Analysis and Transformation

- Analysis:
  - Control Flow Analysis
  - Dataflow Analysis
- Transformation:
  - Register Allocation
  - Optimization
    * Machine dependent/independent
    * Local/Global/Interprocedural
    * Acyclic/Cyclic
  - Scheduling

Dataflow Analysis Motivation

Constant Propagation and Dead Code Elimination:

r1 = 4

r2 = r1 + 5

r2 = 9

r1 = 4

r2 = r1 + 5

r2 = 9

Needs dominator, liveness, and reaching definition information.

Dataflow Analysis Motivation

Register Allocation:

- Infinite number of registers (virtual registers) must be mapped to a limited number of real registers.
- Pseudo-assembly must be examined by *live variable analysis* to determine which virtual registers contain values which may be used later.
- Virtual registers which are not simultaneously *live* may be mapped onto the same real register.

```
1    r2 = r1 + 1

2    r3 = M[r2]

3    r4 = r3 + 4

4    LOAD    r5 = M[r2 + r4]
```

Dataflow Analysis

Three types we will cover:

- Live Variable
  - Live range for register allocation
  - Scheduling
  - Dead code elimination
- Reaching Definitions
  - Constant propagation
  - Constant folding
  - Copy propagation
- Available expressions
  - Common subexpression elimination

Iterative Dataflow Analysis Framework

- These dataflow analyses are all very similar $\rightarrow$ define a framework.
- Specify:
  - Two *set definitions* - $A[n]$ and $B[n]$
  - A *transfer function* - $f\,(A, B, IN/OUT)$
  - A *confluence operator* - $\vee$.
  - A *direction* - FORWARD or REVERSE.
- For forward analyses:

$$IN[n] = \vee_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = f\,(A, B, IN)$$

- For reverse analyses:

$$OUT[n] = \vee_{s \in SUCC[n]} IN[s]$$

$$IN[n] = f\,(A, B, OUT)$$

Definitions

Control Flow Definitions:

- CFG node has *out-edges* leading to *successor nodes*.
- CFG node has *in-edges* coming from *predecessor nodes*.
- For each CFG node n, $PRED[n]$ = set of all predecessors of n.
- For each CFG node n, $SUCC[n]$ = set of all successors of n.

Iterative Dataflow Analysis Framework

- Iterative dataflow analysis equations are applied in an iterative fashion until IN and OUT sets do not change.
- Typically done in (FORWARD or REVERSE) topological sort order of CFG for efficiency.
- IN and OUT sets initialized to $\emptyset$.

```
For each node n {
  IN[n] = OUT[n] = {};
}
Repeat {
  For each node n in forward/reverse topological order {
    IN'[n] = IN[n];
    OUT'[n] = OUT[n];
    IN[n], OUT[n] = (Equations);
  }
} until IN'[n] = IN[n] and OUT'[n] = OUT[n] for all n.
```

Definitions

Liveness Definitions:

- A source (RHS) register t is a *use* of t.
- A destination (LHS) register t is a *definition* of t.
- A register t is *live* on edge e if there exists a path from e to a use of t that does not go through a definition of t.
- Register t is *live-in* at CFG node n if t is live on any in-edge of n.
- Register t is *live-out* at CFG node n if t is live on any out-edge of n.

Live Variable Analysis

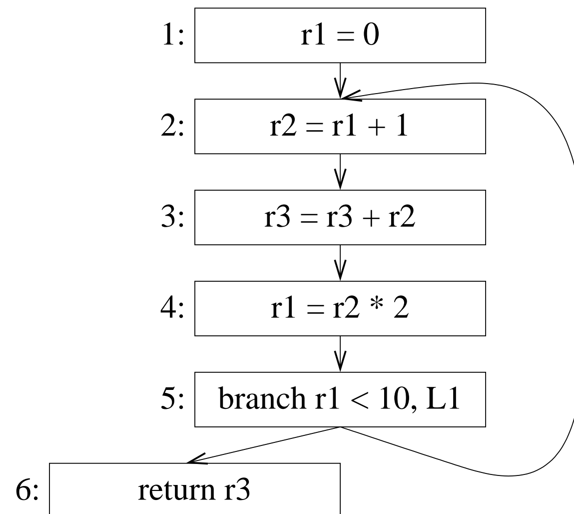
Live Variable Analysis Equation:

- Set definition ($A[n]$): $USE[n]$ - the set of registers that n uses.
- Set definition ($B[n]$): $DEF[n]$ - the set of registers that n defines.
- Transfer function ($f(A, B, OUT)$): $USE[n] \cup (OUT[n] - DEF[n])$
- Confluence operator ($\vee$): $\cup$
- Direction: REVERSE

$$OUT[n] = \cup_{s \in SUCC[n]} IN[s]$$

$$IN[n] = USE[n] \cup (OUT[n] - DEF[n])$$

Live Variable Analysis Example



Node	USE	DEF	OUT	IN	OUT	IN	OUT	IN
1								
2								
3								
4								
5								
6								

Live Variable Application 1: Register Allocation

Register Allocation:

1. Perform live variable analysis.
2. Build *interference graph*.
3. Color interference graph with real registers.

Interference Graph

- Node t corresponds to virtual register t.
- Edge $\langle t_i, t_j \rangle$ exists if registers t_i, t_j have overlapping live ranges.
- For some node n, if $DEF[n] = \{a\}$ and $OUT[n] = \{b_1, b_2, ...b_k\}$, then add interference edges: $\langle a, b_1 \rangle$, $\langle a, b_2 \rangle$, $\langle a, b_k \rangle$

Interference Graph For Example:

Node	DEF	OUT	IN
1	r1	r1,r3	r3
2	r2	r2,r3	r1,r3
3	r3	r2,r3	r2,r3
4	r1	r1,r3	r2,r3
5	-	r1, r3	r1,r3
6	-		r3

Virtual registers r1 and r2 may be mapped to same real registers.

