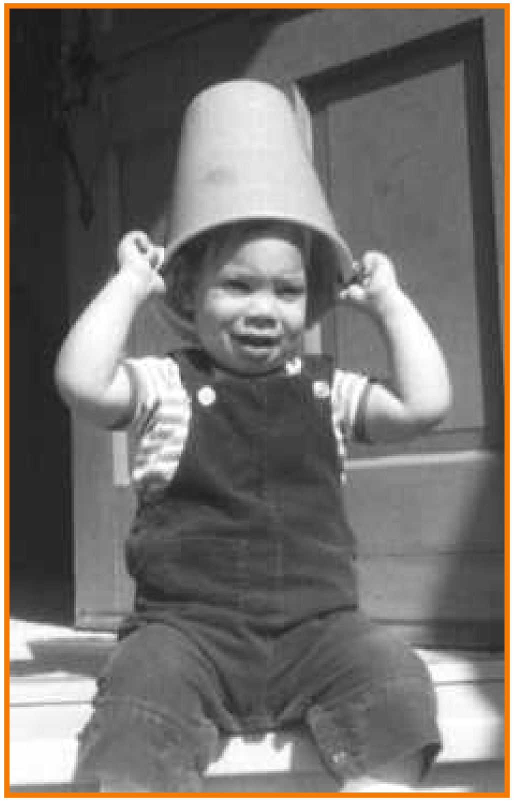
Image Processing

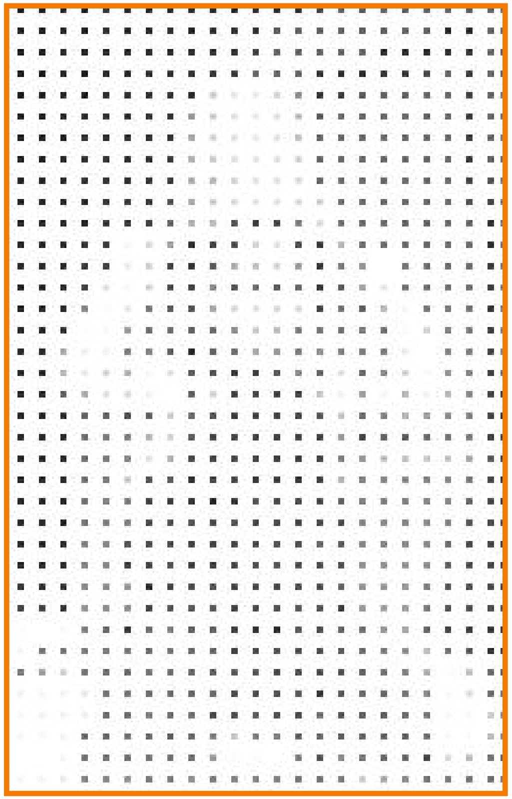
COS 426

What is a Digital Image?

A digital image is a discrete array of samples representing a continuous 2D function



Continuous function



Discrete samples

Limitations on Digital Images

- Spatial discretization
- Quantized intensity
- Approximate color (RGB)
- (Temporally discretized frames for digital video)

Image Processing

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
  - Histogram equalization
- Filtering over neighborhoods
  - Blur & sharpen
  - Detect edges
  - Median
  - Bilateral filter
- Moving image locations
  - Scale
  - Rotate
  - Warp
- Combining images
  - Composite
  - Morph

Similar to Analog / Continuous

- Changing pixel values
  - Linear: scale, offset, etc.
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Account for Limitations of Digital

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
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New Operations

- Changing pixel values
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- Quantization
- Spatial / intensity tradeoff
  - Dithering

Digital Image Processing

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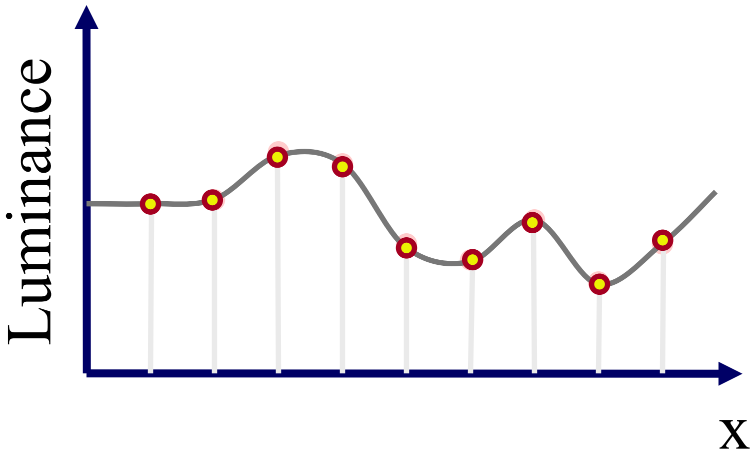
Adjusting Brightness

- What must be done to the RGB values to make this image brighter?

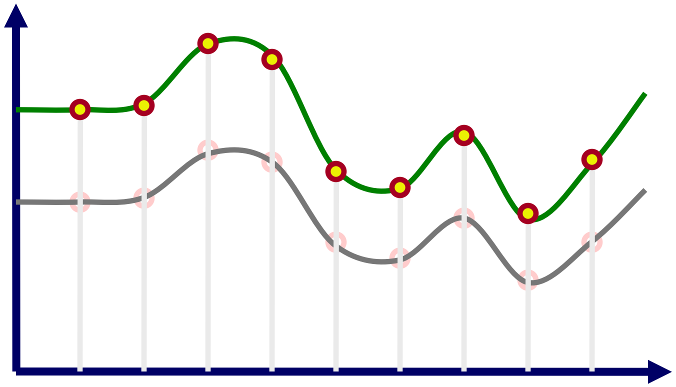


Adjusting Brightness

- Simply scale pixel components
  - o Must clamp to range, e.g. [0..1] or [0..255]


Original


Brighter



Note: this is "contrast" on your monitor!
"Brightness" adjusts black level (offset)

Adjusting Contrast

- Intuitively, “mid-tone” pixels should stay the same, dark ones get darker, light ones get lighter
- Preserve average *luminance*

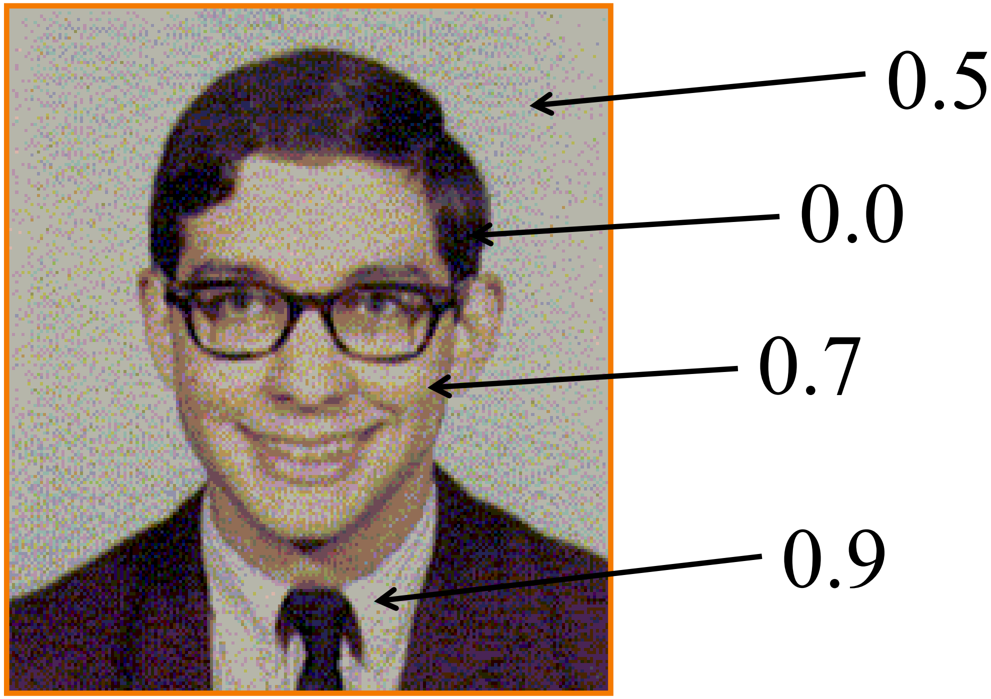
Original

More Contrast

What is Luminance?

Measures perceived “gray-level” of pixel

$$L = 0.30*red + 0.59*green + 0.11*blue$$

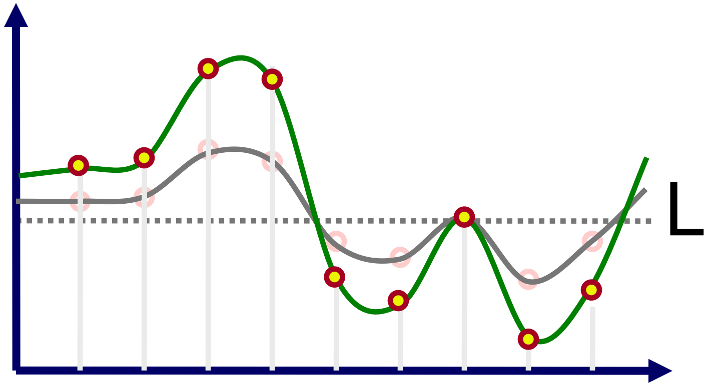


Adjusting Contrast

- Compute mean luminance L for all pixels
  - o luminance = 0.30*r + 0.59*g + 0.11*b
- Scale deviation from L for each pixel component
  - o Must clamp to range (e.g., 0 to 1)

Original

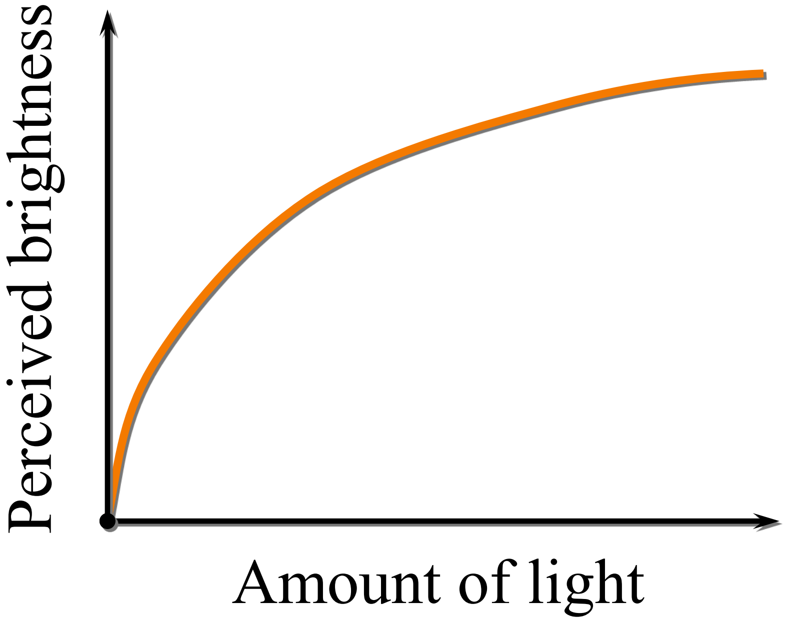
More Contrast



Adjusting Gamma

Function originally accounting for nonlinearity in cameras and displays

$$I_{out} = I_{in}^{\gamma}$$

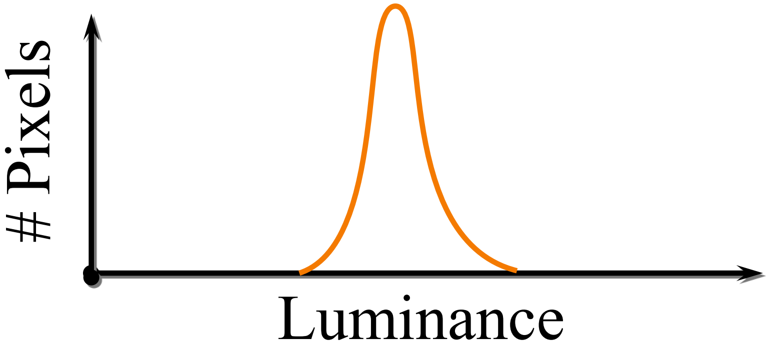


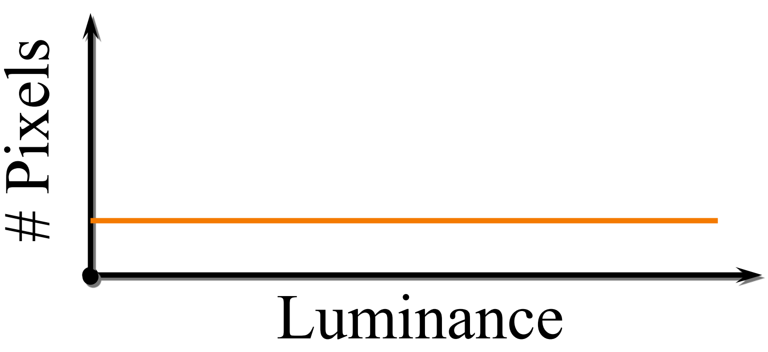
γ depends on camera and monitor

Histogram Equalization

Change distribution of luminance values to cover full range [0-1]

