Spherical Attribute Images (SAI)

Goal

- Define a representation of a 3D model that is canonical
- Independent of translation, rotation
Previous Works

- Gauss Map
- Extended Gaussian Image (EGI)
  - Associate mass with mapping
  - Can’t handle non-convex object
- Variants of EGI

SAI

- Deform a semi-regularly tessellated geodesic dome onto the object

[ Ikeuchi, Hebert 95]
Semi-regular Tessellation
- Subdivide icosahedron into \(20N^2\) triangular faces
- Take the dual
- Every vertex have 3 neighbors

Surface Mapping
- Data Force
- Regularity Constraint
- Iterative deformation
Curvature

- Approximated by simplex angle
- Ranged from $-\pi$ to $+\pi$

Properties

- For given number of nodes, invariant to translation and scaling
- Unique SAI for an object up to a rotation
- Connected patch of surface map to connected patch of the spherical image
Matching of two SAI

- Define $D(S, S', R) = \sum (g(P_s) - g(RP_{s'}))^2$
- Naïve: Try all possible rotations ($\theta, \varphi, \psi$)
- Smart: Only 3K valid rotations for K nodes

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On similarity

$$d_p(S, S', R) = (\sum |g(P_s) - g(RP_{s'})|)^{1/p}$$

$$D_p(S, S') = \min_R d_p(S, S', R)$$

- $D$ is metric
  - $D(A, B) \geq 0$
  - $D(A, A) = 0$
  - $D(A, B) = D(B, A)$
  - $D(A, B) + D(B, C) \geq D(A, C)$
Similarity

[Shum, Ikeuchi, Hebert 96]

Discussion

- Good for matching identical objects
- Applied to merging multiple views, recognition of known surfaces
- Similarity...
- Restricted to genus zero
Point Set Matching

Surface Matching
- Extract “features” (usu. points, maybe labeled)
- Do point matching
- Lots of 2D algorithms, few extend to 3D
Allowed Transformations

- Translation + rotation = **rigid motion** (Euclidean)
- + scaling = similarity
- + shearing = affine

Exact Point Matching

- Numerically very unstable
- 2D:
  - get polar coords w.r.t. centroid
  - sort by $\phi$, $r$ & concatenate => “strings” $A, B$
  - is $A$ substring of $BB$?
  - $O(n \log n)$ $\leftarrow$ sort
- For similarity, first scale object diameter
Exact Point Matching: 3D

- Project points onto center sphere
- Label projections w/ list of projected pts.
- Take 3D convex hull of points on sphere to induce adjacency relationship
- Solve labeled planar graph isomorphism
- \( O(n \log n) \)

Approximate Matching

- Bound Hausdorff distance
- One – to – one mapping
- Can compute optimum or exact answer, but it’s a real pain
Approximate Matching

- If point sets are “close”, is it 1-to-1?
- Network flow on bipartite graph

Approximate Matching

- Find rigid transformation
- Idea: in 2D, 2 points must satisfy Hausdorff distance exactly
  (3 points in 3D)
Approximate Matching: Idea

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Curve of degree 6
Approximate Matching

- Curve has at most 12 intersections with circle (n-2 curves, n-2 circles => O(n²))
- Try all possible 2-2 initial pairing: O(n⁶)
- 3D: fix 3 points ... ugh!

Not One-to-one

- “Pattern matching” (find “object” into “scene”)
- Solution within constant factor of optimum (rigid motion in 3D: 8 + ε)
  - Pick 3 “opposing” points in object
  - Match to every triplet of scene points
  - Keep configuration w/ min Hausdorff distance
Acceleration Techniques

- For each *, find closest □. O(nm)
- Use grid & look only in appropriate box. O(n)

Geometric Hashing

- Goal 1: database query (no transformation)
- In all boxes with *, vote for shapes present
Geometric Hashing

- **Goal:** identify object & recover transformation
  - Objects in database appear multiple times, in all “canonical” representations
    ![Diagram](image.png)
  - Ex: for translation, 1 “interest point” => origin

Geometric Hashing

- **Canonical “basis”**
  - 2D affine basis = 3 “interest points”
  - 3D rigid motion: 3 points

- **Properties**
  - Deals with occlusion
  - Highly parallel
  - Large table (memory requirements) \(O(mn^4)\)