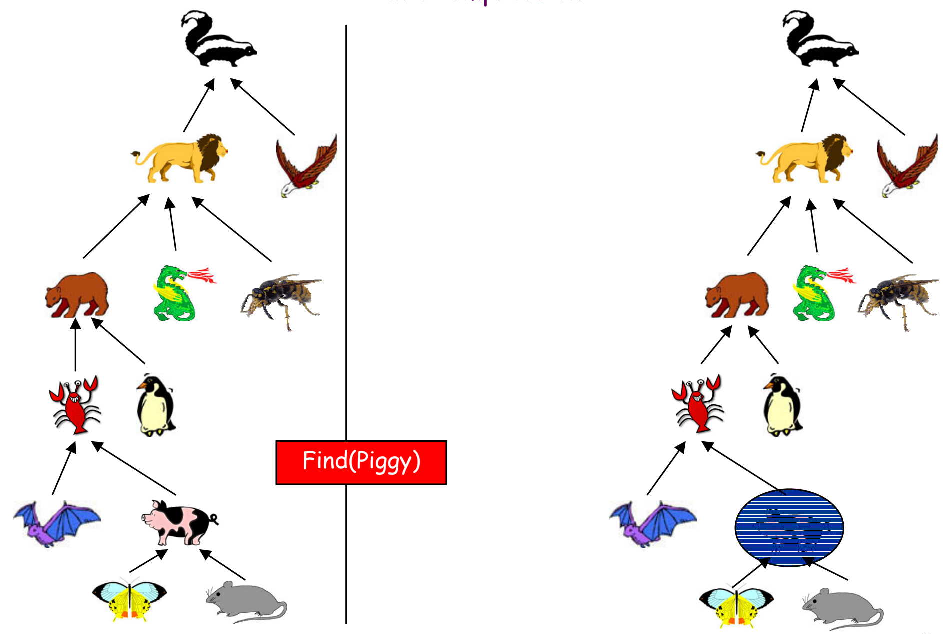
Ex. Huge practical problem.

- 10^{10} edges connecting 10^9 nodes.
- Reduces time from 3,000 years to 1 minute.
- Supercomputer wouldn't help much.
- Good algorithm makes solution possible.

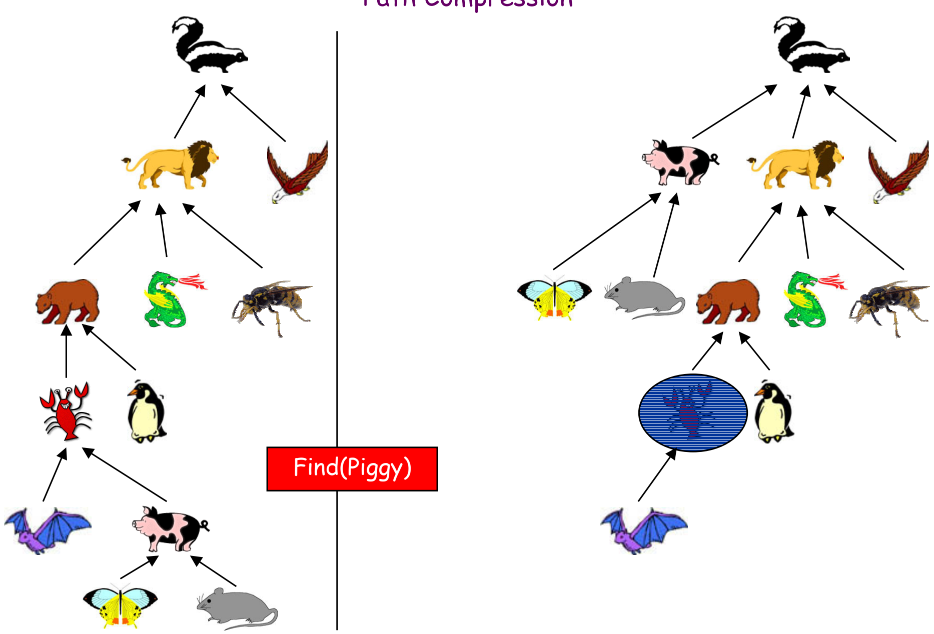
Stop at guaranteed acceptable performance?

- Not hard to improve algorithm further.