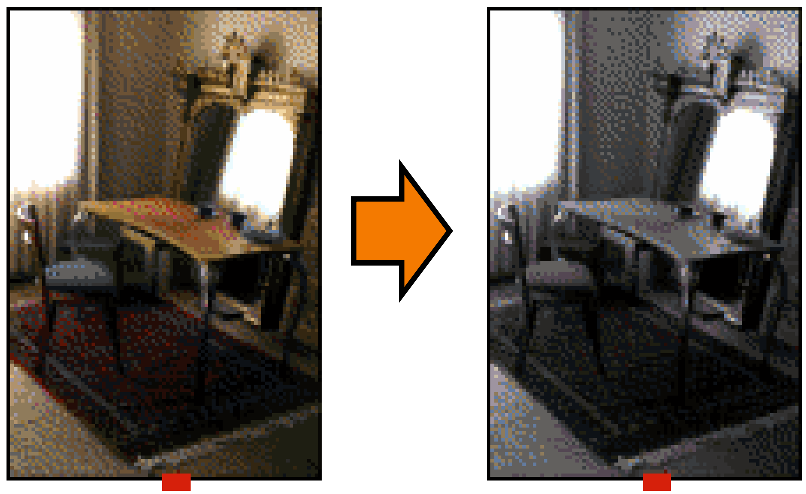
http://en.wikipedia.org/wiki/Histogram_equalization





Grayscale

Convert from color to gray-levels



Original

Grayscale
("black&white" photo)

Compute luminance L, set every pixel to (L,L,L)

Adjusting Saturation

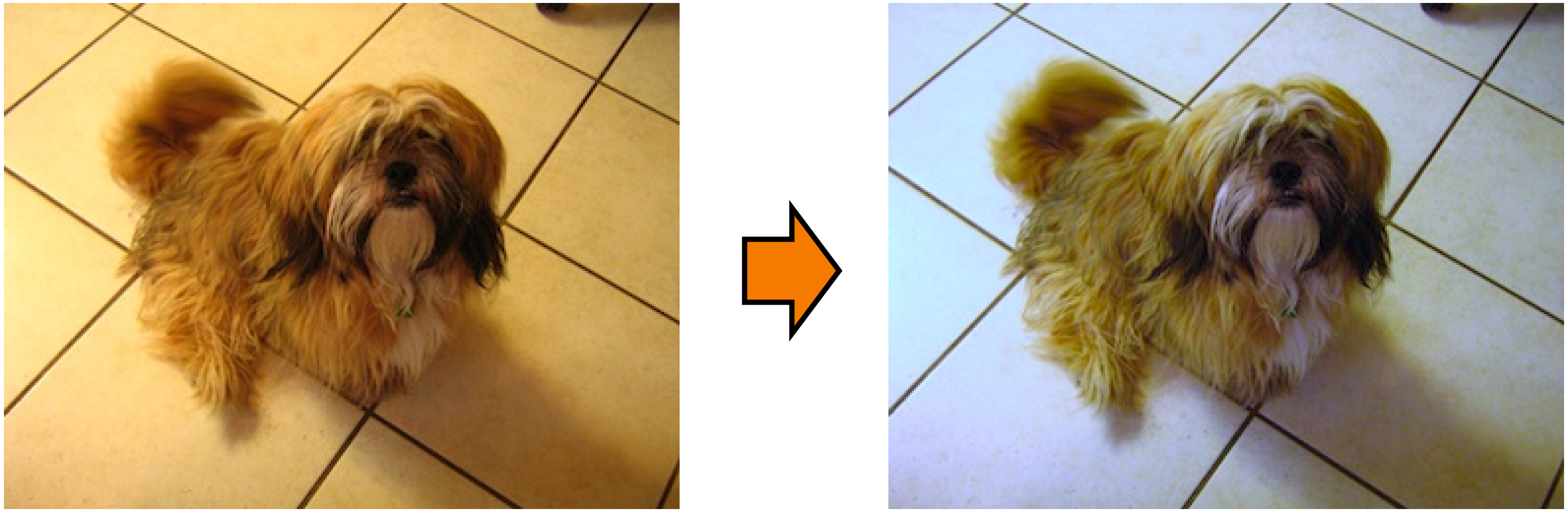
Increase/decrease color saturation of every pixel

-1.0 0.0 0.5 1.0 2.5

Interpolate / extrapolate between image and grayscale version

White Balance

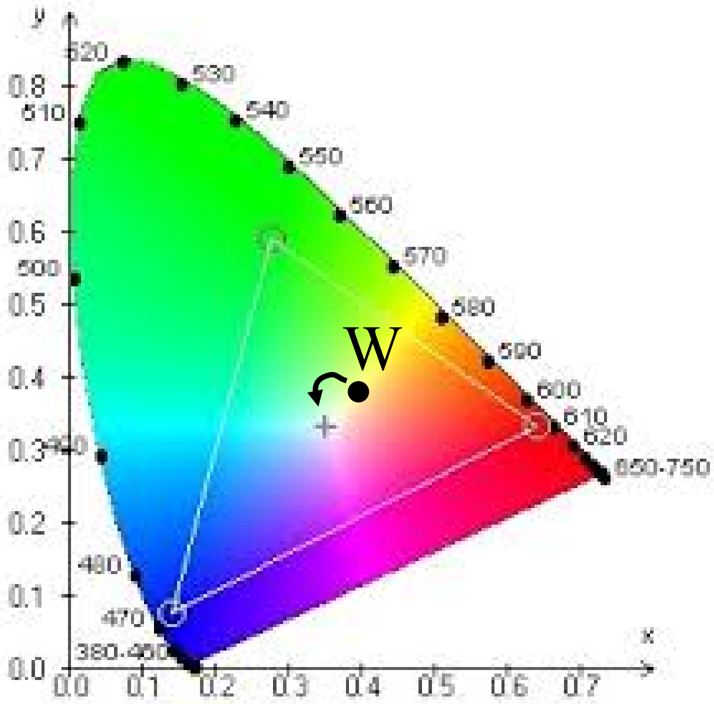
Adjust colors so that a given RGB value is mapped to a neutral color



White Balance

Conceptually:

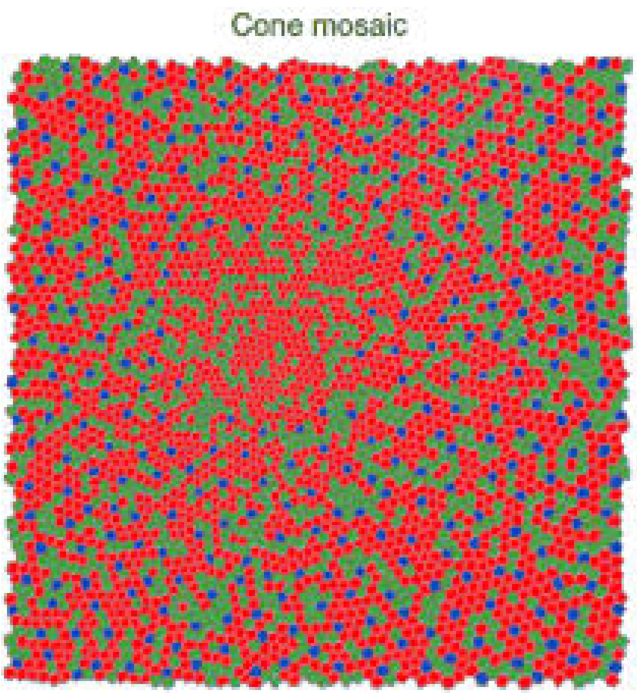
- Provide an RGB value W that should be mapped to white
- Perform transformation of color space

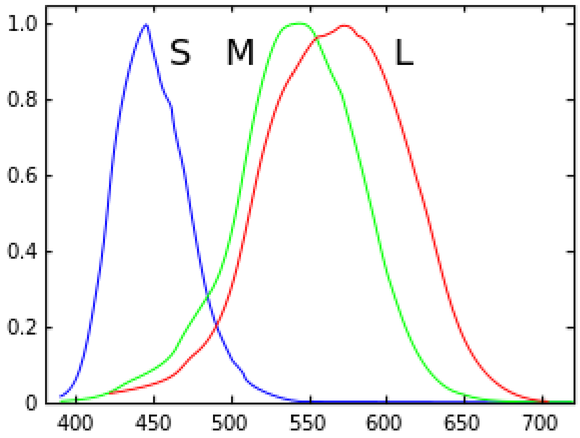


White Balance

Von Kries method: adjust colors in LMS color space

- LMS primaries represent the responses of the three different types of cones in our eyes





White Balance

For each pixel RGB:

1) Convert to XYZ color space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9502 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

2) Convert to LMS color space

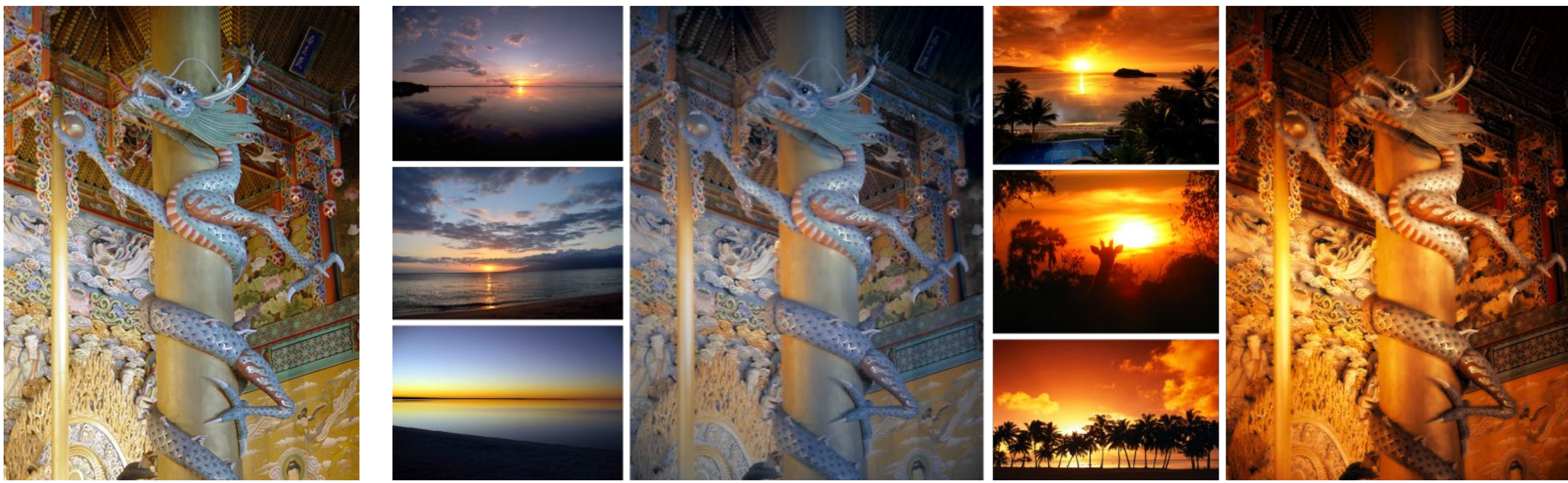
$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.40024 & 0.7076 & -0.08081 \\ -0.2263 & 1.16532 & 0.0457 \\ 0 & 0 & 0.91822 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3) Divide by $L_W M_W S_W$

4) Convert back to RGB

Color Histogram Transfer

Adjust colors so that their distribution (histogram) matches a target distribution



Source image Target colors Result Target colors Result

Fancier version of this idea from "AutoStyle: Automatic Style Transfer from Image Collections to Users' Images" by Princeton student Yiming Liu et al.

Digital Image Processing

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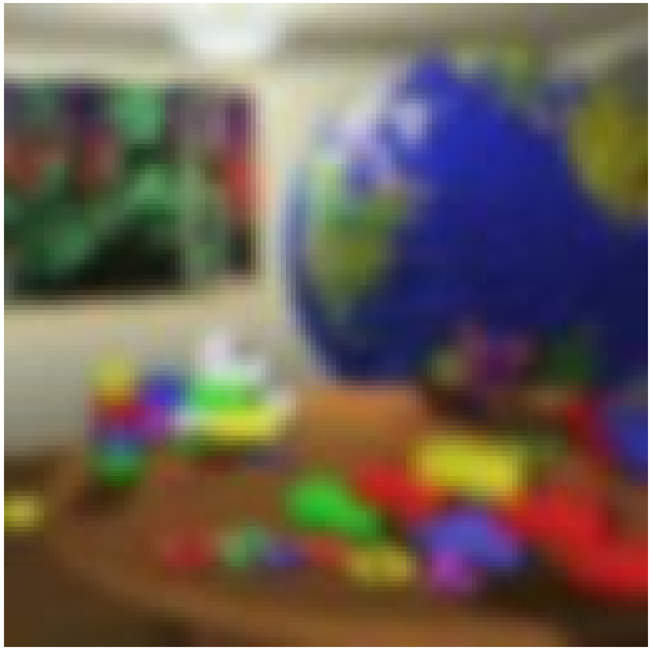
Blur

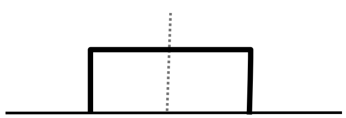
What is the basic operation for each pixel when blurring an image?



