COS 226 Lecture 10: Radix trees and tries

Symbol Table, Dictionary
- records with keys
- INSERT
- SEARCH

Balanced trees, randomized trees
- use O(lgN) comparisons

Hashing
- uses O(1) probes
- but time proportional to key length

Are comparisons necessary?
- (no)

Is time proportional to key length required?
- (no)

Best possible: examine lgN BITS

Digital search trees (DSTs)

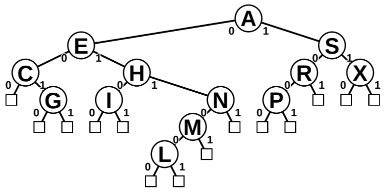
Easy way to balance tree: use bits of key to direct search
Otherwise identical to BST

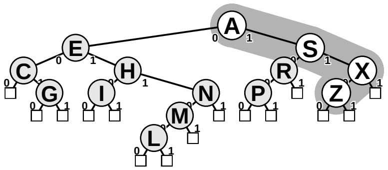
```
#define bit(A, B) digit(A, B)
Item searchR(link h, Key v, int w)
  { Key t = key(h->item);
    if (h == z) return NULLitem;
    if eq(v, t) return h->item;
    if (bit(v, w) == 0)
           return searchR(h->l, v, w+1);
    else  return searchR(h->r, v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

``digit`` macro extracts Bth bit from A (see lecture 6)

Digital tree insertion

A	00001
S	10011
E	00101
R	10010
C	00011
H	01000
I	01001
N	01110
G	00111
X	11000
M	01101
P	10000
L	01100





Trees NOT ordered
- does not support SORT and SELECT

DST insertion code

```
link insertR(link h, Item item, int w)
  { Key v = key(item), t = key(h->item);
    if (h == z) return NEW(item, z, z, 1);
    if (bit(v, w) == 0)
           h->l = insertR(h->l, item, w+1);
    else  h->r = insertR(h->r, item, w+1);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item, 0); }
```

Tries

Branch according to bits in keys

No keys in internal nodes

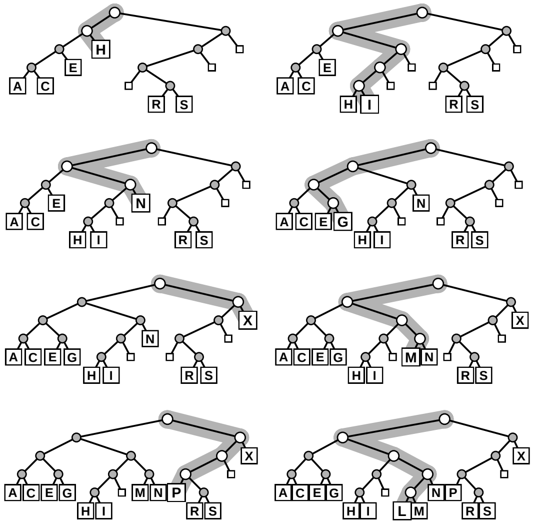
Records/keys in external nodes

Structure depends on keys, not insertion order

Examine lgN BITS to distinguish key
- independent of key length!

Problems:
- 44% space waste from 1-way branching
- multiple node types

Trie example



Trie search implementation

Branch according to bits, as in DST

Three outcomes:
- null link
- key in external node matches search key
- key in external node differs from search key

```
#define external(A) ((h->l == z) && (h->r == z))
Item searchR(link h, Key v, int w)
  { Key t = key(h->item);
    if (h == z) return NULLitem;
    if (external(h))
      return eq(v, t) ? h->item : NULLitem;
    if (digit(v, w) == 0)
           return searchR(h->l, v, w+1);
    else  return searchR(h->r, v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