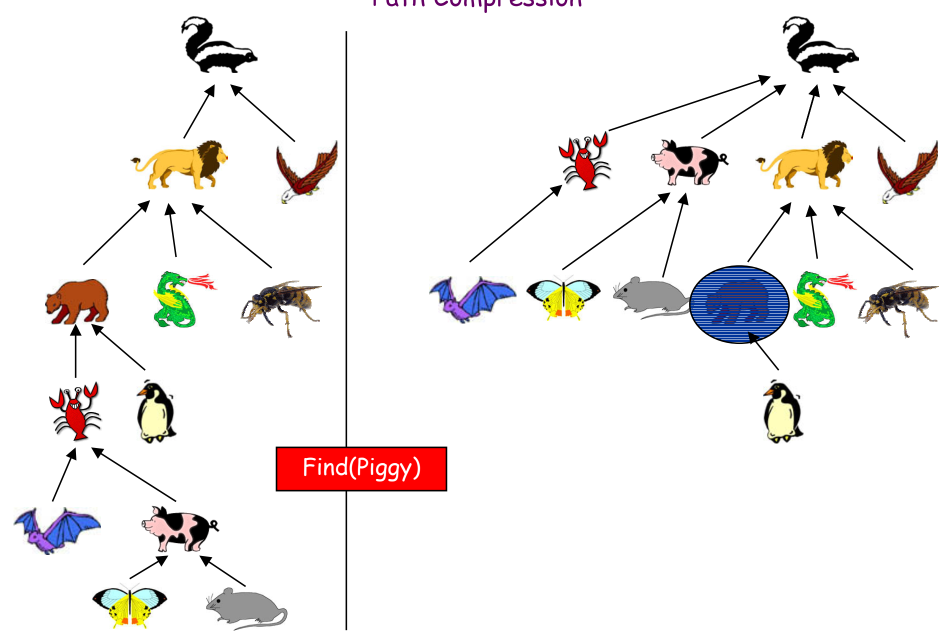
Path Compression



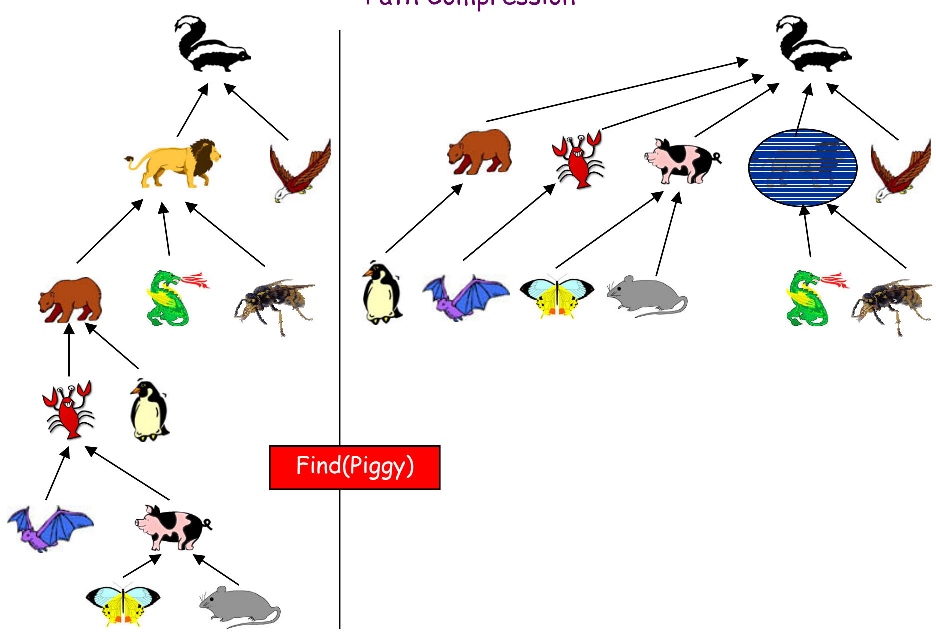
Path Compression



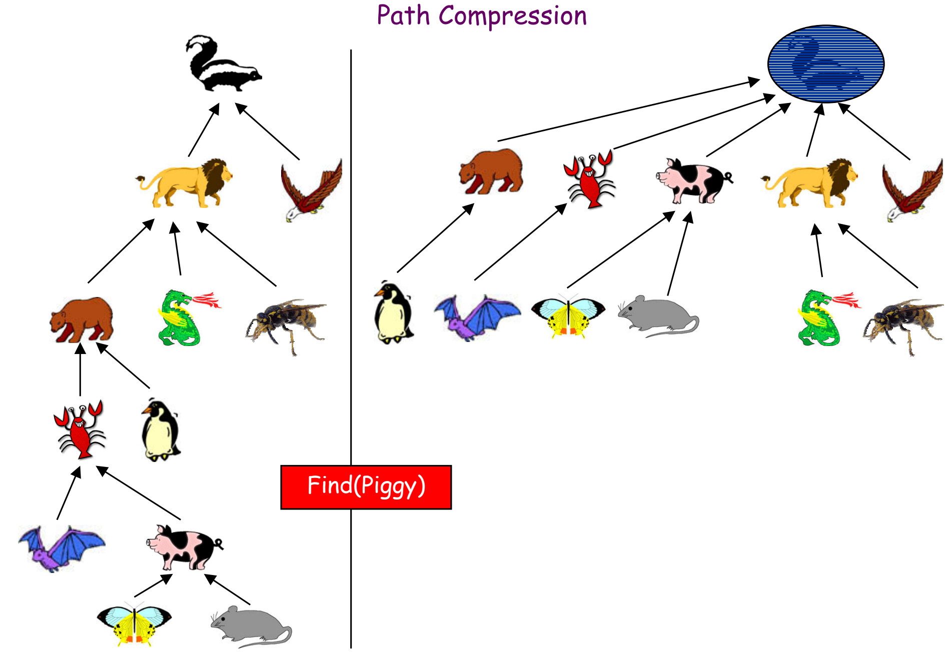
Path Compression



Path Compression

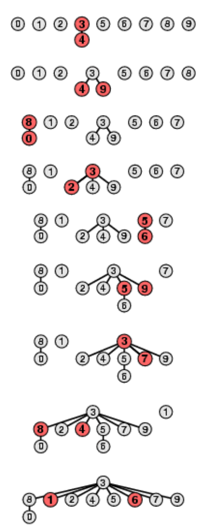


Path Compression



Weighted Quick-Union with Path Compression

3-4	0	1	2	3	3	5	6	7	8	9
4-9	0	1	2	3	3	5	6	7	8	3
8-0	8	1	2	3	3	5	6	7	8	3
2-3	8	1	3	3	3	5	6	7	8	3
5-6	8	1	3	3	3	5	5	7	8	3
5-9	8	1	3	3	3	3	5	7	8	3
7-3	8	1	3	3	3	3	5	3	8	3
4-8	8	1	3	3	3	3	5	3	3	3
6-1	8	3	3	3	3	3	3	3	3	3



Weighted Quick-Union with Path Compression

Path compression.

- Add second loop to `root` to compress tree that sets the id of every examined node to the root.
- Simple one-pass variant: make each element point to grandparent.

```
public int root(int x) {
   while (x != id[x]) {
      id[x] = id[id[x]];
      x = id[x];
   }
   return x;
}
```

← only one extra line of code

- No reason not to!
- In practice, keeps tree almost completely flat.

Weighted Quick-Union with Path Compression

Theorem. A sequence of M union and find operations on N elements takes O(N + M lg* N) time.

- Proof is very difficult.
- But the algorithm is still simple!

N	lg* N
2	1
4	2
16	3
65536	4
2^{65536}	5

Remark. lg* N is a constant in this universe.

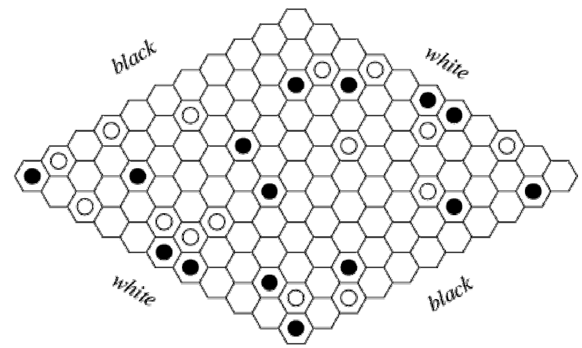
Linear algorithm?

- Cost within constant factor of reading in the data.
- Theory: WQUPC is not quite linear.
- Practice: WQUPC is linear.

Another Application: Hex

Hex. (Piet Hein 1942, John Nash 1948, Parker Brothers 1962)