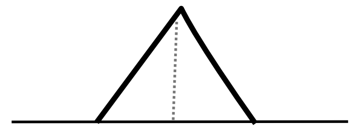
Basic Operation: Convolution

Output value is weighted sum of values in neighborhood of input image

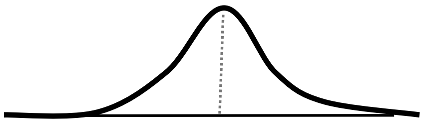
- Pattern of weights is the “filter” or “kernel”



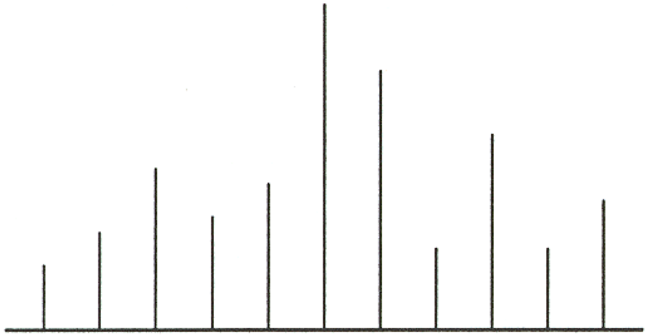
Box Filter



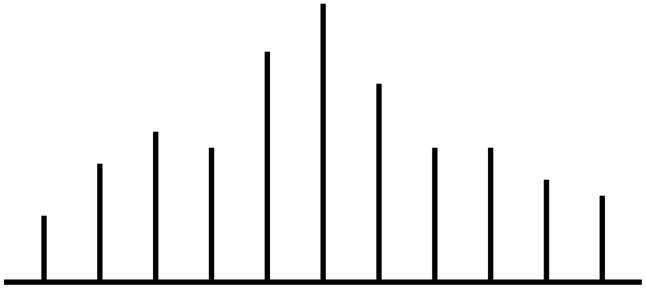
Triangle Filter



Gaussian Filter



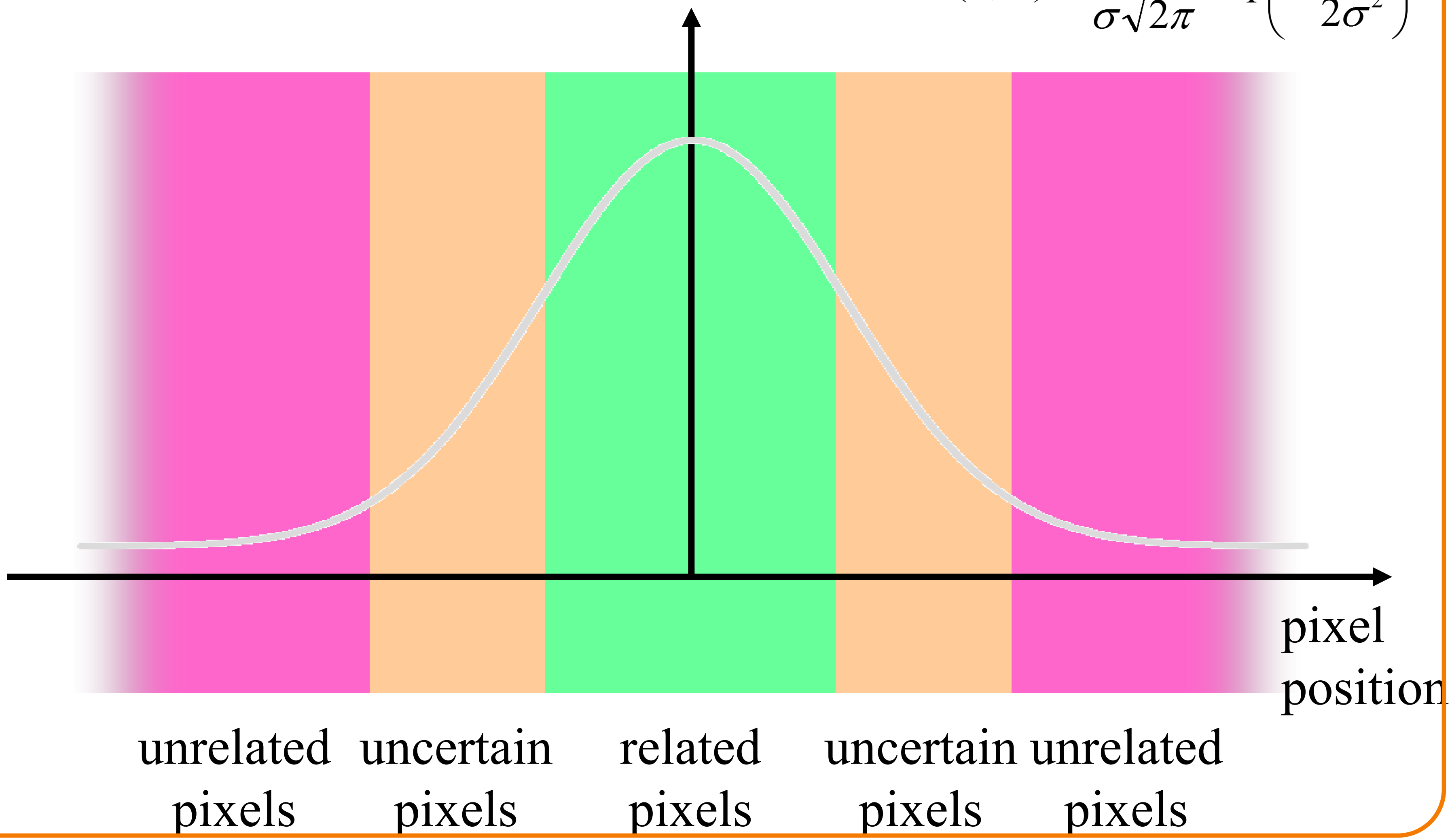
Input



Output

Convolution with a Gaussian Filter

$$G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



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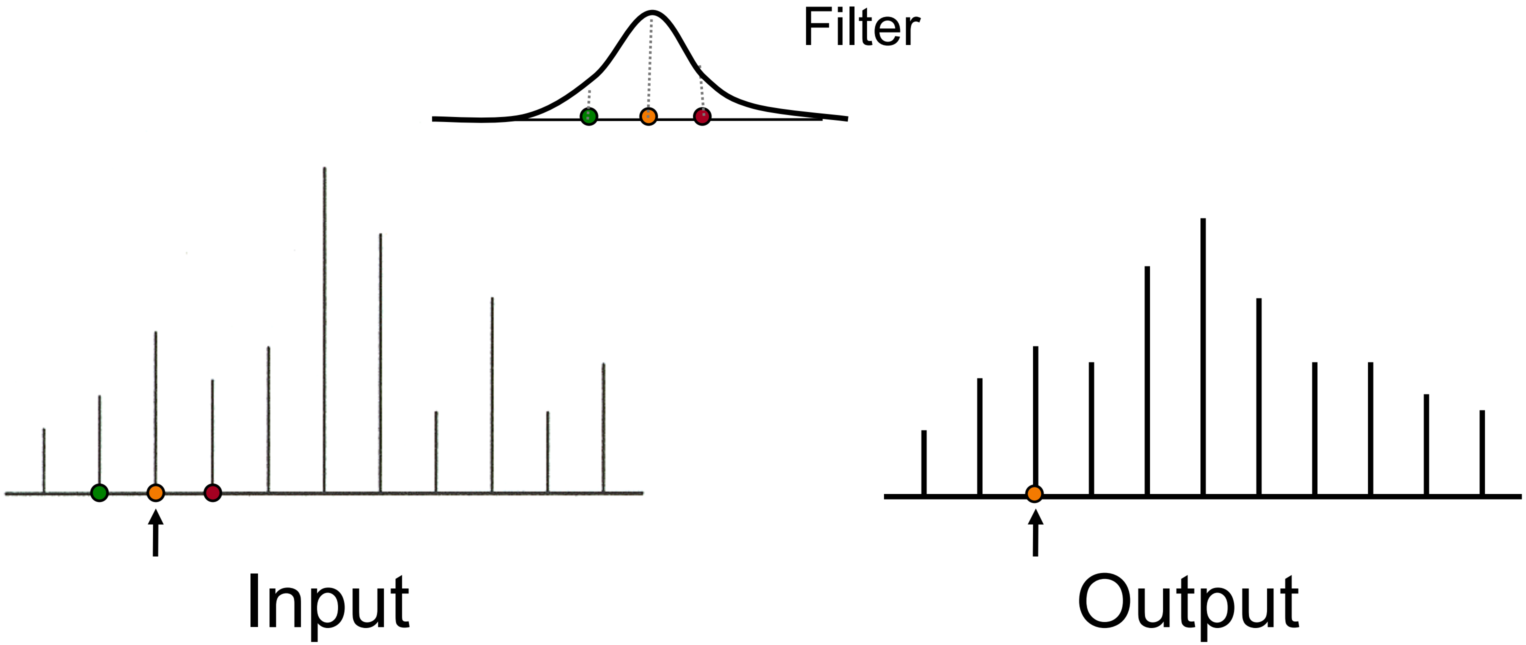

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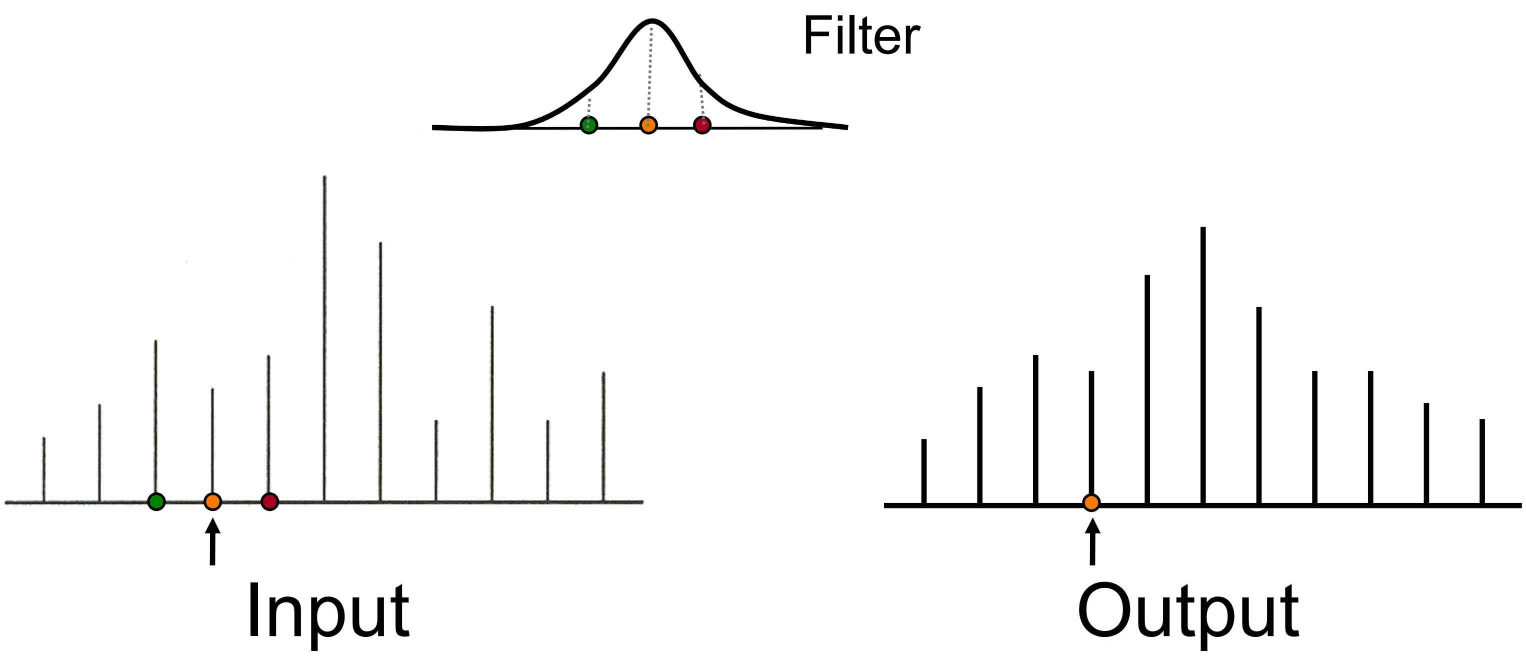

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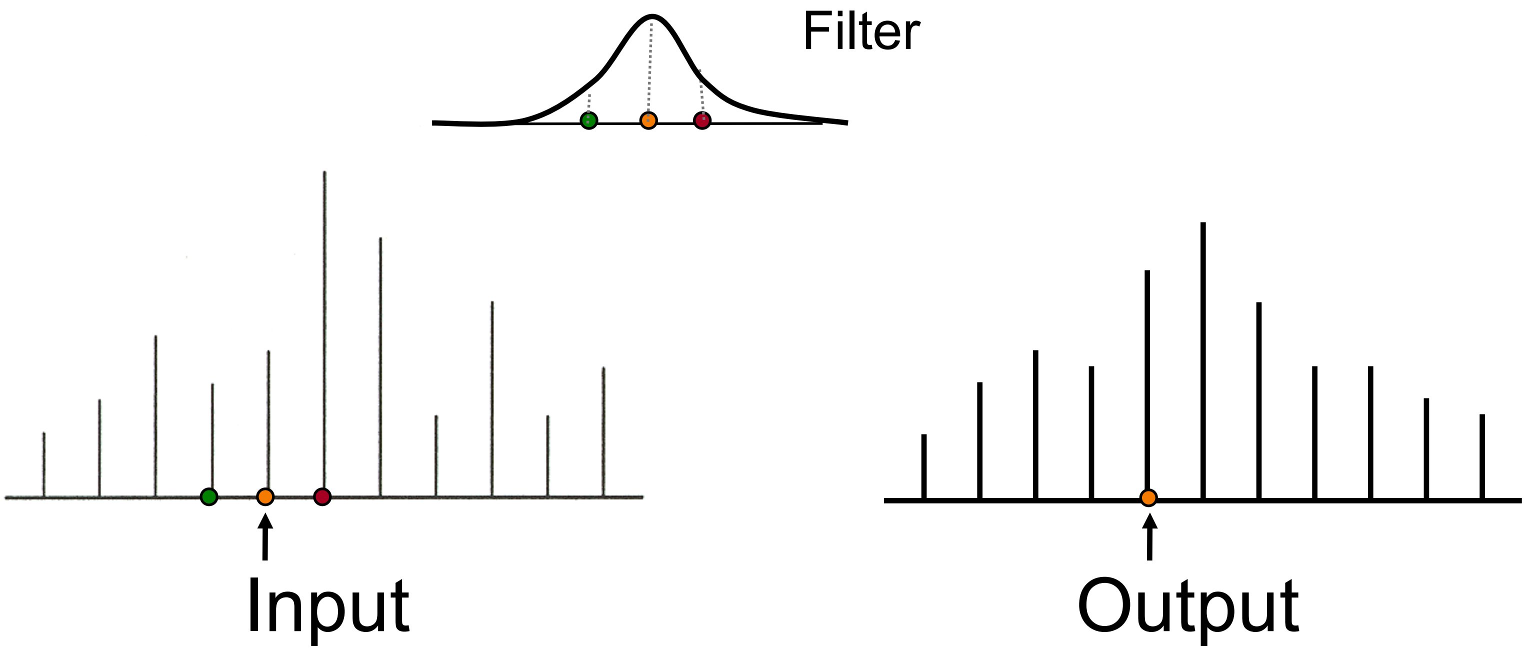

