# COS126 Number Systems Activity — Booksite **6.1**

## Binary and Decimal

1. What is the binary integer 101, represented in decimal?   $4 + 1 = \mathbf{5}$

2. What is the binary integer 1010, represented in decimal?   $8 + 2 = \mathbf{10}$.
   (How is this related to the previous answer?) Twice as much as 101

3. What is the binary integer 10100, represented in decimal?   **20**.
   (What is the pattern?) Again twice as much since all ones became twice as valuable

4. What is the binary integer 101001, represented in decimal?   **41**. Twice as much plus one.
   (Could you write a program to use this approach?) Yes, and it is useful in LFSR!

5. What is the decimal integer 116, represented in binary?

   Right to left: see "Converting from decimal to base b" on booksite §5.1. 116 is even, so ends in a
   0, preceded by representation of $116/2 = 58$. 58 even so it ends in a 0, etc. $\Rightarrow$ **1110100**
   Left to right: biggest power of 2 that fits ($\leq 116$) is 64, leaving 116-64 = 52. Biggest power of 2
   in this remainder is 32. Keep going with remainders, 116=64+32+16+4 = binary **1110100**.

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## Binary and Hex

6. What are the hexadecimal numbers C, D, and E, expressed in binary? These are twelve, thirteen,
   fourteen, which are **1100, 1101, 1110**.

7. Express the hexadecimal number C0DE as a sum of 4 terms corresponding to the 4 digits.
   What is the value of this expression when converted to binary? Note that $16 = 2^4, 16^3 = 2^{12}$
   and $\times 2$ shifts us left by one position. C0DE is $12 \times 16^3 + 0 \times 16^2 + 13 \times 16^1 + 14 \times 16^0$
   = **1100 0000 1101 1110**  ($C$ 0 $D$ $E$)

8. What is the binary number 100100110, represented in hexadecimal? (Avoid using decimal.) Reverse the previous process. 1 0010 0110 and converting each 4 bits to a hex digit, **126**

## Bitwise Operators (In Q9 thru Q13, all numbers are in binary)

9. What is the binary value of 1010 | 110?     **1110**

10. What is the binary value of 1010 & 110?     **10**

11. What is the binary value of 1010 << 10?     **101000**

12. What is the binary value of 1010 >> 10?     **10**

13. What is the binary value of 1010 ∧ 110?     **1100**

14. What is the value, expressed in hexadecimal, of C05126 ∧ CBE245 ∧ C05126? (What is the trick?)
    Since the order of inputs to xor doesn't matter, this equals CBE245 ∧ C05126 ∧ C05126. Since anything xor'ed with itself is 0, this is CBE245 ∧ 0 = **CBE245**

## 16-bit Two's-Complement Representations

15. What is the complement of 0101 0000 1100 1111?     **1010 1111 0011 0000**

16. Give the **16-bit two's-complement** binary representation of the decimal integer 116
    (Use question 5) **0000 0000 0111 0100**

17. Give the 16-bit two's-complement binary representation of the decimal integer $-116$ First complement the bits of $+116$, then add one, giving **1111 1111 1000 1100**

18. What is the 16-bit two's-complement **hexadecimal** representation of the decimal integer $-116$?
    Like Q8 (converting each 4 bits to a hex digit) **FF8C**

19. What is the decimal representation of the 16-bit two's-complement hexadecimal number FFFE?
    Since the first bit is 1, this number is negative. Call this negative number $X$. Then the binary representation of the positive number $-X$ is obtained by flipping bits (0000 0000 0000 0001) and adding one (0000 0000 0000 0010). So $-X$ is 2, i.e. $X$ is $-\mathbf{2}$.

## Challenges (Read Booksite § **6.1**)

20. What should the binary numbers 0.1 and 0.01 represent? In decimal these are $10^{-1}$ and $10^{-2}$. In binary these are likewise $2^{-1} = 1/2$ and $2^{-2} = 1/4$