Lecture 18: Theory of Computation

Introduction to Theoretical CS

Two fundamental questions.

- What can a computer do?
- What can a computer do with limited resources?

General approach.

Pentium IV running Linux kernel 2.4.22

- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory

In theory . . .

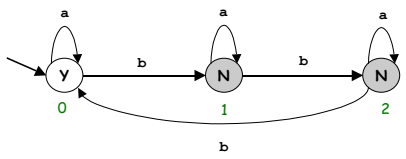
- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice . . .

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

Regular Expressions and DFAs

```
a* | (a*ba*ba*)*
```



Pattern Matching Applications

Test if a string matches some pattern.

- Scan for virus signatures.
- Process natural language.
- Search for information using Google.
- Search for markers in human genome.
- Access information in digital libraries.
- Retrieve information from Lexis/Nexis.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

Regular Expressions: Basic Operations

Regular expression.

- Notation to specify a set of strings.

Operation	Regular Expression	Yes	No
Concatenation	`aabaab`	`aabaab`	every other string
Logical Or	`aa | baab`	`aa` `baab`	every other string

Replication	`ab*a`	`aa`
		`aba`
		`abbba`
			ε
			`ab`
			`ababa`

Grouping	`a(a|b)aab`	`aaaab` `abaab`	every other string

	`(ab)*a`	`a`
		`aba`
		`ababa`
			ε
			`aa`
			`abbba`

Regular Expressions: Examples

Regular expression examples.

- Notation is surprisingly expressive.

Regular Expression	Yes	No

`a* | (a*ba*ba*ba*)*`
multiple of three b's
	ε
	`bbb`
	`aaa`
	`abbbaaa`
	`bbbaababbaa`
		`b`
		`bb`
		`abbaaaa`
		`baabbbaa`

`a | a(a|b)*a`
begins and ends with a
	`a`
	`aba`
	`aa`
	`abbaabba`
		ε
		`ab`
		`ba`

`(a|b)* abba (a|b)*`
contains the substring abba
	`abba`
	`bbabbabb`
	`abbaabba`
		ε
		`abb`
		`bbaaba`

Using Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ...
- Additional operations typically added for convenience.
- Ex: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.

Operation	Regular Expression	Yes	No
Any single character	`..oo..oo.`	`bloodroot` `spoonfood`	`cookbook` `choochoo`
One or more	`a(bc)+de`	`abcde` `abcbcde`	`ade` `bcde`
Character classes	`[a-e]+`	`decade` `accede`	`Upper45`

Regular Expressions in Java

Ex: pattern match.

- Is `text` in the set described by the `pattern`?

```
public class RE {
   public static void main(String[] args) {
      String pattern = args[0];
      String text    = args[1];
      System.out.println(text.matches(pattern));
   }
}
```

```
% java RE "..oo.oo." bloodroot
true

% java RE "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true

% java RE "[a-z]+@([a-z]+\.)+(edu|com)" doug@cs.princeton.edu
true
```

need help solving crosswords?

legal Java identifier

legal email address (simplified)

need quotes to "escape" the shell

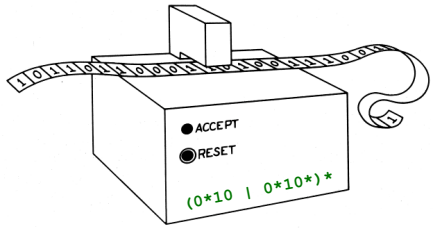
Solving the Pattern Match Problem

Regular expressions are a concise way to describe patterns.

- How would we implement `String.matches` ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

DFA: simple machine that solves the pattern match problem.

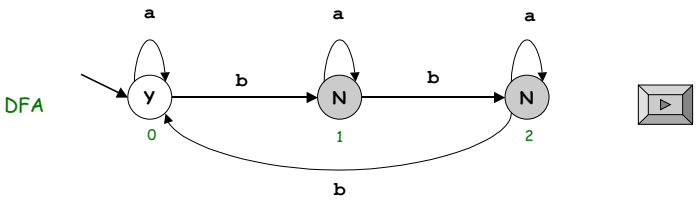
- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on `true` or `false` questions for simplicity.