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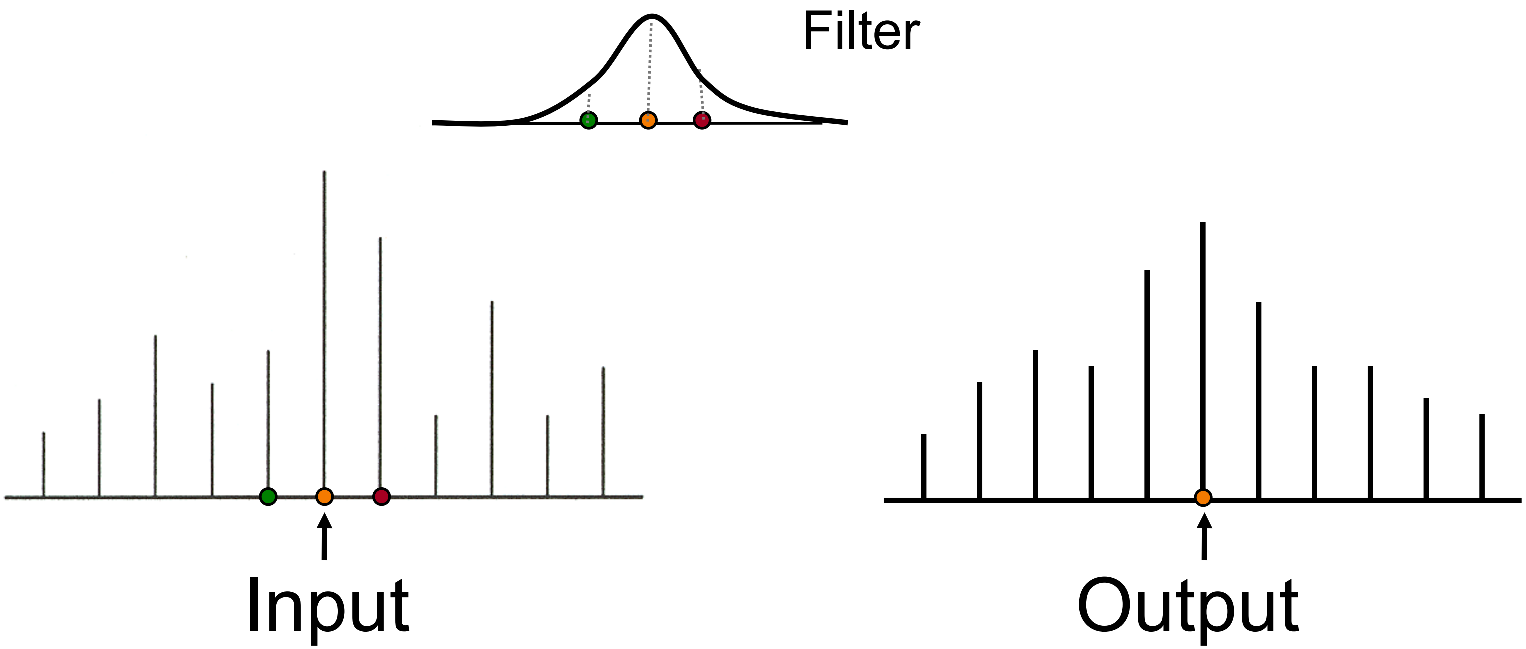

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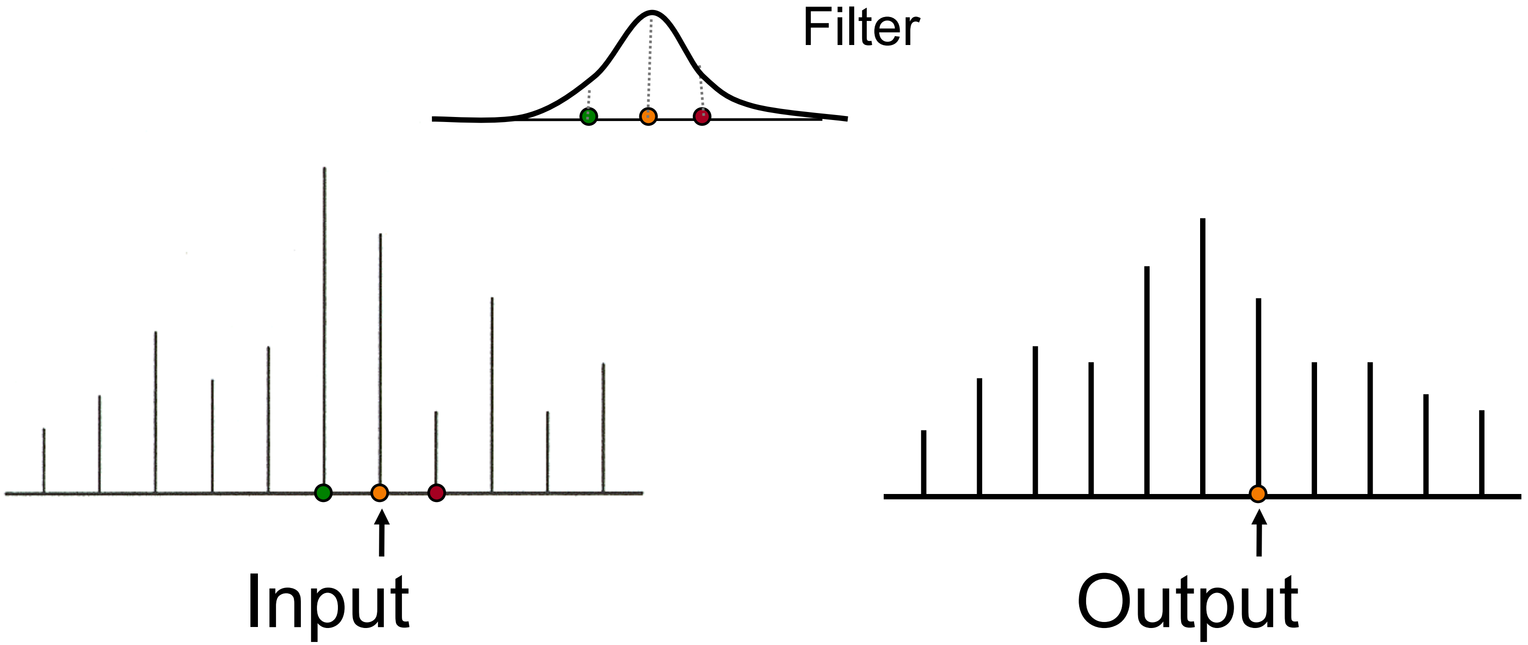
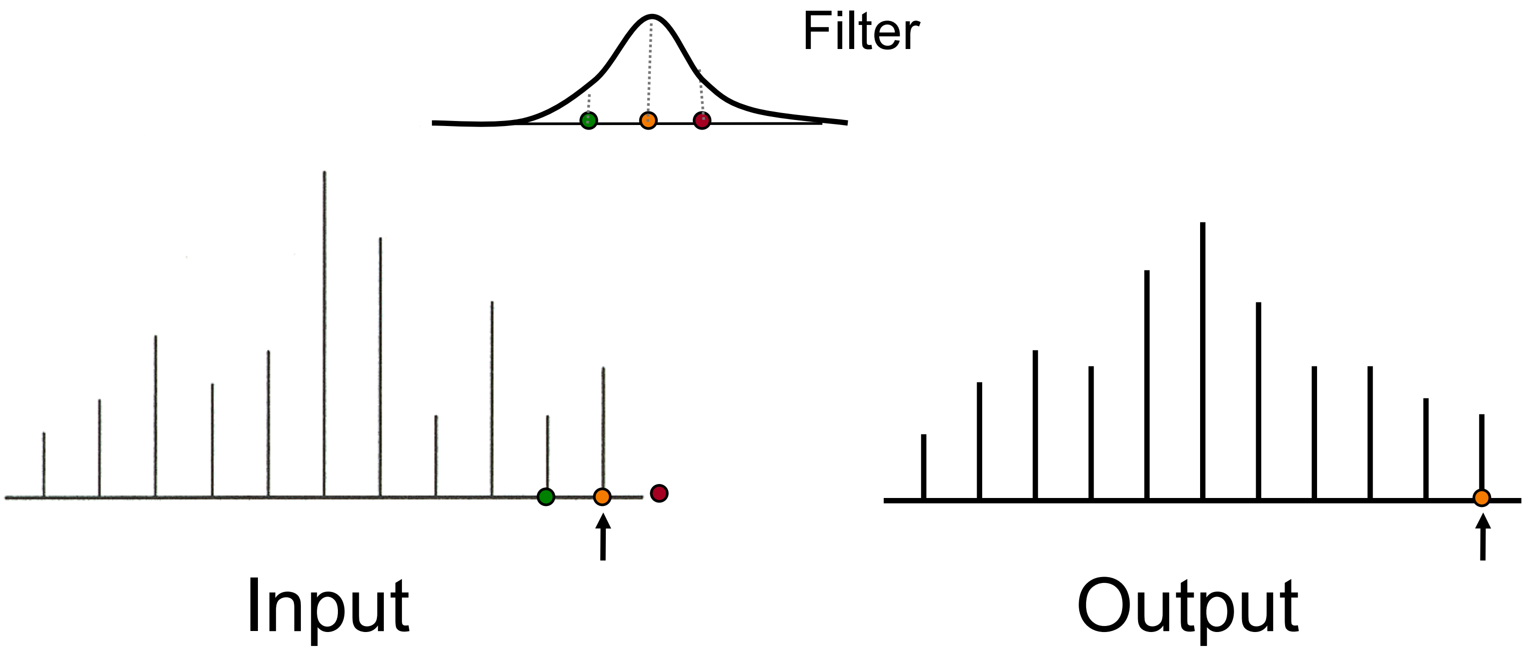


Convolution with a Gaussian Filter

What if filter extends beyond boundary?

$$G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



Convolution with a Gaussian Filter

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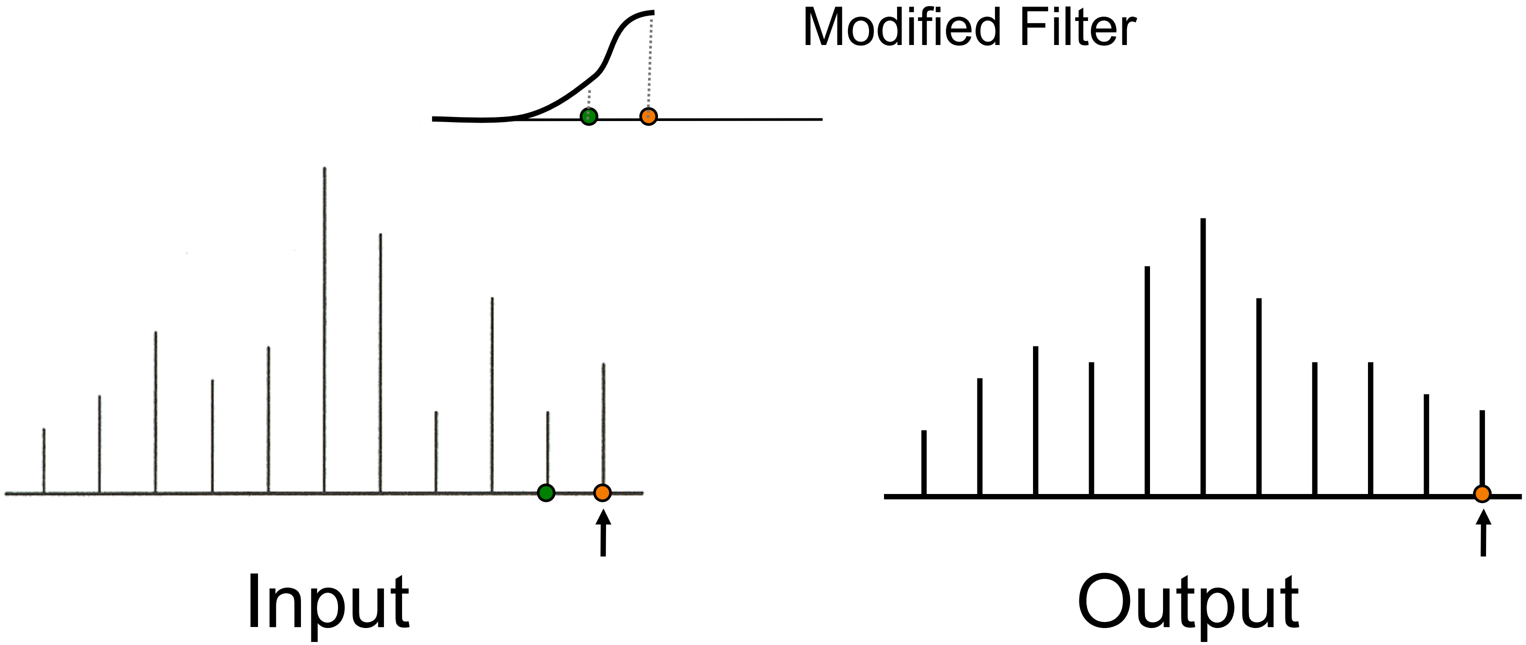


