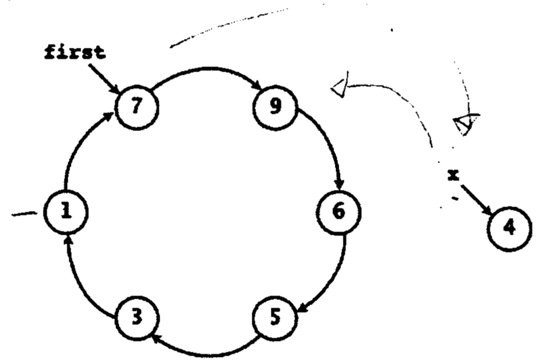
COS 126
Spring 2016
Written Exam 2 solutions

Q11. Searching and Sorting (7 points). To the right of each option, fill in the circle corresponding to the one-word characterization that best describes the order of growth of the ***worst-case*** running time. *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

	logarithmic ($\log n$)	linear (n)	linearithmic ($n \log n$)	quadratic (n^2)
A. mergesort	○	○	●	○
B. merge	○	●	○	○
C. binary search	●	○	○	○
D. BST search	○	●	○	○
E. insertion sort	○	○	○	●
F. sequential search	○	●	○	○
G. bubble sort	○	○	○	●

Q12. Linked Structures (7 points). Suppose that the Node data type is defined as:

```
private class Node {
    private int item;
    private Node next;
}
```



and that first is a variable of type Node that refers to one node in a circular linked list. Let x be a variable that refers to a newly created Node.

```
Node x = new Node();
x.item = 4;
```

For each of the **independent** code fragments below, fill in the circle that describes its effect. Fill in "neither" if the code invalidates the property that first refers to a node in a circular linked list. *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

	inserts x after first	inserts x before first	neither
`x.next = first.next;` `first.next = x;`	●	○	○

`Node z = first;`
`while(z.next != first) z = z.next;`
`z.next = x;`
`x.next = first;`
	○	●	○

`first.next = x;` `x.next = first.next;`	○	○	●

`x = first;`
`x.next = first;`
x refers to same Node as first so setting next to itself?
	○	○	? → ●

`first.next = x;` breaks circle	○	○	●

`Node z = first;`
`x.next = first.next;`
`z.next = x;` ?
	●	○	○

`x.item = first.item;`
`first.item = x.item;`
`first.next = x;`
first.item = 7
first.next = x
	○	○	●

Q13. Binary Search Trees (7 points). *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

A. Suppose that we create a BST by inserting integers into an initially empty tree. For each of the following insertion orders, fill in the bubble corresponding to the height of the binary tree that is produced (max distance from the root to any node) when keys are inserted in that order into an initially empty tree. The first answer is provided for you.

	1	2	3	4	5	6	7
`10, 11, 20, 30, 32, 48, 60`	○	○	○	○	○	○	●
`10, 20, 32, 11, 30, 48, 60`	○	○	○	○	●	○	○
`30, 10, 20, 11, 60, 48, 32`	○	○	○	●	○	○	○
`20, 11, 10, 30, 32, 60, 48`	○	○	○	○	●	○	○

B. Suppose that you are searching for the key `70` in a binary search tree. In the following list, indicate for each sequence whether it could be the sequence of keys examined, by filling in the corresponding bubble.

	Possible	Not possible
`77, 41, 99, 20, 85, 70`	○	●
`99, 10, 80, 20, 60, 70`	●	○
`5, 10, 80, 40, 32, 50, 70`	○	●
`22, 58, 81, 70`	●	○

Q14. REs (7 points). Let $L = \{aaaba, aabaa, abbba, ababa, aaaaa\}$. For each of the regular expressions below fill in the only answer that applies. *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

The possible options (and their shortnames) are:

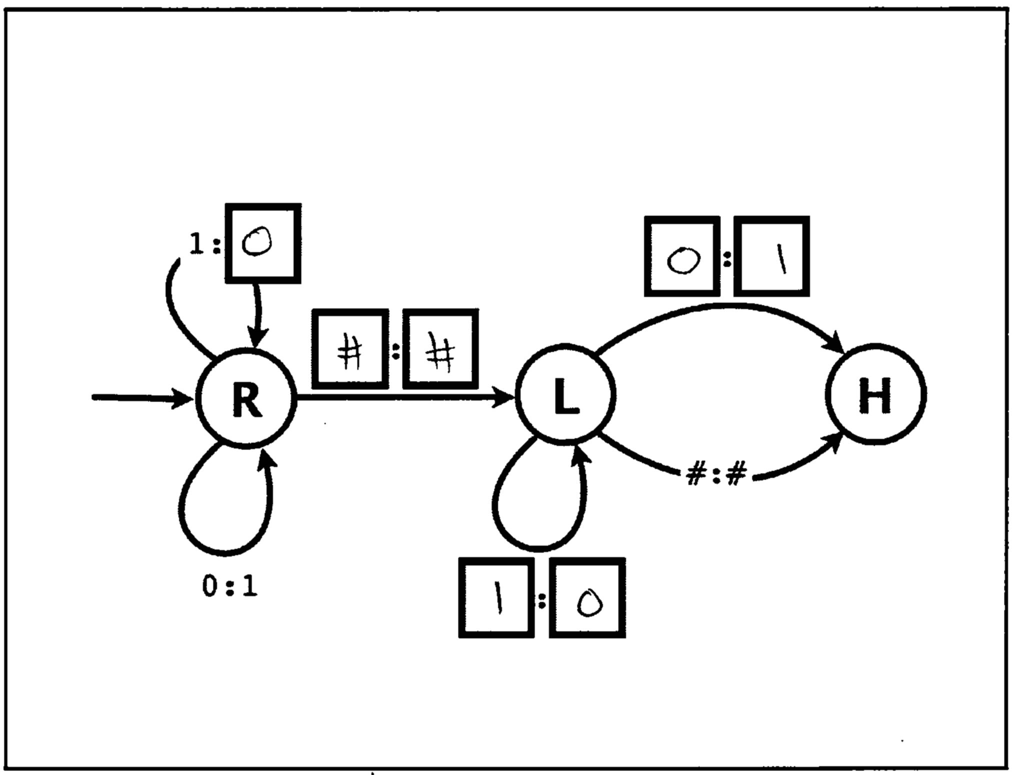
- [NONE] Matches no strings in L.
- [SOME] Matches only some strings in L and some other strings.
- [MORE] Matches all strings in L and some other strings.
- [EXACT] Matches all strings in L and no other strings.

	NONE	SOME	MORE	EXACT
A. `a(a|b)*abb(a|b)*`	●	○	○	○
B. `a(a|b)*a`	○	○	●	○
C. `a*b*aba`	○	●	○	○
D. `a((a*|b*)|(b*aba*))a`	○	●	○	○
E. `a*b*aa*b*ba*a*a*b*b*a*a*b*`	○	●	○	○
F. `(a|b)(a|b)(a|b)(a|b)a`	○	○	●	○
G. `(a|aa|aaa)(ba|aa|bbb)a`	○	●	○	○

Q15. Turing Machines (7 points). Below is an incomplete diagram of a Turing Machine. Complete the diagram so that it satisfies the following specification:

- Assume the initial tape consists of a binary string with infinitely many # symbols on both sides.
- Assume the initial head location is the leftmost bit of the binary string.
- Assume the initial state is the state labelled R on the left, and recall that the Turing Machine's first step is reading/writing, not moving the head.
- Interpret the input as a two's-complement binary integer. After the Turing machine halts, the binary integer that remains on the tape should be the negative of the original input. (Recall that computing the negative of a number involves flipping all of the bits and then adding one.)

Fill in exactly one symbol in each of the seven empty square boxes below so as to satisfy this specification. Do not add new states or new transitions, and do not use any tape symbols other than #, `0`, or `1`.



flip

right of tape

#10100#

then #01011#

+1 01100 #

01011

01100

Q16. Intractability (7 points). For each of the computational problems below, indicate its difficulty by filling in the most appropriate choice among *True*, *False* or *Nobody Knows*. *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

	True	False	Nobody Knows
A. If P is not equal to NP there is no polynomial-time algorithm for integer linear programming (ILP).	●	○	○
B. If P = NP, every problem in P is NP-complete.	●	○	○
C. If P = NP there is a polynomial-time algorithm for factoring.	●	○	○
D. No problem is in both P and NP.	○	●	○
E. There exists a deterministic Turing machine that can solve every problem in NP.	●	○	○
F. A Universal Turing Machine can simulate the operation of any Turing machine, including itself, in polynomial time.	○	●	○
G. No polynomial-time algorithm can solve the Halting Problem.	●	○	○

all P are in NP

brute force

nothing can

Q17. TOY (7 points). Fill in the blanks in the following table.

Hint: $7 \times 16^3 = 28,672$.

Note: The notation here and in the text and lectures is slightly different than the notation used in precepts and online. For example, `R[2] = R[3] + R[4]` means precisely the same thing as `R[2] <- R[3] + R[4]`.

hex	decimal integer	16-bit two's complement	TOY instruction pseudo-code
FFFE	-2	1111111111111110	R[F] = PC; PC = FE
1234	4,660	0001 0010 0011 0100	R[2] = R[3] + R[4]
1101	4,353	0001000100000001	R[1] = R[0] + R[1]
77FF	30,719	0111011111111111	R[7] = 00FF
FF01	-255	1111 1111 0000 0001	R[F] = PC; PC = 01
7A00	31232	0111101000000000	R[A] = 0000

TYPO

Q18. TOY Programming (7 points). Consider the following TOY program:

```
20: 81FF  R[1] = stdin
21:       SEE BELOW
22: 1211  R[2] = R[1] + R[1]
23: 0000  halt
```

To the right of each of the possibilities below for the instruction at M[21], write the contents of R[1] when the machine halts after being started at 20 with 1111 on standard input. Your answers must each be four hex digits.