Trie insertion implementation

Two possible search miss outcomes
- null link: replace with link to new node
- external node: recursive split to distinguish new key

```
void STinit()
  { head = (z = NEW(NULLitem, 0, 0, 0)); }
link insertR(link h, Item item, int w)
  { Key v = key(item), t = key(h->item);
    if (h == z) return NEW(item, z, z, 1);
    if (external(h))
    { return split(NEW(item, z, z, 1), h, w); }
    if (digit(v, w) == 0)
           h->l = insertR(h->l, item, w+1);
    else  h->r = insertR(h->r, item, w+1);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item, 0); }
```

Trie insertion recursive node split

```
link split(link p, link q, int w)
  { link t = NEW(NULLitem, z, z, 2);
    switch(bit(p->item, w)*2+bit(q->item, w))
    { case 0: t->l = split(p, q, w+1); break;
      case 1: t->l = p; t->r = q; break;
      case 2: t->r = p; t->l = q; break;
      case 3: t->r = split(p, q, w+1); break; }
    return t;
  }
```

Patricia tries

Digression: cute nomenclature

P ractical
A lgorithm
T o
R etrieve
I nformation
C oded
I n
A lphanumeric

information reTRIEval (but pronounced "try")

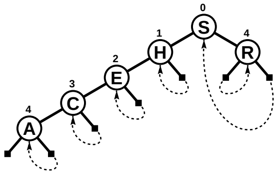
Patricia:

- collapse one-way branches in tries
- "thread" tree to eliminate multiple node types

Quintessential search algorithm

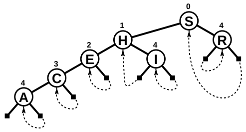
Patricia trie search and insertion

SEARCH: branch according to specified

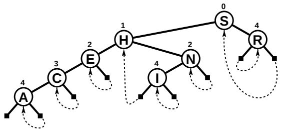


INSERT: search, then find leftmost bit that
distinguishes new key from key found

Easy (external) case



Harder (internal) case



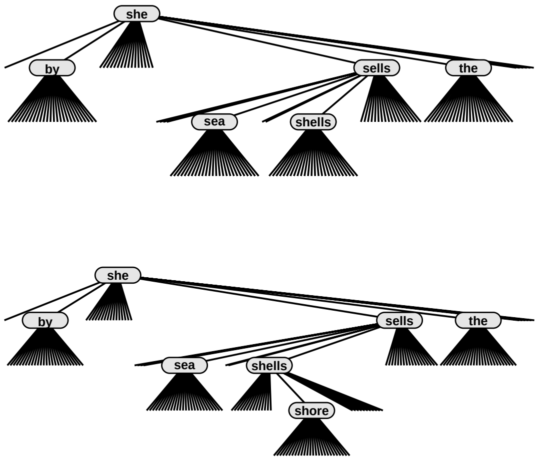
R-way digital tree

Generalize DIGITAL TREE to R links per node
R-way branching gives fast search

```
struct STnode
  { Item item; link next[26]; };
Item searchR(link h, Key v, int w)
  { int i = digit(v, w);
    if (h == z) return NULLitem;
    if eq(v, key(h->item)) return h->item;
    return searchR(h->next[i], v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

Simple, fast (but uses a lot of space)

R-way digital tree example



R-way digital tree analysis

N keys: N internal nodes, R links per node

Space: N*R

Time: lgN/lgR comparisons

Ex: R=26, N=20000

500,000 links

tree height 3-4

Ex: R=16, N=1M

16M links

tree height 5

Plus: one node type, easy implementation

Minus:

- full comparison at each node
- does not support SORT and SELECT

R-way trie

Generalize TRIE to R-way branching

Nodes contain characters

R-way branching on next character

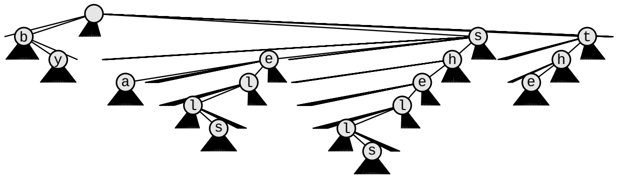
End-of-key options
- fixed-length keys
- prefix match
- prefix-free keys
    - end-of-key character
    - suffix trie

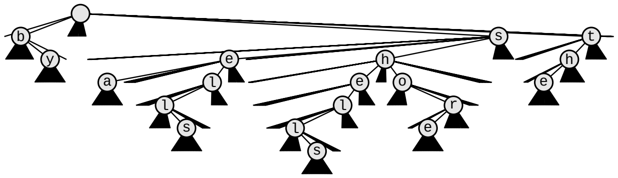
Maintain order in tree to support SORT and SELECT

R-way trie search code

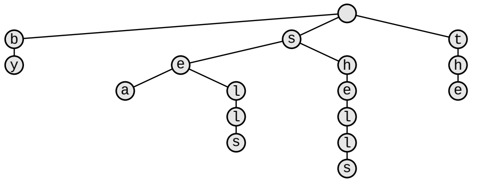
```
typedef struct STnode* link;
struct STnode
  { Item item; link next[R]; };
Item searchR(link h, Key v, int w)
  { int i = digit(v, w);
    if (h == z) return NULLitem;
    if (internal(h))
      return searchR(h->next[i], v, w+1);
    if eq(v, key(h->item)) return h->item;
    return NULLitem;
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