Convolution with a Gaussian Filter

Output contains samples from smoothed input

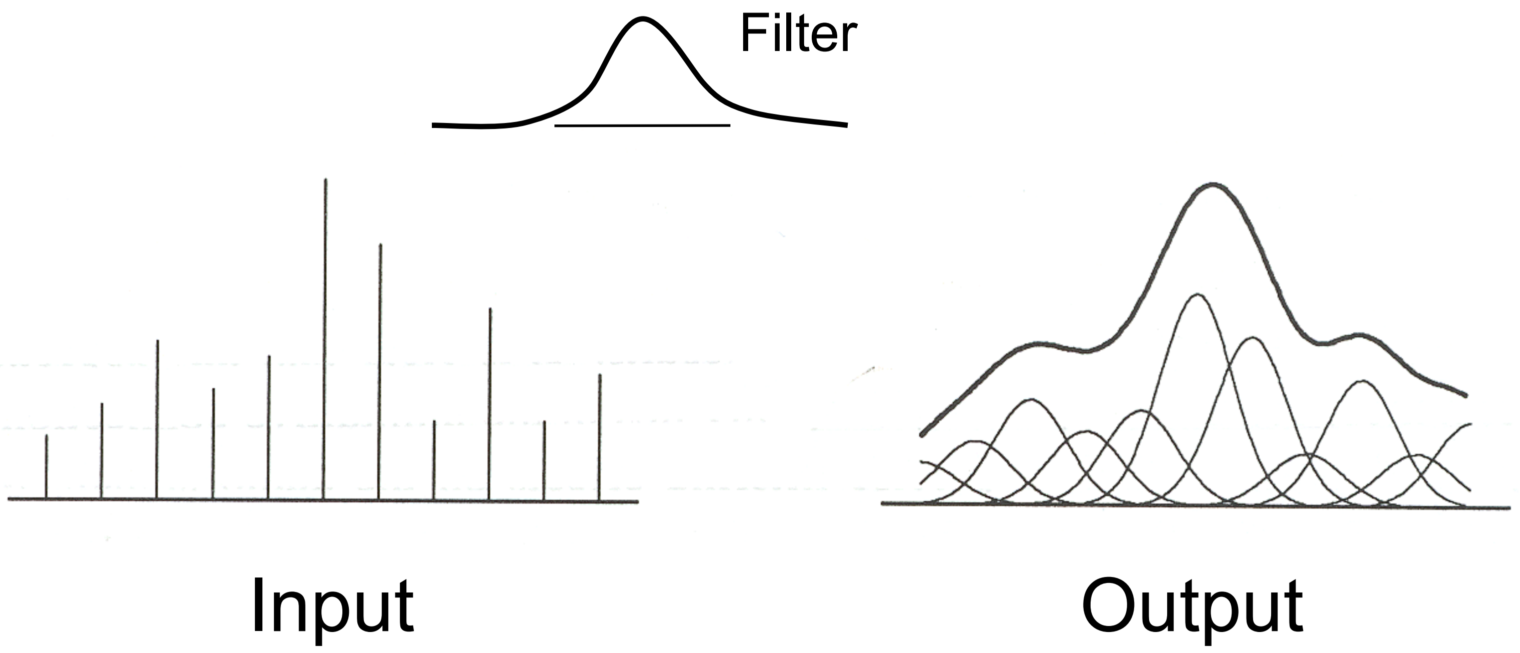
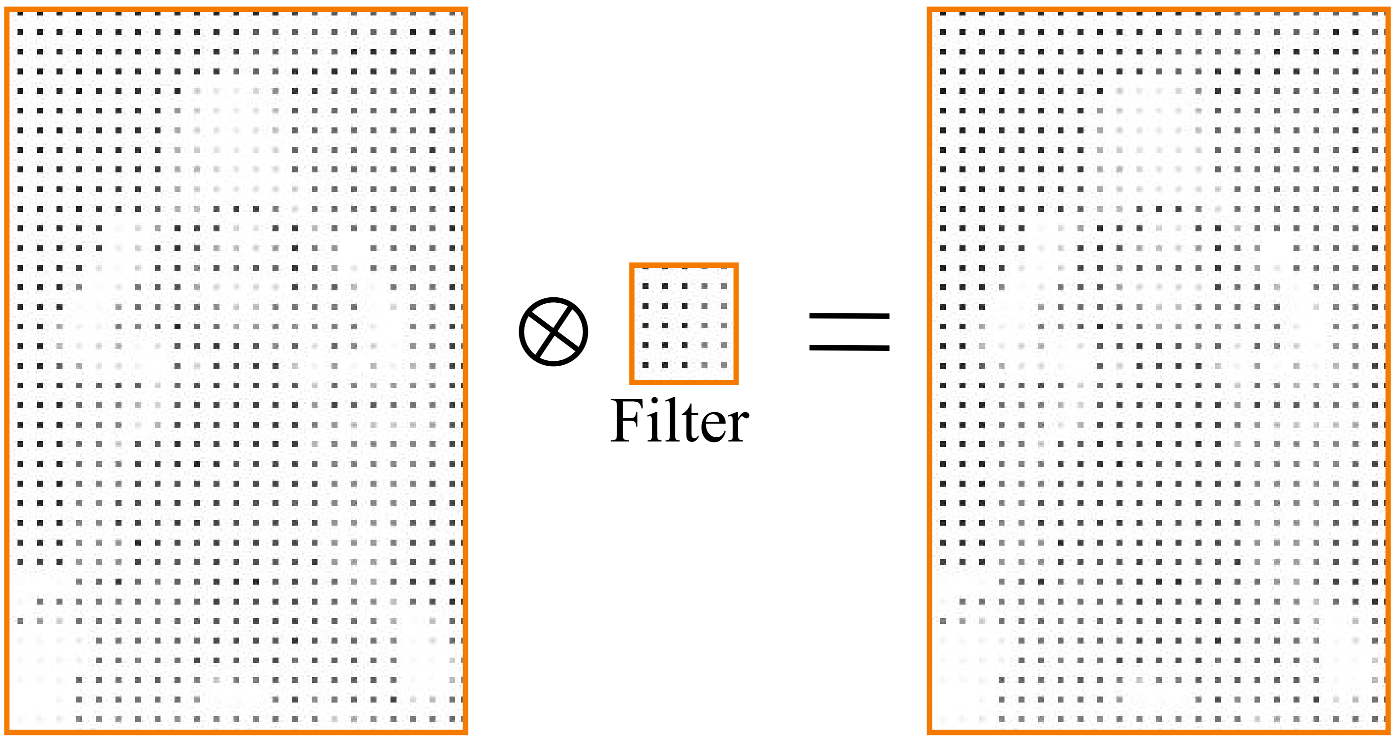


Figure 2.4 Wolberg

Linear Filtering

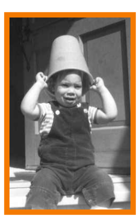
2D Convolution

- Each output pixel is a linear combination of input pixels in 2D neighborhood with weights prescribed by a filter



Input Image

Output Image



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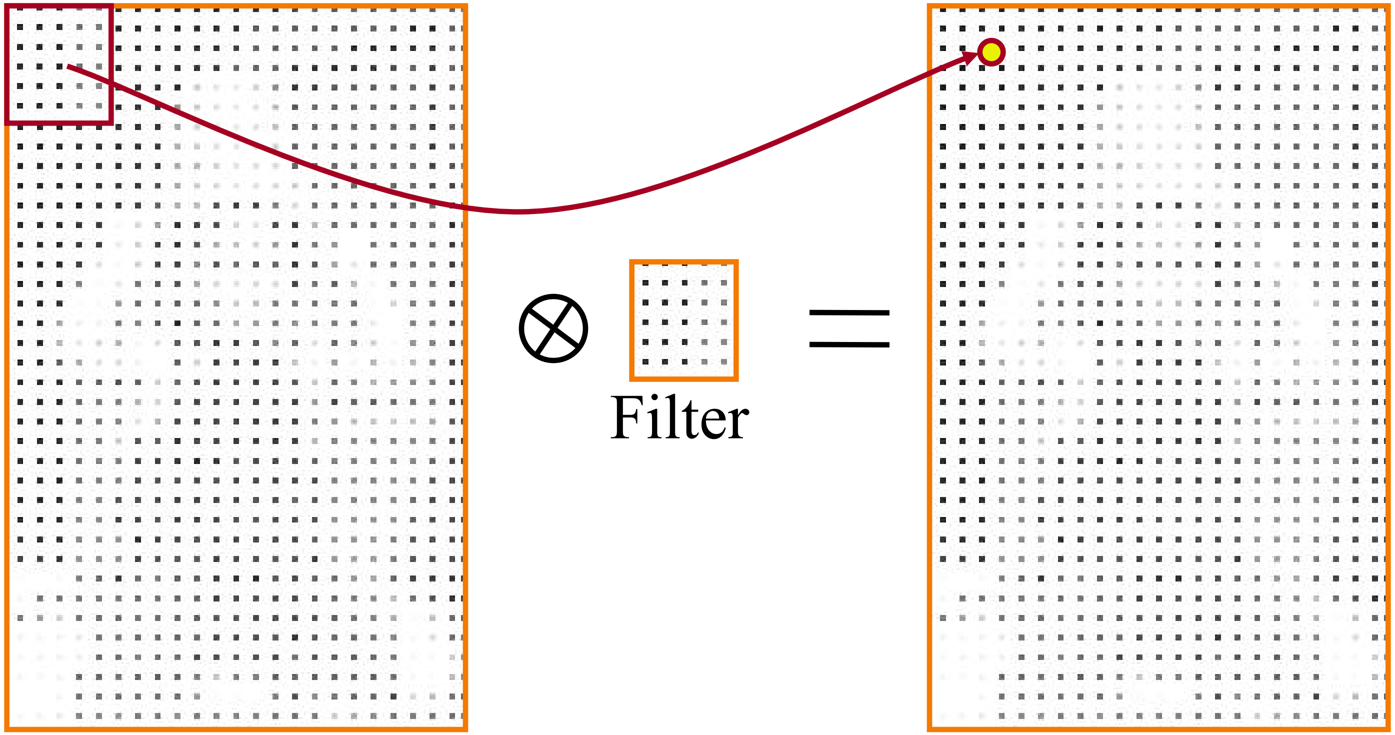
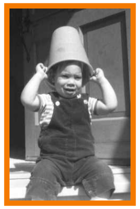


Input Image

Output Image



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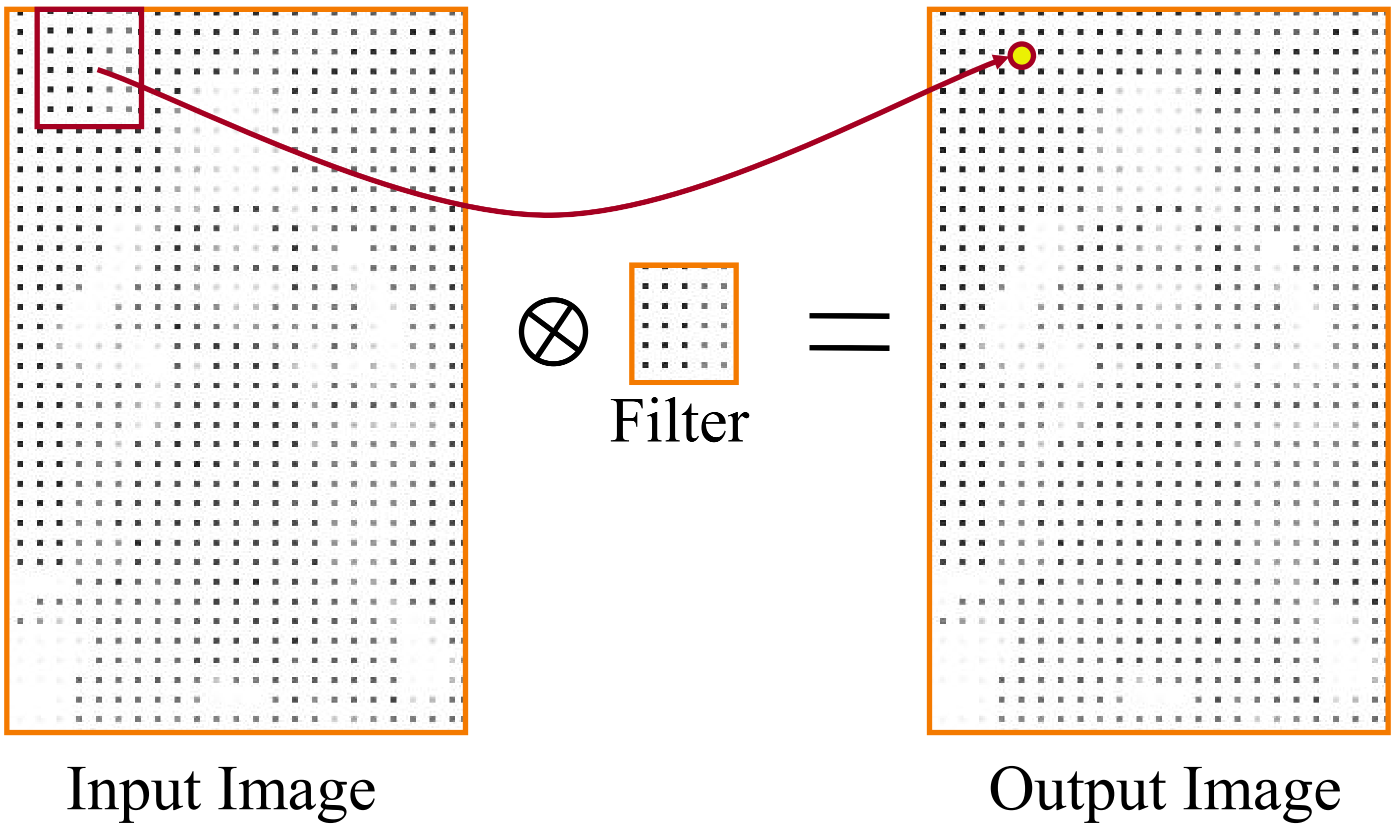