Live Variable Application 2: Dead Code Elimination

- Given statement s with a definition and no side-effects:

  `r1 = r2 + r3, r1 = M[r2], or r1 = r2`

  If r1 is *not* live at the end of s, then the s is *dead*

- Dead statements can be deleted.

- Given statement s without a definition or side-effects:

  `r1 = call FUN_NAME, M[r1] = r2`

  Even if r1 is not live at the end of s, it is not dead.

Example:

```
r1 = r2 + 1
r2 = r2 + 2
r1 = r2 + 3
M[r1] = r2
```

Reaching Definition Analysis

Determines whether definition of register t directly affects use of t at some point in program.

Reaching Definition Definitions:

- *unambiguous* - instruction explicitly defines register t.
- *ambiguous* - instruction may or may not define register t.
  - Global variables in a function call.
  - No ambiguous definitions in tiger since all globals are stored in memory.
- Definition of d (of t) *reaches* statement u if a path of CFG edges exists from d to u that does not pass through an unambiguous definition of t.
- One unambiguous and many ambiguous definitions of t may reach u on a single path.

Reaching Definition Analysis

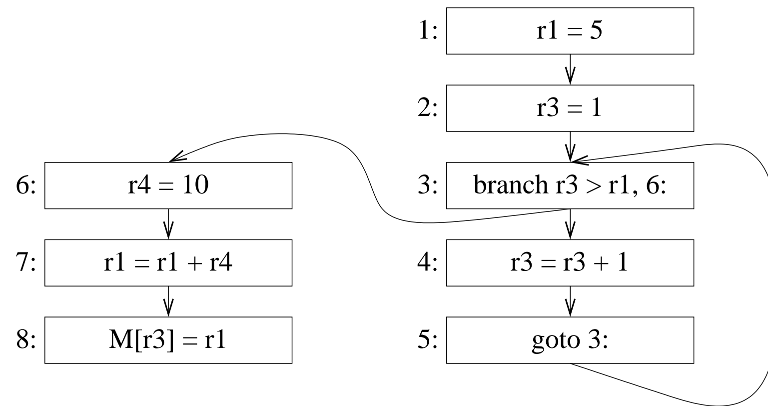
Reaching Definition Analysis Equation:

- Set definition ($A[n]$): $GEN[n]$ - the set of *definition id's* that n creates.
- Set definition ($B[n]$): $KILL[n]$ - the set of *definition id's* that n kills.
  - $defs(t)$ - set of all *definition id's* of register t.
- Transfer function ($f(A, B, IN)$): $GEN[n] \cup (IN[n] - KILL[n])$
- Confluence operator ($\vee$): $\cup$
- Direction: FORWARD

$$IN[n] = \cup_{p \in PRED[n]} OUT[p]$$

$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

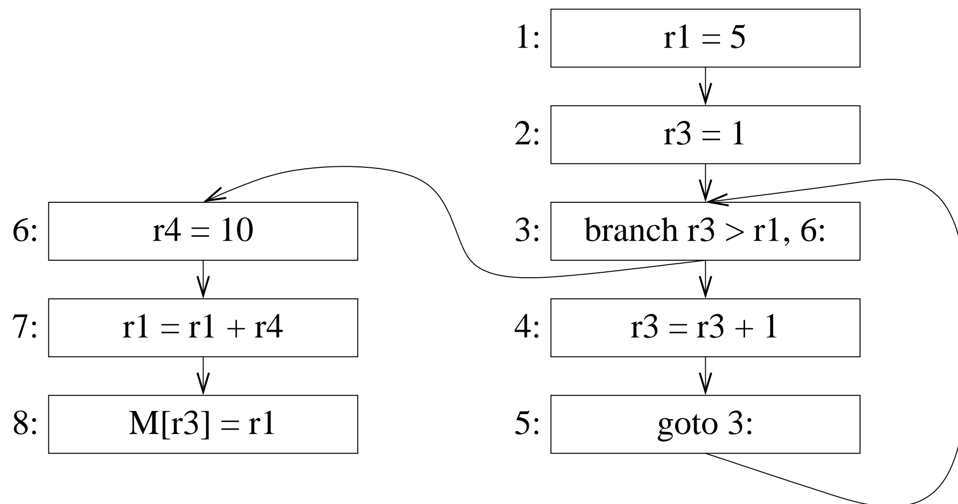
Reaching Definition Analysis Example



Node	GEN	$KILL$	IN	OUT	IN	OUT	IN	OUT
1								
2								
3								
4								
5								
6								
7								
8								

Reaching Definition Application 1: Constant Propagation

- Given Statement d: `a = c` where `a` is constant
- Given Statement u: `t = a op b`
- If statement d reach u and no other definition of `a` reaches u, then replace u by `t = c op b`.



Statements 1 and 6 are dead.

Constant Folding

- Given Statement d: `t = a op b`
- If `a` and `b` are constant, compute `c` as `a op b`, replace d by `t = c`