COS 522, February 29, 2000
Due: March 21, 2000

## Homework Set 2

Do Exercises 5.13, 5.15, and 5.21 in the book by Du and Ko, plus the following special problems.

**Problem 1** Let $A = \{K_1, K_2, K_3, K_4, K_5\}$ be a family of affine subspaces in $R^2$, where $K_1, K_2, K_3$, $K_4$ and $K_5$ are respectively the lines $x = 0$, $y = 0$, $x + y = 0$, $x + y = 1$ and $x + y = -1$.
(a) Let $V_A = \cup_{1 \leq i \leq 5} K_i$, regarded as a set in the compactified $R^2$. What is $\beta_0(V_A)$? What is $\beta_1(V_A)$?
(b) Draw the partially ordered set $L_A$. Calculate $\sum_{\vec{0} < x \in L_A} |\mu(\vec{0}, x)|$.

**Problem 2** Let $P$ be the property of an $n$-vertex graph being Hamiltonian. Use the following adversary strategy to prove $C(P) \geq \Omega(n^2)$. An *$n$-cycle* is a graph consisting exactly of a cycle of length $n$. Clearly, there are $(n-1)!$ such $n$-cycles. Note that any $n$-cycle is Hamiltonian. At any time, the adversary keeps a count of the number of $n$-cycles consistent with the revealed information about the graph so far. When a query "$x_{i,j} = 0$?" is asked, the adversary picks the answer to maximize this count consistent with this answer.

**Problem 3** Let $p > 1$ be a prime number and $n \geq 2p$. Let $P$ be the $n$-vertex graph property such that $P(G) = 1$ if and only at least one of the following holds: (a) $G$ is connected, (b) $G$ contains a clique of size $p$. Prove that property $P$ is elusive.

*Hint:* Use an extension of the Kahn-Saks-Sturtevant method.