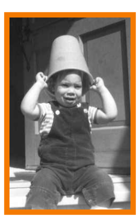

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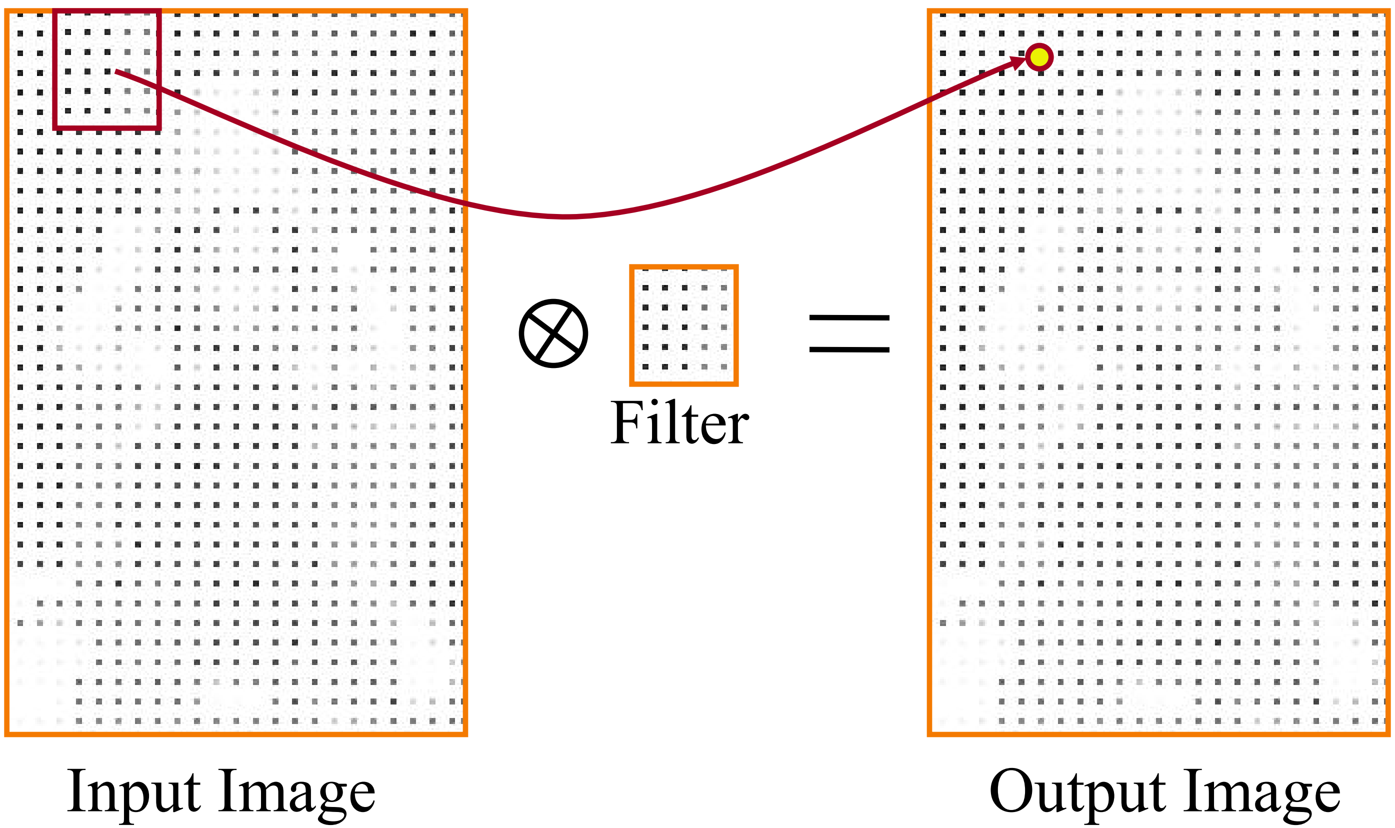

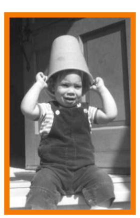

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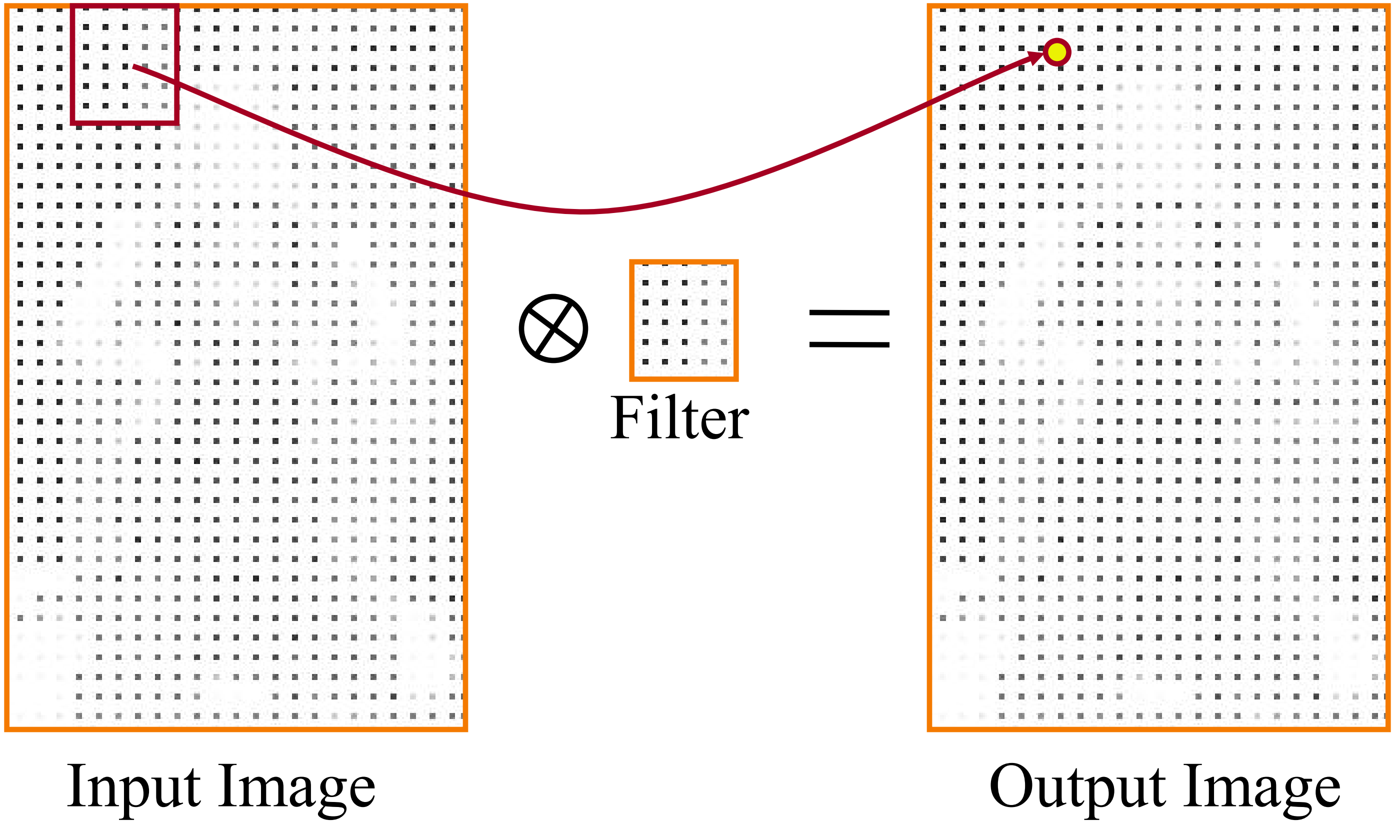
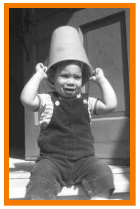
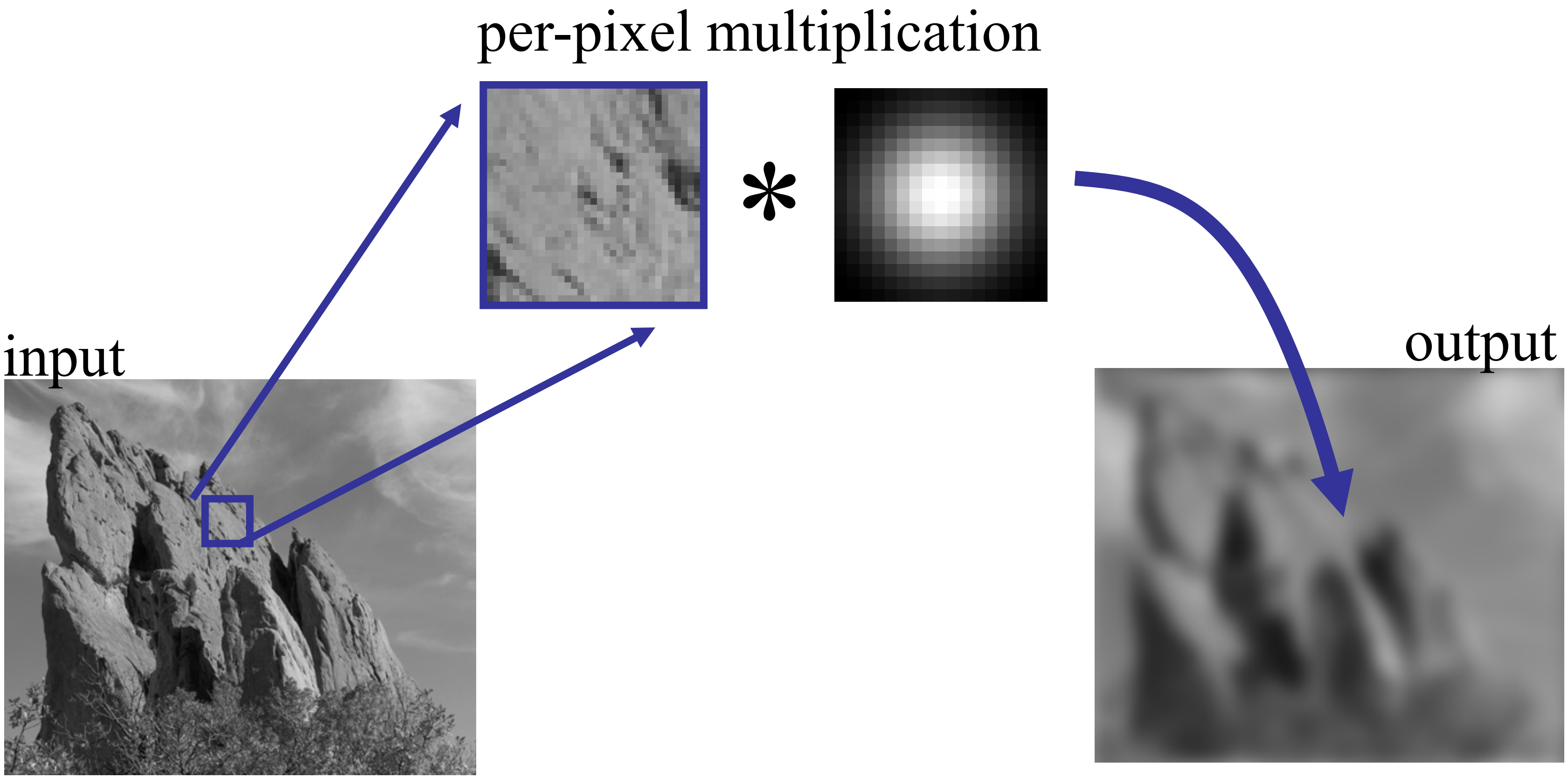