Deterministic Finite State Automaton (DFA)

Simple machine with N states.

- Begin in *start state*.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- *Accept* or *reject* string depending on label of last state.

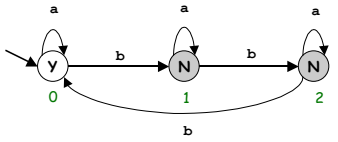


Theory of DFAs and REs

RE. Concise way to *describe* a set of strings.

DFA. Machine to *recognize* whether a given string is in a given set.

Duality: for any DFA, there exists a regular expression to describe the same set of strings; for any regular expression, there exists a DFA that recognizes the same set.



multiple of 3 b's

`a* | (a*ba*ba*ba*)*`

multiple of 3 b's

Practical consequence of duality proof: to match regular expression patterns, (i) build DFA and (ii) simulate DFA on input string.

Implementing a Pattern Matcher

Problem: given a regular expression, create program that tests whether given input is in set of strings described.

Step 1: build the DFA.

- A compiler!
- See COS 226 or COS 320.

Step 2: simulate it with given input.

- Easy.

```
State state = start;
while (!CharStdIn.isEmpty()) {
   char c = CharStdIn.readChar();
   state = state.next(c);
}
System.out.println(state.accept());
```


Application: Email Harvester

Harvest email addresses from web for spam campaign.

- User enters name of file and program prints email addresses.

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;
public class EmailHarvester {
   public static void main(String[] args) {
      In in = new In(args[0]);
      String input = in.readAll();
      String regexp = "[a-z]+@([a-z]+\\.)+(edu|com|net|tv)";
      Pattern pattern = Pattern.compile(regexp);
      Matcher matcher = pattern.matcher(input);
      while (matcher.find())
         System.out.println(matcher.group());
   }
}
```

simple email validator

```
% java EmailHarvester http://www.cs.princeton.edu/courses/cs126/precepts.html
pcalamia@cs.princeton.edu
dgabai@cs.princeton.edu
sgaw@cs.princeton.edu
wayne@cs.princeton.edu
```

OK to enter URL instead of file!

Application: Parsing a Data File

Parsing input files: TOY, Internet movie database, NCBI genome file.

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus (platypus)
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