Note: The notation used here and in the text and lectures is slightly different than the notation used in precepts and online. For example, R[1] = M[21] means precisely the same thing as R[1] <- mem[21].

Possibilities for M[21]	**Contents of R[1] when halting**
21: 1111 R[1] = R[1] + R[1]	2222
21: 0000 halt	1111
21: 1211 R[2] = R[1] + R[1]	1111
21: 7100 R[1] = 0000	0000
21: C023 PC = 23	1111
21: 8121 R[1] = M[21]	8121
21: 9122 M[22] = R[1]	2222

Q19. Combinational Circuits (7 points). The *IS-XOR?* function of 3 boolean variables is **x**, **y** and **z** is 1 if and only if **x XOR y == z.**

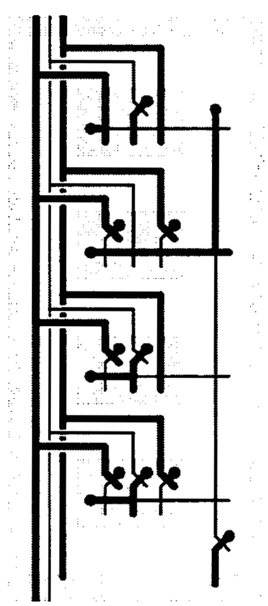
A. (2 points) Fill in the missing entries in this truth table for the 3-variable *IS-XOR?* function.

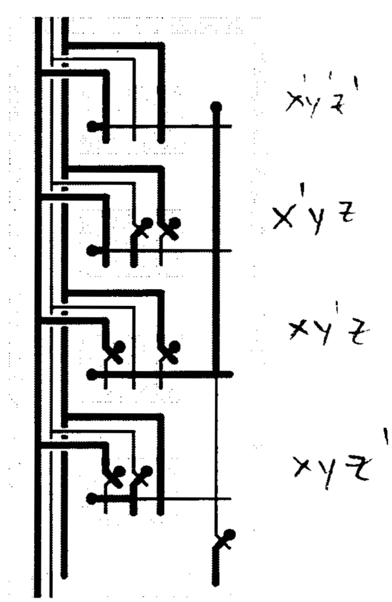
x	x XOR y	y	z	IS-XOR?
0	0	0	0	1
0	0	0	1	0
0	1	1	0	0
0	1	1	1	1
1	1	0	0	0
1	1	0	1	1
1	0	1	0	1
1	0	1	1	0

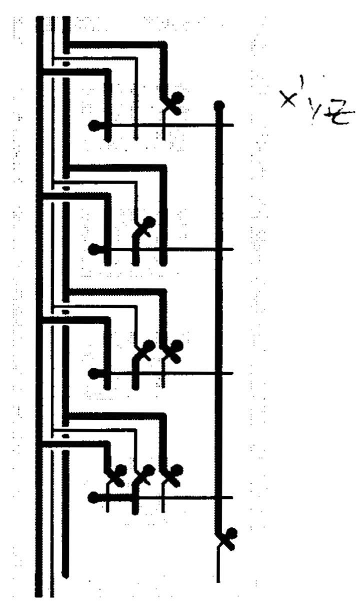
B. (3 points) In the box, write out the sum-of-products form of the 3-variable *IS-XOR?* function.

$x'y'z' + x'yz + xy'z + xyz'$

C. (2 points) Which of the circuits below is computing *IS-XOR?* for 3 variables with the inputs `1 0 1`? In each circuit, assume that the inputs `x y z` are provided in that order to the three lines at the upper left and the output is the line at the bottom right. Fill in the circle above the correct circuit. *Correct answers are worth 1 point; incorrect answers are worth -1 point; blank answers are worth 0 points.*

○ 

● 

○ 

Q20. Central Processing Unit (7 points). Consider the following list of CPU components. In the box to the right of each option, write the letter corresponding to the description at right that best matches. Each letter should be used **at most** once. *Correct answers are worth 1 point.*

Component	Answer
Bus connection	I
ALU	G
MUX ?	F
IR	H
Control line	C
PC	D
Clock	J

A. Sequential circuit that carries information.

B. Combinational circuit that holds data.

C. Component input that determines behavior.

D. Holds address of current instruction.

E. Allows switching among component inputs.

F. Allows switching among component outputs.

G. Computes boolean function values.

H. Holds instruction being executed.

I. Path carrying data between components.

J. Sequential circuit that produces periodic pulses.

K. Memory unit extension.