CS 594: Approximation Algorithms
Homework 1
Due: Mon, Mar 10

1. The following problem arises in telecommunications networks, and is known as the SONET ring loading problem. The network consists of a cycle on n nodes, numbered 0 through $n-1$ clockwise around the cycle. Some set C of calls is given; each call is a pair (i, j) originating at node i and destined to node j. The call can be routed either clockwise or counterclockwise around the ring. The objective is to route the calls so as to minimize the total load on the network. The load L_i on link $(i, i+1 (\text{mod } n))$ is the number of calls routed through the link, and the total load is $\max_{1 \leq i \leq n} L_i$. Give a 2-approximation algorithm for the SONET ring loading problem.

2. Consider the *maximum coverage* problem: Given a collection of subsets, and a number k, we would like to pick k sets from the collection so as to cover the maximum number of elements. Analyze the approximation ratio achieved by the greedy algorithm for this problem. (*Bonus:* Show that your analysis is tight).

3. Consider the following variant of *maximum coverage*: We have k different collections of subsets of some universe U of elements. We would like to pick one set from each collection so as to maximize the number of elements of U covered. Generalize the greedy algorithm to this problem and analyze the approximation ratio achieved. (*Bonus:* Show that your analysis is tight).

4. Using randomized rounding and first fit together with the Karmarkar-Karp LP, devise an algorithm for bin packing that uses $\rho \cdot OPT + k$ bins where $\rho < 2$ and k is a constant.

5. Consider the LP for MAX SAT we used in class. Suppose we use randomized rounding where variable x_i is set to TRUE with probability $\frac{1}{2}y_i + \frac{1}{4}$. Prove that this gives a $\frac{3}{4}$ approximation algorithm.