

**Problem Set No. 6**  
Collaboration is not allowed

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1. (Greedy Matching) Given an undirected, unweighted graph, consider the following algorithm for finding a large matching. Begin with the empty matching. Repeat the following step until it no longer applies: find an edge both of whose ends are unmatched, and add it to the matching.
  - (a) Describe an implementation of this algorithm that runs in linear time (in the number of vertices and edges of the graph).
  - (b) Prove that this algorithm finds a matching that is within a factor of two of maximum size. Give an example to show that this constant factor of two is tight (an example on which the algorithm finds a matching only half the size of maximum).
2. (Scheduling with profits and deadlines: see CLRS Problem 34-4) Suppose you have one machine and  $n$  tasks  $a_1, a_2, \dots, a_n$ . Each task  $a_j$  has a processing time  $t_j$ , a profit  $p_j$  and a deadline  $d_j$ . The machine can process only one task at a time, and the task must run without interruption for  $t_j$  consecutive time units. If you complete task  $a_j$  by its deadline  $d_j$ , you receive a profit  $p_j$ , but if you complete it after its deadline you receive nothing. As an optimization problem, you wish to find a schedule that completes all the tasks and returns the greatest profit.

Formulate a “yes-no” version of the problem and show that it is NP-complete.

3. (Bin-packing: see CLRS Problem 35-1) Given a set of  $n$  objects, where the size  $s_i$  of the  $i$ th object satisfies  $0 < s_i < 1$ , we wish to pack all the objects into the minimum number of bins. Each bin can hold any subset of the objects whose total size does not exceed 1. The “first-fit” algorithm puts the objects into bins sequentially according to the following rule: add the next object to the first bin into which it fits.
  - (a) Give an efficient implementation of the first-fit algorithm, and analyze its running time.
  - (b) Prove that the first-fit algorithm comes within a factor of 2 of minimizing the number of bins used.