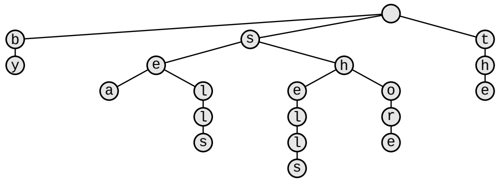
R-way trie example





R-way trie example (without null links)





R-way trie existence table

EXISTENCE TABLE

- no data in trie
- keys encoded in trie structure

Easy to implement, but may use excessive space

```
Item searchR(link h, Key v, int w)
  { int i = digit(v, w);
    if (h == z) return NULLitem;
    if (i == NULLdigit) return v;
    return searchR(h->next[i], v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

Extend digit(v, w) implementation to

- return NULLdigit if v has fewer than w digits

R-way trie insert code

```
link split(link p, link q, int w)
  { link t = NEW(NULLitem);
    int pd = digit(p->item, w),
        qd = digit(q->item, w);
    if (pd == qd)
          { t->next[pd] = split(p, q, w+1); }
    else  { t->next[pd] = p; t->next[qd] = q; }
    return t;
  }
link insertR(link h, Item item, int w)
  { Key v = key(item);
    int i = digit(v, w);
    if (h == z) return NEW(item);
    if (!internal(h))
      { return split(NEW(item), h, w); }
    h->next[i] = insertR(h->next[i], v, w+1);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item, 0); }
```

R-way trie analysis

Assumptions
- one-way links collapsed
- link to remainder of key in external nodes
- N keys, total of C characters in keys
- approx. N internal nodes
- R links per node

Space: N*R + C

Time: lgN/lgR CHARACTER comparisons

Ex: R=26, N=20000
        520,000 links
        tree height 3-4

Ex: R=16, N=1M
        16M links
        tree height 5

R-way trie summary

Faster than hashing
- successful search: no arithmetic
- unsuccessful search: don't need to examine whole key

Supports SORT, SELECT, other ADT operations

Problems
- eliminating 1-way branches
- multiple node types
- too much space for null links

Correspondence with sorting algs

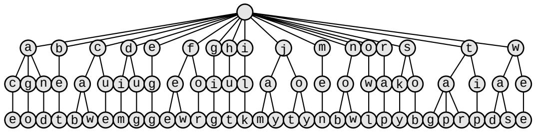
BSTs correspond to Quicksort recursive structure

Roles of TIME and SPACE interchanged

TIME:
- path in tree
- size of subfile in sort

SPACE:
- stack size in sort
- branching factor in tree

Tries correspond to MSD radix sort



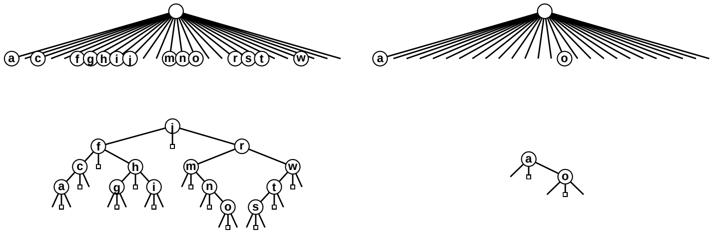
3-way tries correspond to 3-way Quicksort

Three-way radix tries

Nodes contain characters

three-way branching on next character
- left: key character less
- middle: key character equal
- right: key character greater

Equivalent to replacing trie node with BST on character



Ternary search tries (TSTs)

Existence trie implementation OK

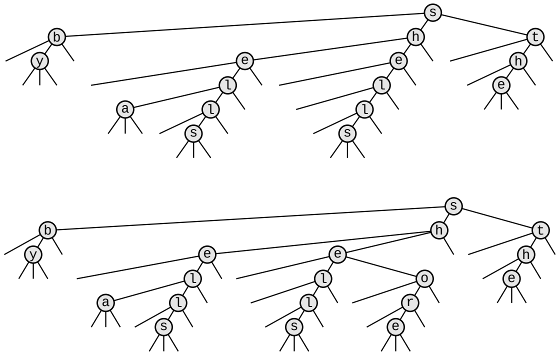
Adds factor of 2lnM to search cost
- constant no. of BYTES for unsucc. search

Easy recursive sort, selection

Most important advantages
- adapts well to strange keys
- uses just linear space
- supports full array of ADT operations

Faster than hashing, without wasting space

TST example



TST search implementation

Code writes itself

