## COS 226 Lecture 16: Geometric search

Extend search ADT to geometric data

PROBLEMS

- Range search
- Intersections among geometric objects
- Near neighbor search
- Point location

TWO-DIMENSIONAL

MULTIDIMENSIONAL

APPROACHES

- trees
- divide-and-conquer
- discretized algorithms



## Range searching

Possible addition to symbol-table ADT:

```
.       void STinit();
.       void STinsert(Item x);
.       Item STsearch(Key v);
.        int STempty();
.        int STrange(Key v1, Key v2);
```

Options to actually process the records

- pass a procedure to call for each record in the range
- return list of records (possibly sorted)

Depends on how many records expected (count them first)

ARRAY implementation: do binary search on both keys

HASH TABLE implementation: no easy algorithm

BST, TRIE implementations: recursive traversal works

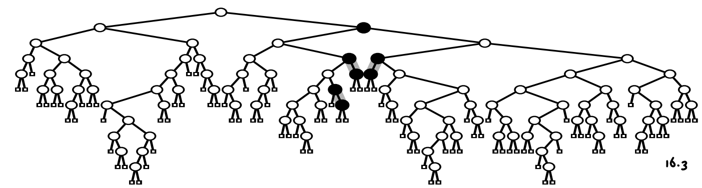


## BST 1D range searching

Recursively search subtrees that COULD HAVE keys in interval

- root may or may not be in interval
- search BOTH subtrees if it is

```
Key v1, v2; int count = 0;
int BSTrangeR(link h)
  { int tx1 = (h->key >= v1);
    int tx2 = (h->key <= v2);
    if (tx1 && (h->l != z)) BSTrangeR(h->l);
    if (tx1 && tx2) count++;
    if (tx2 && (h->r != z)) BSTrangeR(h->r);
  }
```





## 2D Range searching

Same basic method works in higher dimensions (!!)

- discovered by an undergraduate

INTERVAL in 1D is RECTANGLE in 2D

2D TREE: alternate x and y

Recursively search subtrees that COULD HAVE keys in interval

- root may or may not be in rectangle
- search BOTH subtrees if it is

Corresponds to recursive subdivision of the plane

- alternating horizontal and vertical lines

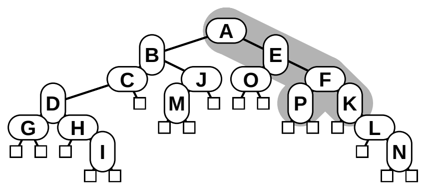
kD tree: trivial generalization

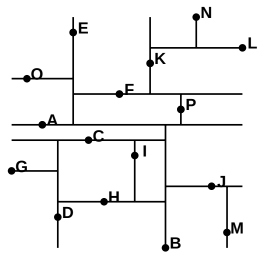
## 2D tree example

Each EXTERNAL node corresponds to an area in the plane

Each INTERNAL node divides its area into two subdivisions

Switch between horizontal and vertical dividing lines



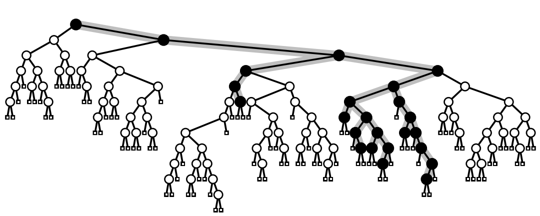

**Quad tree**

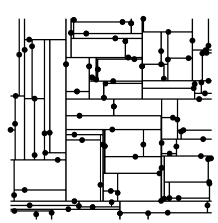
- use 4-way tree (divide on both coordinates at once)



## 2D tree range searching

```
int x1, y1, x2, y2, count = 0;
TDTrangeR(link h, int d)
  { int t1,t2,tx1,tx2,ty1,ty2;
    if (h == z) return;
    tx1 = x1 < h->p.x; tx2 = h->p.x <= x2;
    ty1 = y1 < h->p.y; ty2 = h->p.y <= y2;
    t1 = d ? tx1 : ty1; t2 = d ? tx2 : ty2;
    if (t1 && (h->l != z)) TDTrangeR(h->l, !d);
    if (tx1 && tx2 && ty1 && ty2) count++;
    if (t2 && (h->r != z)) TDTrangeR(h->r, !d);
  }
```