```
String  regexp  = "[ ]*[0-9]+([actg ]*).*";
Pattern pattern = Pattern.compile(regexp);
In in = new In(filename);
String line;
while ((line = in.readLine()) != null) {
   Matcher matcher = pattern.matcher(line);
   if (matcher.find()) {
      String s = matcher.group(1).replaceAll(" ", "");
      // do something with s
   }
}
```

just the RE part in parentheses

replace this RE with this string

Fundamental Questions

Which languages CANNOT be described by any RE?

- Set of all bit strings with equal number of 0s and 1s.
- Set of all decimal strings that represent prime numbers.
- Many more. . . .

How can we extend REs to describe richer sets of strings?

- Context free grammar. ⬅ see COS 320
- Ex: Java language. ⬅ http://java.sun.com/docs/books/jls/second_edition/html/syntax.doc.html

How can we make simple machines more powerful?

Are there any limits on what kinds of problems machines can solve?

Summary

Programmer.

- Regular expressions are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.

- Regular expression is a compact description of a set of strings.
- DFA is an abstract machine that solves pattern match problem for regular expressions.
- DFAs and regular expressions have limitations.

You. Practical application of core CS principles.

Turing Machines

Challenge: Design simplest machine that is "as powerful" as conventional computers.

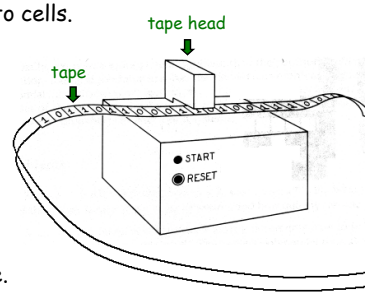
Alan Turing (1912-1954)

Turing Machine: Components

Alan Turing sought the most primitive model of a computing device.

Tape.

- Stores input, output, and intermediate results.
- One arbitrarily long strip, divided into cells.
- Finite alphabet of symbols.



Tape head.

- Points to one cell of tape.
- Reads a symbol from active cell.
- Writes a symbol to active cell.
- Moves left or right one cell at a time.

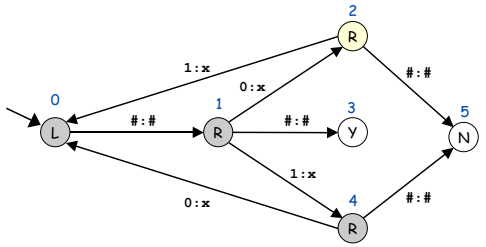
Turing Machine: Fetch, Execute

States.

- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.

- Ex. if in state 2 and input symbol is `1` then: overwrite the `1` with `x`, move to state 0, move tape head to left.



Before

…	#	#	x	x	x	1	1	0	#	#	…

Turing Machine: Fetch, Execute

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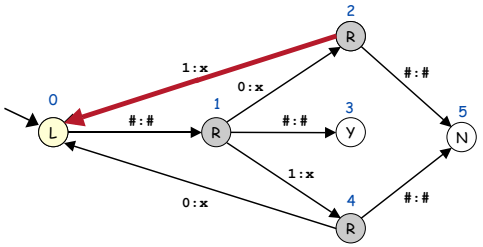


After

…	#	#	x	x	x	x	1	0	#	#	…

Turing Machine: Initialization and Termination

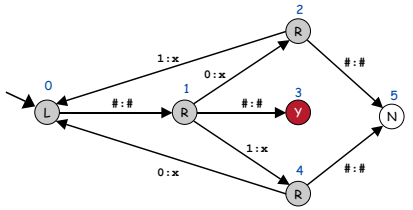
Initialization.

- Set input on some portion of tape.
- Set tape head.
- Set initial state.

…	#	#	0	0	1	1	1	0	#	#	…

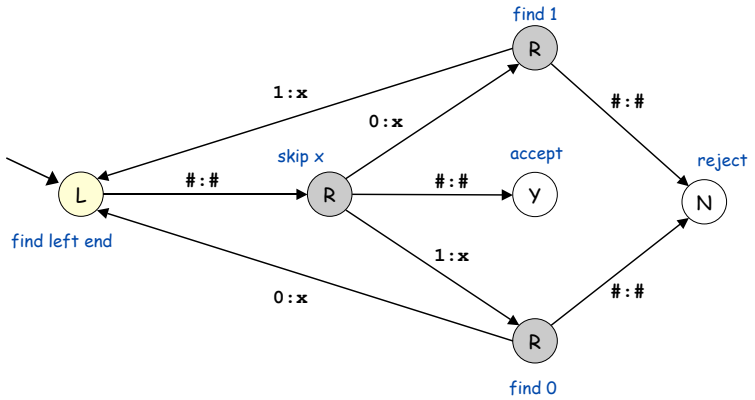
Termination.

- Stop if enter `yes`, `no`, or `halt` state.
- Infinite loop possible.



…	#	#	x	x	x	x	x	x	#	#	…

Example: Equal Number of 0's and 1's



…	#	#	0	0	1	1	1	0	#	#	…

Turing Machine Summary

Goal: simplest machine that is "as powerful" as conventional computers.

Surprising Fact 1. Such machines are very simple.

Surprising Fact 2. Some problems cannot be solved by ANY computer.

next lecture

Consequences.

- Precursor to general purpose programmable machines.
- Exposes fundamental limitations of all computers.
- Enables us to study the physics and universality of computation.
- No need to seek more powerful machines!