Gaussian Blur

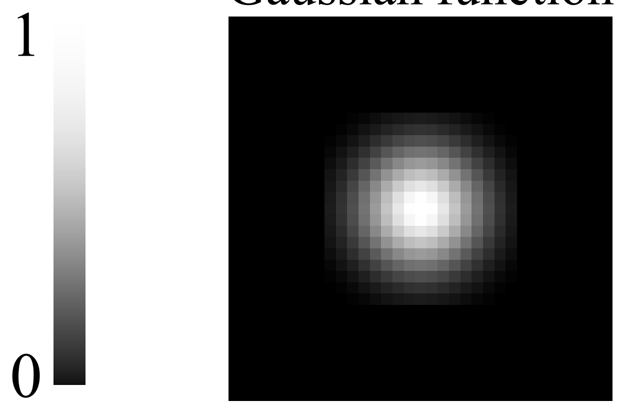


Gaussian Blur

Output value is weighted sum of values in neighborhood of input image

$$Blur(I_{\mathbf{p}}, \sigma) = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G(\| \mathbf{p} - \mathbf{q} \|, \sigma) \, I_{\mathbf{q}}$$

normalized Gaussian function




input


Gaussian blur

Linear Filtering

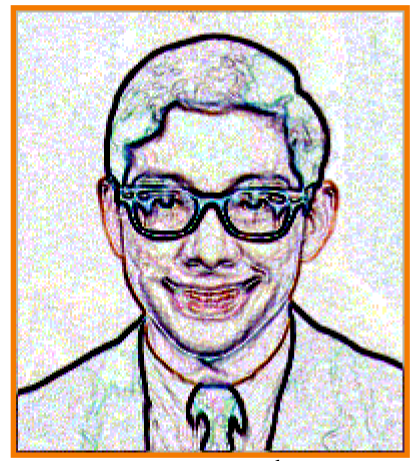
- Many interesting linear filters
  - Blur
  - Edge detect
  - Sharpen
  - Emboss
  - etc.

$$\text{Filter} = \quad ?$$

Edge Detection

Convolve with a 2D Laplacian filter that finds differences between neighbor pixels

Original



Detect edges

$$\text{Filter} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & +8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Sharpen

Sum detected edges with original image

Original

Sharpened

$$\text{Filter} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Emboss

Convolve with a filter that highlights gradients in particular directions

Original

Embossed

$$\text{Filter} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Side Note: Separable Filters

Some filters are separable (e.g., Gaussian)

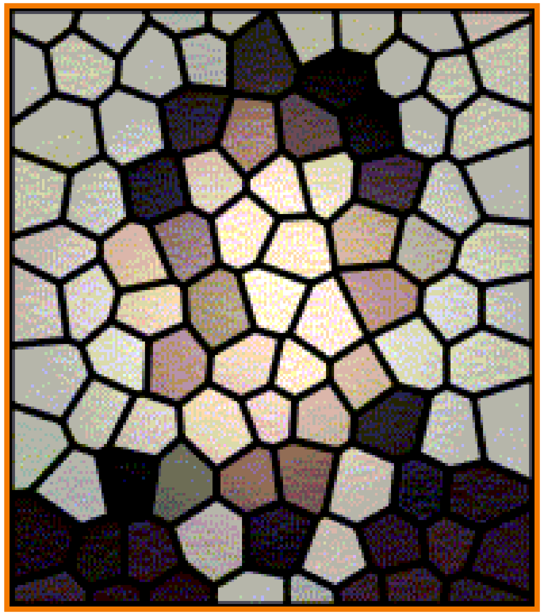
- First, apply 1-D convolution across every row
- Then, apply 1-D convolution across every column
- **HUGE** impact on performance (when kernel is big)

Non-Linear Filtering

Each output pixel is a non-linear function of input pixels in neighborhood (filter depends on input)

Original

Paint



Stained Glass

Median or “Despeckling” Filter

Each output pixel is median of input pixels in neighborhood

original image

1px median filter

3px median filter

10px median filter

Bilateral Filter

Gaussian blur uses same filter for all pixels

- Blurs across edges as much as other areas

Original

Gaussian Blur

Bilateral Filter

Gaussian blur uses same filter for all pixels

Prefer a filter that preserves edges (adapts to content)

Original

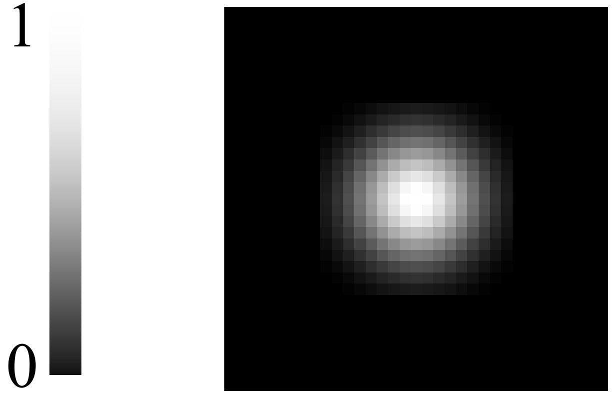
Bilateral Filter

Recall: Gaussian Blur

Output value is weighted sum of values in neighborhood of input image

$$Blur(I_{\mathbf{p}},\sigma)=\frac{1}{W_{\mathbf{p}}}\sum_{\mathbf{q}\in S}\boxed{G(\|\mathbf{p}-\mathbf{q}\|,\sigma)}\,I_{\mathbf{q}}$$

normalized Gaussian function



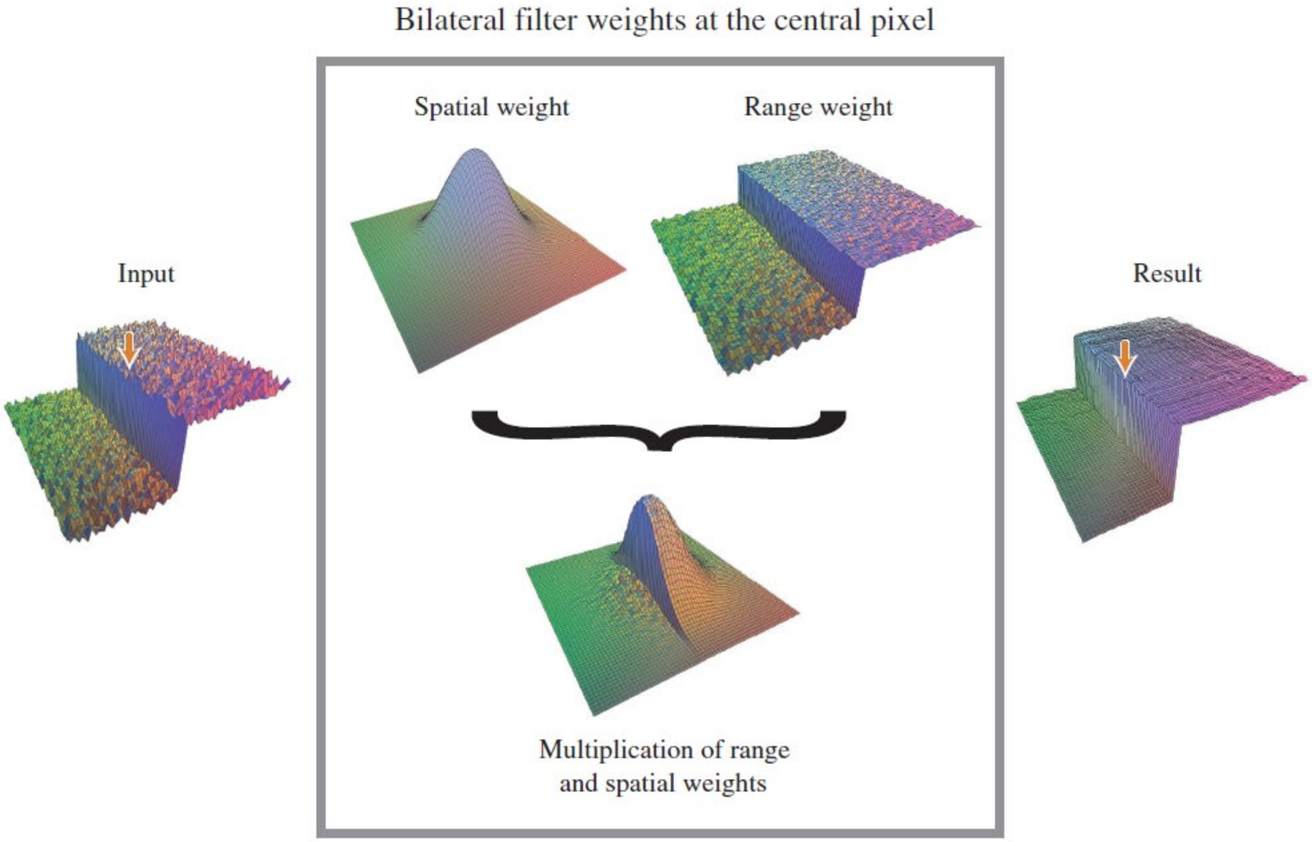
Bilateral Filter

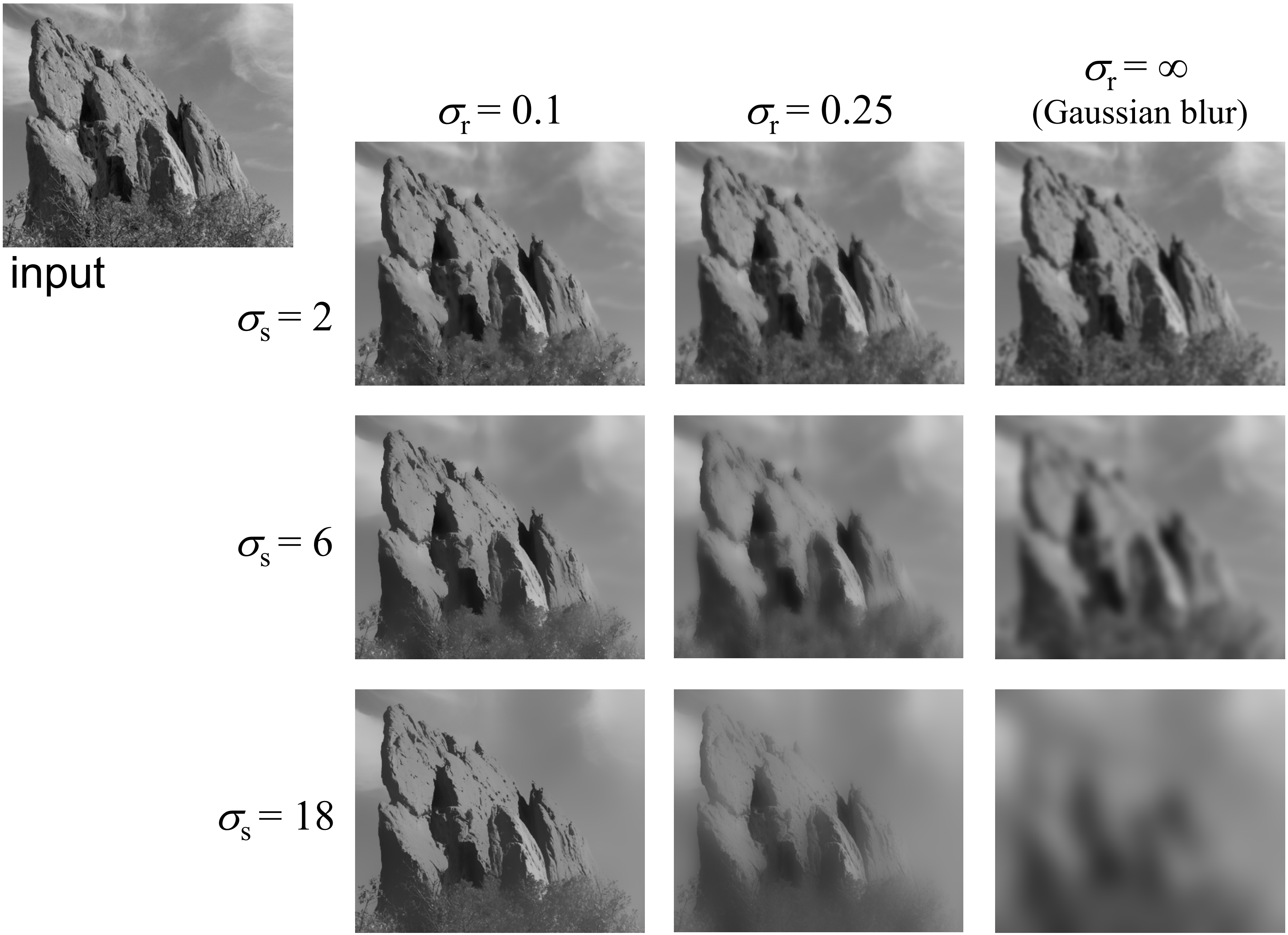
Combine Gaussian filtering in both spatial domain and color domain

$$Bilateral[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{Spatial Proximity Weight}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)}_{\text{Color Proximity Weight}} I_{\mathbf{q}}$$