- Two players alternate in picking a cell in a hex grid.
- Black: make a black path from upper left to lower right.
- White: make a white path from lower left to upper right.
- Goal: algorithm to detect when a player has won?



Yet Another Application: Percolation

Percolation phase-transition.

- Two parallel conducting bars (top and bottom).
- Electricity flows from a site to one of its 4 neighbors if both are occupied by conductors.
- Suppose each site is randomly chosen to be a conductor or insulator with probability p. What is percolation threshold p* at which charge carriers can percolate from top to bottom?

~ 0.592746 for square lattices, but constant only known via simulation

0	0	0	0	0	0	0	0	0	0	0	0
2	3	4	0	6	0	8	9	10	11	12	0
14	15	0	0	0	0	20	21	22	23	24	0
14	14	28	29	30	31	32	33	34	35	36	0
14	39	40	1	42	43	32	45	46	1	1	49
50	1	52	1	54	55	56	57	58	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

insulator

Lessons

Union-find summary. Online algorithm can solve problem while collecting data for "free."

Algorithm	Time
Quick-find	M N
Quick-union	M N
Weighted	N + M log N
Path compression	N + M log N
Weighted + path	5 (M + N)

M union-find ops on a set of N elements

Simple algorithms can be very useful.

- Start with brute force approach.
  - don't use for large problems
  - can't use for huge problems
- Strive for worst-case performance guarantees.
- Identify fundamental abstractions. union-find, disjoint forests

might be nontrivial to analyze