Subdivision Surfaces

COS 426, Spring 2017
Princeton University
3D Object Representations

- Raw data
  - Range image
  - Point cloud

- Surfaces
  - Polygonal mesh
    - Subdivision
  - Parametric
  - Implicit

- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep

- High-level structures
  - Scene graph
  - Application specific
Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software
Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software
Geri’s Game

• “served as a demonstration of a new animation tool called subdivision surfaces” (Wikipedia)

• Subdivision used for head, hands & some clothing

• Academy Award winner
Geri’s Game

• Guest performance in Toy Story 2
Subdivision Surfaces

- An alternative to NURBS, overcoming:
  - Many patches
  - Difficult to mark sharp features

- Irregularities after deformation

Stanford Graphics course notes
Subdivision Surfaces

• What makes a good surface representation?
  • Accurate
  • Concise
  • Intuitive specification
  • Local support
  • Affine invariant
  • Arbitrary topology
  • Guaranteed continuity
  • Natural parameterization
  • Efficient display
  • Efficient intersections

Reif & Schroeder 2000
Subdivision Surfaces

- What makes a good surface representation?
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Reif & Schroeder 2000
A curve / surface with $G^k$ continuity has a continuous k-th derivative, geometrically.
Continuity

Similar, but not exactly $C^k$ continuity

- Geometric, not of algebraic (e.g.: $f_x(u) = r_x \cos(2\pi u)$)
- Different only when algebraic gradient $\nabla f = 0$
Subdivision

• How do you make a curve with guaranteed continuity?
Subdivision

- How do you make a curve with guaranteed continuity? …
Subdivision

• How do you make a surface with guaranteed continuity?
Subdivision Surfaces

- Repeated application of
  - Topology refinement (splitting faces)
  - Geometry refinement (weighted averaging)
Subdivision Surfaces – Examples

• Base mesh
Subdivision Surfaces – Examples

• Topology refinement
Subdivision Surfaces – Examples

- Geometry refinement
Subdivision Surfaces – Examples

- Topology refinement
Subdivision Surfaces – Examples

- Geometry refinement
Subdivision Surfaces – Examples

• Topology refinement
Subdivision Surfaces – Examples

- Geometry refinement
Subdivision Surfaces – Examples

• Limit surface
Subdivision Surfaces – Examples

- Base mesh + limit surface

Meshlab demo
Design of Subdivision Rules

• What types of input?
  • Quad meshes, triangle meshes, etc.

• How to refine topology?
  • Simple implementations

• How to refine geometry?
  • Smoothness guarantees in limit surface
    » Continuity ($C^0, C^1, C^2, \ldots$?)
  • Provable relationships between limit surface and original control mesh
    » Interpolation of vertices?
Linear Subdivision

• Type of input
  • Quad mesh -- four-sided polygons (quads)
  • Any number of quads may touch each vertex

• Topology refinement rule
  • Split every quad into four at midpoints

• Geometry refinement rule
  • Average vertex positions

This is a simple example to demonstrate how subdivision schemes work
Linear Subdivision
Linear Subdivision

- Topology refinement
Linear Subdivision

• Geometry refinement
Linear Subdivision

LinearSubivision \((F_0, V_0, k)\)

for \(i = 1 \ldots k\) levels

\((F_i, V_i) = \text{RefineTopology}(F_{i-1}, V_{i-1})\)

\text{RefineGeometry}(F_i, V_i)

return \((F_k, V_k)\)
Linear Subdivision

RefineTopology (F, V)

newV = V
newF = {}

for each face Fi
  Insert new vertex c at centroid of Fi into newV
  for j = 1 to 4
    Insert in newV new vertex ej at
    centroid of each edge (Fi,j, Fi,j+1)
    for j = 1 to 4
      Insert new face (Fi,j, ej, c, ej-1) into newF

return (newF, newV)
Linear Subdivision

RefineGeometry( F, V )

newV = V
newF = F

for each vertex \( V_i \) in newV

weight = 0;
newV[i] = (0,0,0)

for each face \( F_j \) connected to \( V_i \)

\[ \text{newV}[i] += \text{centroid of } F_j \]
weight += 1.0;

newV[i] /= weight

return (newF, newV)
Linear Subdivision

• Example

Input mesh
Linear Subdivision

• Example
Linear Subdivision

• Example

Geometry refinement
Linear Subdivision

• Example

Topology refinement
Linear Subdivision

• Example

Geometry refinement
Linear Subdivision

- Example

Topology refinement

Scott Schaefer
Linear Subdivision

• Example

Geometry refinement

Scott Schaefer
Linear Subdivision

- Example

Topology refinement

Scott Schaefer
Linear Subdivision

• Example

Final result
Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others

- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

... which makes differences in ...
- Provable properties
Catmull-Clark Subdivision
Catmull-Clark Subdivision

Scott Schaefer
Catmull-Clark Subdivision

Scott Schaefer
Catmull-Clark Subdivision
Catmull-Clark Subdivision
Catmull-Clark Subdivision

Scott Schaefer
Catmull-Clark Subdivision

New $\bullet = \left( 4 \times \text{avg of } \bullet - 1 \times \text{avg of } \bullet + (n-3) \times \bullet \right) / n$
Catmull-Clark Subdivision
Catmull-Clark Subdivision

Scott Schaefer
Catmull-Clark Subdivision

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Linear Subdivision

Catmull-Clark Subdivision

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https://www.youtube.com/watch?v=4hbHa8deT90
Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
  - $C^2$ almost everywhere
  - $C^1$ at vertices with valence $\neq 4$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
- Most commonly used subdivision scheme in the movies…
Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others

- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

... which makes differences in ...
  - Provable properties
Loop Subdivision

• Operates on pure triangle meshes

• Subdivision rules
  • Linear subdivision
  • Averaging rules for “even / odd” (white / black) vertices
Loop Subdivision

- Averaging rules
  - Weights for “odd” and “even” vertices

Odd:

Even:
Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:

  a. *Masks for odd vertices*

  b. *Masks for even vertices*
Loop Subdivision

• How to choose $\beta$?
  • Analyze properties of limit surface
  • Interested in continuity of surface and smoothness
  • Involves calculating eigenvalues of matrices

» Original Loop

$$\beta = \frac{1}{n} \left( \frac{5}{8} - \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right)$$

» Warren

$$\beta = \begin{cases} 
\frac{3}{8n} & n > 3 \\
\frac{3}{16} & n = 3 
\end{cases}$$
Loop Subdivision

- Operates only on triangle meshes
- Smoothness of limit surface
  - $C^2$ almost everywhere
  - $C^1$ at vertices with valence $\neq 6$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
Subdivision Schemes

Loop

Catmull-Clark
Subdivision Schemes

Loop

Catmull-Clark
Subdivision Schemes

https://vimeo.com/118340176
Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others

- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

... which makes differences in ...
- Provable properties
Subdivision Schemes

- Other subdivision schemes

<table>
<thead>
<tr>
<th>Primal (face split)</th>
<th>Triangular meshes</th>
<th>Quad Meshes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approximating</td>
<td>Loop($C^2$)</td>
<td>Catmull-Clark($C^2$)</td>
</tr>
<tr>
<td>Interpolating</td>
<td>Mod. Butterfly($C^1$)</td>
<td>Kobbelt($C^1$)</td>
</tr>
</tbody>
</table>

**Dual (vertex split)**

- Doo-Sabin, Midedge($C^1$)
- Biquartic($C^2$)

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Other Subdivision Schemes

- Butterfly subdivision
Other Subdivision Schemes

- Butterfly subdivision
Other Subdivision Schemes

• Butterfly subdivision
Other Subdivision Schemes

- Butterfly subdivision

\[
E_1 = \frac{1}{2} \left( \mathbf{d}_1 + \mathbf{d}_2 \right) + \omega \left( \mathbf{d}_3 + \mathbf{d}_4 \right) - \frac{\omega}{2} \left( \mathbf{d}_5 + \mathbf{d}_6 + \mathbf{d}_7 + \mathbf{d}_8 \right)
\]

\[d'_i = d_i\]
Other Subdivision Schemes

• Butterfly subdivision

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Other Subdivision Schemes

- Vertex-split subdivision (Doo-Sabin, Midedge, Biquartic)

\[ V_2 = \frac{1}{n} \times \sum_{j=1}^{n} d_j \]

\[ E_i = \frac{1}{2} (d_1 + d_{2i}) \]

\[ d'_{1,j} = \frac{1}{4} (d_1 + E_j + E_{j-1} + V_j) \]

One step of Doo-Sabin subdivision

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Other Subdivision Schemes

• Doo-Sabin Subdivision

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Other Subdivision Schemes

Comparison

Meshlab demo

Doo-Sabin
Catmull-Clark
Loop
Butterfly

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Other Subdivision Schemes

- Comparisons:
  - Interpolation vs. smoothness
  - Triangulation dependent asymmetry
  - Shrinking for approximating schemes
  - Similar for smooth meshes with uniform triangles

Loop Butterfly Catmull-Clark

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Drawing Subdivision Surfaces

• Goal:
  • Draw best approximation of smooth limit surface
  • With limited triangle budget
Drawing Subdivision Surfaces

• Goal:
  • Draw best approximation of smooth limit surface
  • With limited triangle budget

• Solution:
  • Stop subdivision at different levels across the surface
  • Stop-criterion depending on quality measure

• Quality of approximation can be defined by
  • Projected (screen) area of final triangles
  • Local surface curvature
Adaptive Subdivision

10072 Triangles

228654 Triangles

[Kobbelt 2000]
Subdivision Surface Summary

• Advantages:
  • Simple method for describing complex surfaces
  • Relatively easy to implement
  • Arbitrary topology
  • Intuitive specification
  • Local support
  • Guaranteed continuity
  • Multi-resolution

• Difficulties:
  • Parameterization
  • Intersections
Comparison

Parametric surfaces
- Provide parameterization
- More restriction on topology of control mesh
- Some require careful placement of control mesh vertices to guarantee continuity (e.g., Bezier)

Subdivision surfaces
- No parameterization
- Subdivision rules can be defined for arbitrary topologies
- Provable continuity for all placements of control mesh vertices
## Comparison

<table>
<thead>
<tr>
<th>Feature</th>
<th>Polygonal Mesh</th>
<th>Parametric Surface</th>
<th>Subdivision Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accurate</td>
<td>No</td>
<td>Yes</td>
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</tr>
<tr>
<td>Concise</td>
<td>No</td>
<td>Yes</td>
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<tr>
<td>Affine invariant</td>
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<tr>
<td>Arbitrary topology</td>
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</tr>
<tr>
<td>Guaranteed continuity</td>
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<td>Yes</td>
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<td>Natural parameterization</td>
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<td>Yes</td>
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</tr>
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