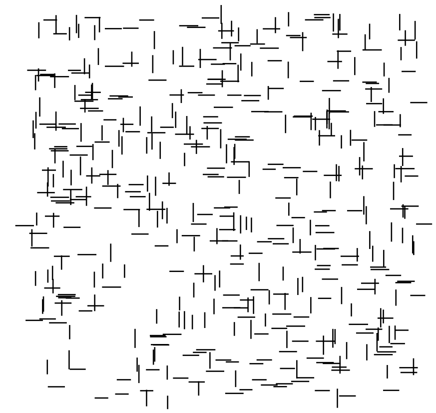




## Manhattan line intersection problem

N lines, all either horizontal or vertical

How many pairs intersect?



As with other search problems

- usually no harder to REPORT all intersections
- (call a given function for each)

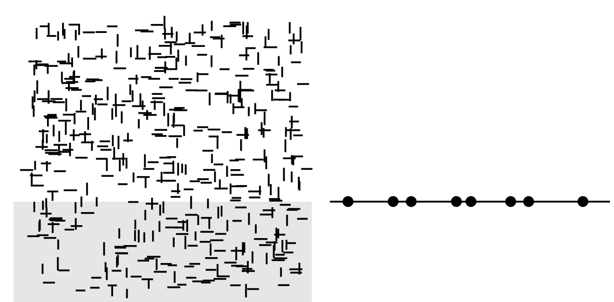


## Manhattan line intersection

Dynamic SWEEP LINE algorithm

Horizontal line sweeps from bottom to top

- vertical data line represents "point"
- horizontal data line represents "interval"



There is an h-v intersection if "point" is in "interval"

Reduces 2D line intersection problem to 1D range searching!

## Sweep line implementation

Uses both PQ and ST (with range search) ADT

- PQ: get y coordinates in increasing order
- ST: range search on x coordinates for intersection

Three types of "events"

B: bottom of vertical line [INSERT x]

T: top of vertical line [DELETE x]

H: horizontal line [RANGE (x1, x2)]

Generalizes to give fast algorithms for

- rectangles, general lines, circles, convex polygons

Generalizes to higher dimensions

- "sweep hyperplane"



## Near neighbor searching

Another possible addition to Search ADT:

```
.        void STinit();
.        void STinsert(Item x);
.        Item STsearch(Key v);
.         int STempty();
.        Item STnearest(Key v);
```

Find the record with key value closest to v

Need a concept of "distance", not just "less"

- easy if keys are numbers, or points in space

ARRAY implementation: scan both ways after binary search

HASH TABLE implementation: no easy algorithm

BST, TRIE implementations: recursive traversal works



## 1D BST near neighbor searching

Recursively search subtrees that COULD HAVE near neighbor

- may search BOTH subtrees

```
void BSTnear(link h)
  {
    if (h == z) return;
    if (dist(v, h->key) < min)
      { best = h; min = dist(v, best->key); }
    if (v < h->key || (v - h->key) < min)
      BSTnear(h->l);
    if (v > h->key || (h->key - v) < min)
      BSTnear(h->r);
  }
```

Multidimensional near neighbor searching:

- same algorithm on kD tree

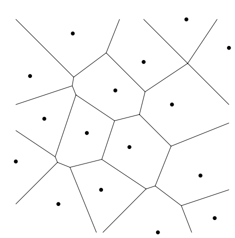


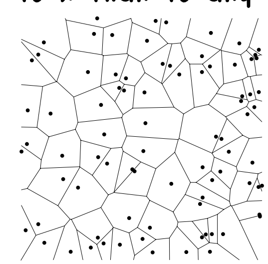
## Voronoi diagram

Given: set S of N points

point x's Voronoi REGION:

- set of points closer to x than to any other y in S





Voronoi EDGES: perpendicular bisectors of point pairs

- intersect at centroids of point triple triangles

Voronoi DIAGRAM: union of Voronoi edges

Challenge to compute

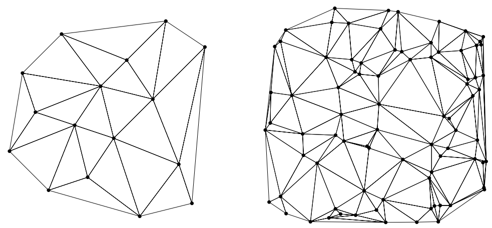
- Representation?
- Degenerate cases?

## Delaunay triagulation

Given: set S of N points

DELAUNAY TRIANGULATION

- edge x-y iff Voronoi edge separates x and y



Outer boundary is convex hull

Representation easier: no extra points

THM: Voronoi diagram and Delauney triangulation can be computed in N log N steps (!!)

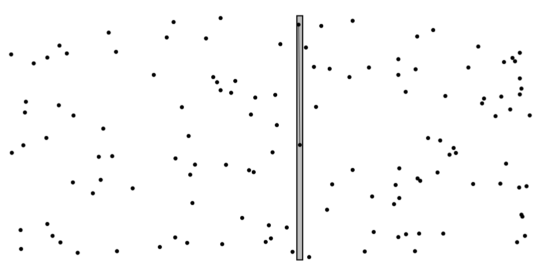
- divide and conquer
- sweep line
- randomize
- discretize



## 2D divide-and-conquer

Ex: CLOSEST PAIR algorithm

- sort on x
- divide into two sets of N/2 points
- find closest pair in each half
- find closest pair crossing boundary



Boundary check MUST be efficient (terminates recursion)

- sort on y to make boundary check easy
- y sort comes for free (!!)

Implementation: tricky exercise in recursion (see text)



## Grid methods

Grids : geometric search :: tries : search ADT

Grid method

- define uniform grid of fixed-size squares
- put points in lists associated with squares
- ignore points in faraway grid squares

Time-space tradeoff like MSD sort

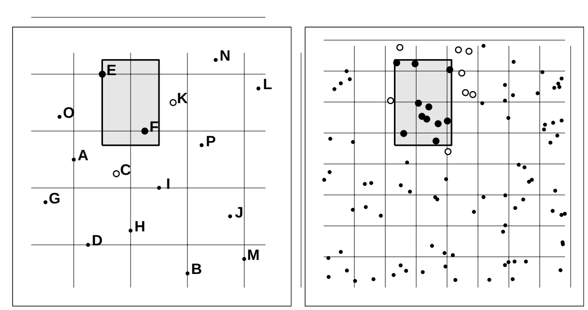
- grid too fine: empty cells
- grid too coarse: lists too long

Use 2- or 3-level grids, or recurse ala quad trees



## Grid methods (continued)

Ex: range searching




For graphics applications

- ultimate grid is PIXEL ARRAY
- leads to ``discretized algorithms``

## Point location problem

Ex: find state corresponding to point on map

Planar subdivision

- 2D tree planar decomposition
- N lines
- Voronoi diagram
- grid
- pixel array

Which division contains the given point?

Difficult in general
if only because of difficulty of
representing planar subdivisions



## Discretized line intersection

p-by-p bit raster, p^2 pixels

N lines

Draw rasterized verson of line

- report intersection if pixel already 1

Cost:

- p^2 to initialize pixels to 0
- number of pixels on lines

Cost dominated by p^2

Line intersection same cost as drawing blank picture!



## Discretized Voronoi diagram

put 1 pixels on a priority queue
priority: distance to closest point

ALGORITHM

- remove pixel from priority queue
- check all neighbor pixels
  - if closer or same: ignore
  - if farther: check pixel value
    - if 0, set to 1 and put back on pq
    - if 1, must be on a voronoi edge!

Time proportional to initialize plus product of

- number of pixels on diagram
- diameter of largest cell

Idea: refine discretized diagram to compute real diagram