```
Item searchR(link h, Key v, int w)
  { int i = digit(v, w);
    if (h == z) return NULLitem;
    if (i == NULLdigit) return v;
    if (i < h->d) return searchR(h->l, v, w);
    if (i == h->d) return searchR(h->m, v, w+1);
    if (i > h->d) return searchR(h->r, v, w);
  }
Item STsearch(char *v)
  { return searchR(head, v, 0); }
```

TST insertion implementation

```
typedef struct STnode* link;
struct STnode
  { Item item; int d; link l, m, r; };
void STinit() { head = z; }
link NEW(int d)
  { link x = malloc(sizeof *x);
    x->d = d; x->l = z; x->m = z; x->r = z;
    return x;
  }
link insertR(link h, Item item, int w)
  { Key v = key(item);
    int i = digit(v, w);
    if (h == z) h = NEW(i);
    if (i == NULLdigit) return h;
    if (i < h->d) h->l = insertR(h->l, v, w);
    if (i == h->d) h->m = insertR(h->m, v, w+1);
    if (i > h->d) h->r = insertR(h->r, v, w);
    return h;
  }
void STinsert(Key key)
  { head = insertR(head, key, 0); }
```

Empirical studies

random 32-bit keys

	BUILD				SEARCH			
	bst	dst	trie	pat	bst	dst	trie	pat
5000	4	5	7	7	3	2	3	2
12500	18	15	20	18	8	7	9	7
25000	40	36	44	41	20	17	20	17
50000	81	80	99	90	43	41	47	36
100000	176	167	269	242	103	85	101	92
200000	411	360	544	448	228	179	211	182

Empirical studies (continued)

words in Moby Dick

	BUILD				SEARCH			
	bst	hash	tst	fast	bst	hash	tst	fast
5000	5	4	3	3	3	2	2	1
12500	11	8	9	7	9	5	5	3
25000	23	15	17	13	19	12	10	7
50000	50	29	31	25	43	25	21	15

library call numbers

	BUILD				SEARCH			
	bst	hash	tst	fast	bst	hash	tst	fast
5000	19	16	21	20	10	8	6	4
12500	48	48	54	97	29	27	15	14
25000	118	99	188	156	67	59	36	30
50000	230	191	333	255	137	113	70	65

fast: TST with R*R-way branching at the root