Bilateral Filtering

Combine Gaussian filtering in both spatial domain and color domain




input
σr = 0.1
σr = 0.25
σr = ∞
(Gaussian blur)
σs = 2
σs = 6
σs = 18

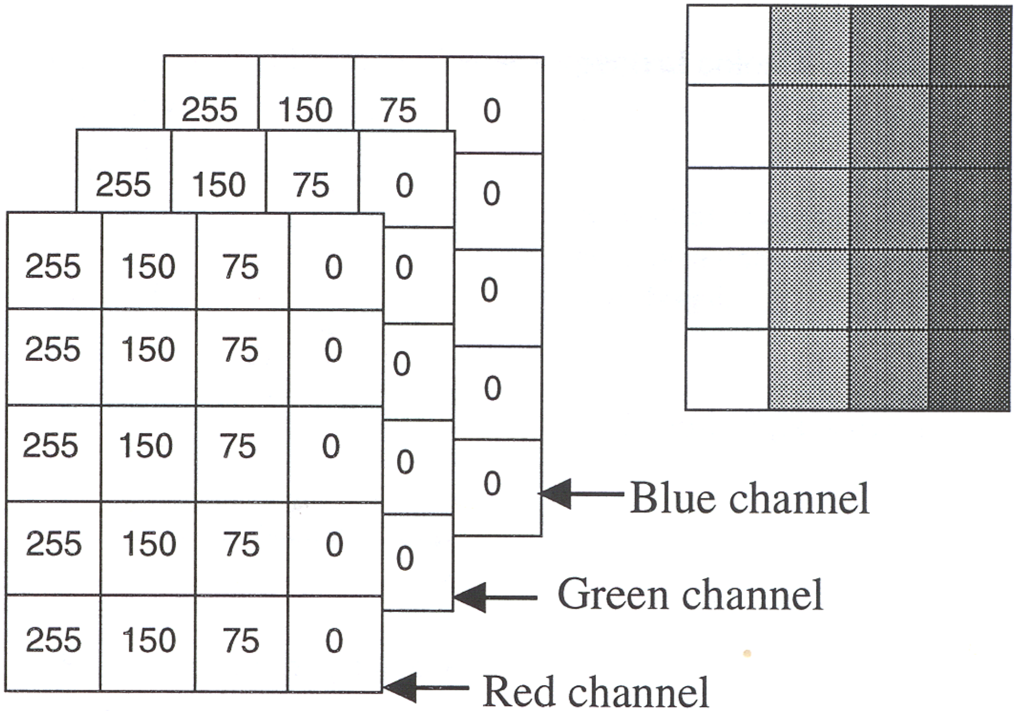
Digital Image Processing

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
  - Histogram equalization
- Filtering over neighborhoods
  - Blur & sharpen
  - Detect edges
  - Median
  - Bilateral filter
- Moving image locations
  - Scale
  - Rotate
  - Warp
- Combining images
  - Composite
  - Morph
- Quantization
- Spatial / intensity tradeoff
  - Dithering

Quantization

Reduced intensity resolution

- Frame buffers have limited number of bits per pixel
- Physical devices have limited dynamic range



Effects of Quantization

8 bits / pixel / color

6 bits / pixel / color

Marc Levoy / Hanna-Barbera

Effects of Quantization

5 bits / pixel / color

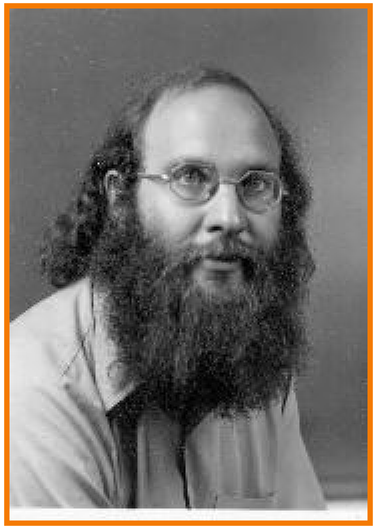
4 bits / pixel / color

Marc Levoy / Hanna-Barbera

Dithering

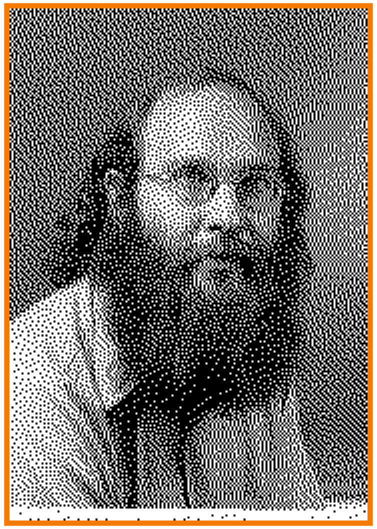
Distribute errors among pixels

- Exploit spatial integration in our eye
- Display greater range of perceptible intensities
- Trade off spatial resolution for intensity resolution



Original
(8 bits)

Uniform
Quantization
(1 bit)



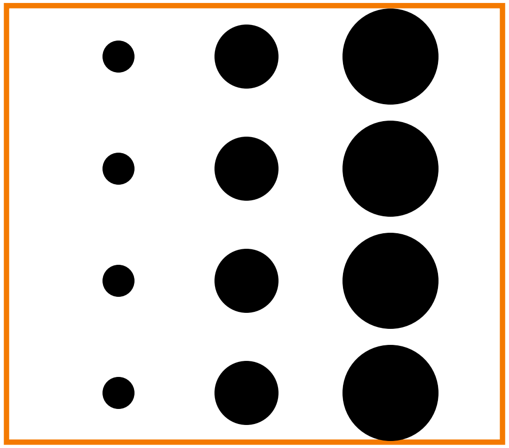
Floyd-Steinberg
Dither
(1 bit)

Classical Halftoning

Use dots of varying size to represent intensities

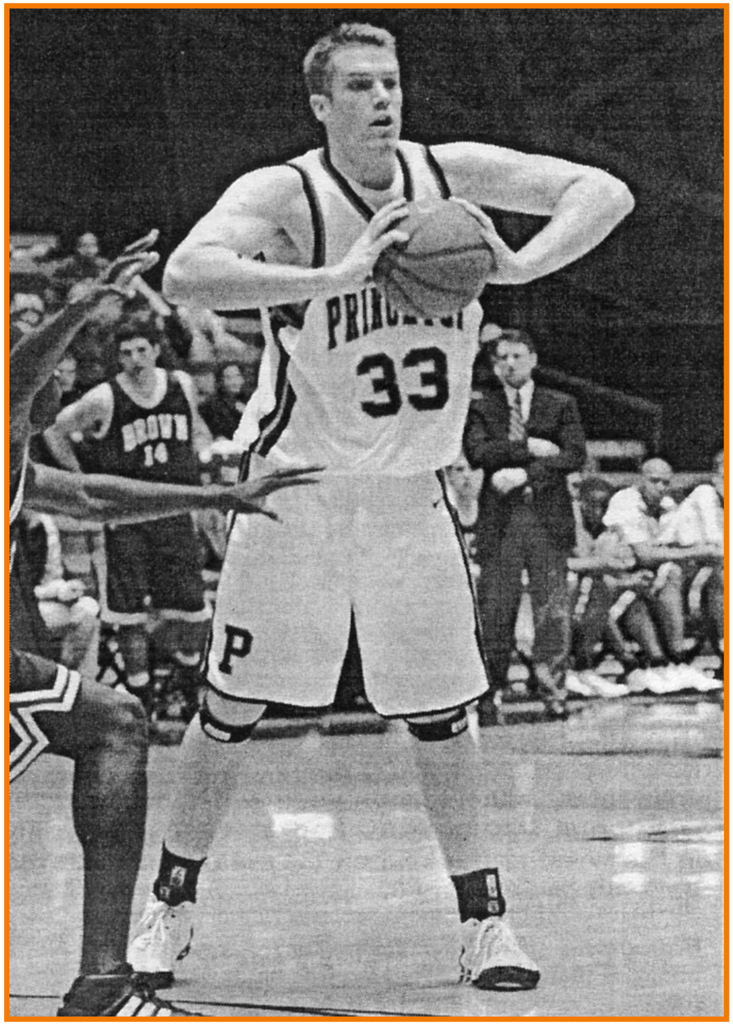
- Area of dots proportional to intensity in image

I(x,y)

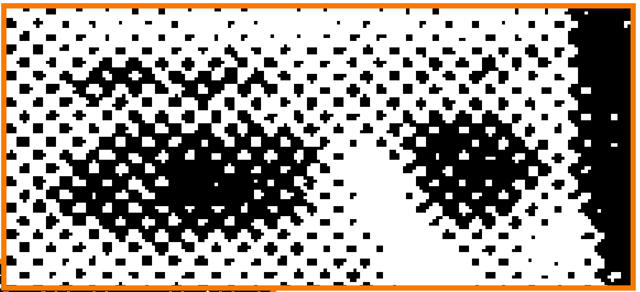


P(x,y)

Classical Halftoning



From *Town Topics*, Princeton



Digital Halftone Patterns

Use cluster of pixels to represent intensity

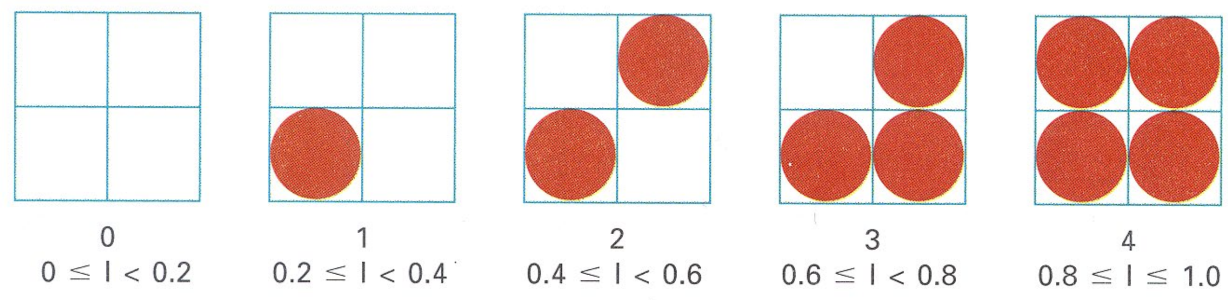
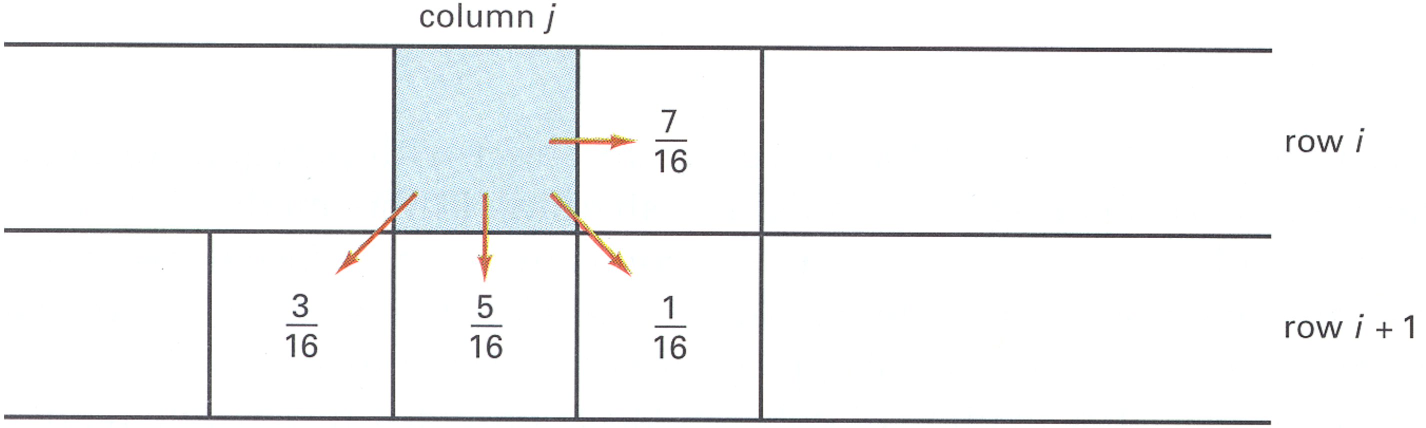


Figure 14.37 from H&B

Error Diffusion Dither

Spread quantization error over neighbor pixels

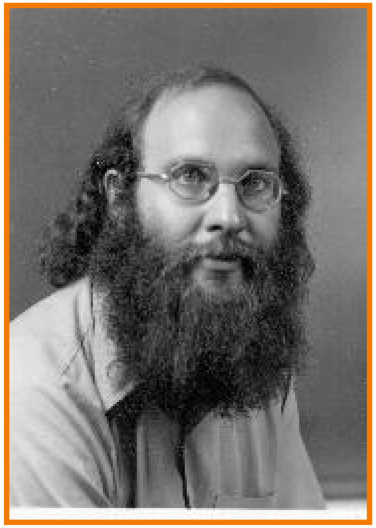
- Error dispersed to pixels right and below
- Floyd-Steinberg weights:



$$3/16 + 5/16 + 1/16 + 7/16 = 1.0$$

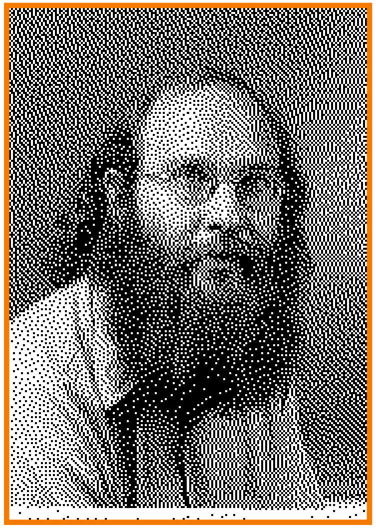
Figure 14.42 from H&B

Error Diffusion Dither



Original
(8 bits)

Uniform
Quantization
(1 bit)



Floyd-Steinberg
Dither
(1 bit)

Next Time…

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
  - Histogram equalization
- Filtering over neighborhoods
  - Blur & sharpen
  - Detect edges
  - Median
  - Bilateral filter
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