



# Polygonal Meshes

COS 426, Spring 2015  
Princeton University

# 3D Object Representations



## Points

- Range image
- Point cloud

## Surfaces

- Polygonal mesh
- Subdivision
- Parametric
- Implicit

## Solids

- Voxels
- BSP tree
- CSG
- Sweep

## High-level structures

- Scene graph
- Application specific

# 3D Polygonal Mesh



Set of polygons representing a 2D surface embedded in 3D

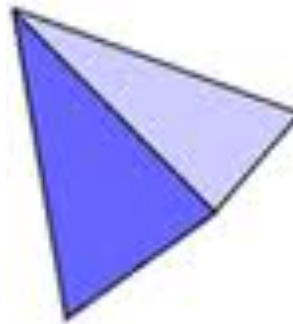
Platonic Solids



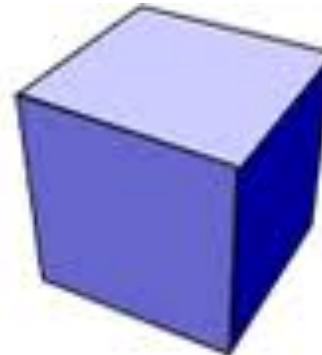
Dodecahedron



Icosahedron



Tetrahedron



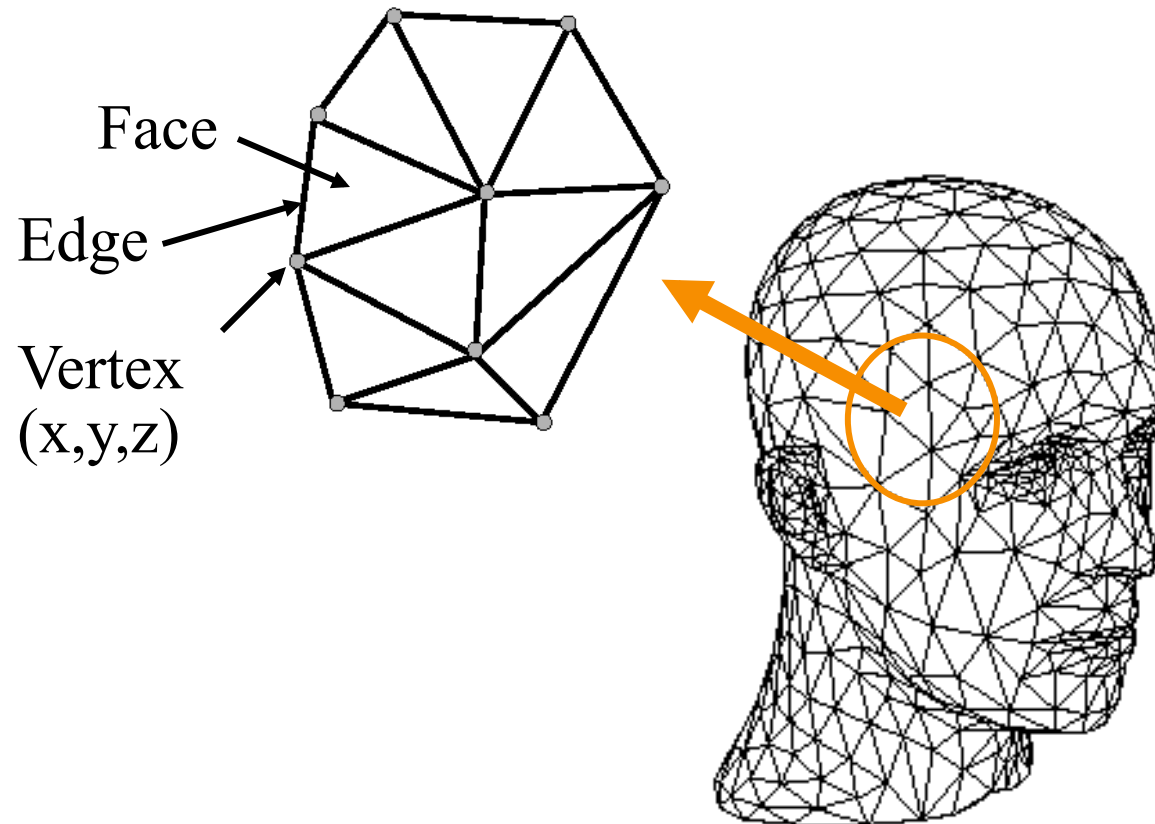
Cube



Octahedron

# 3D Polygonal Mesh

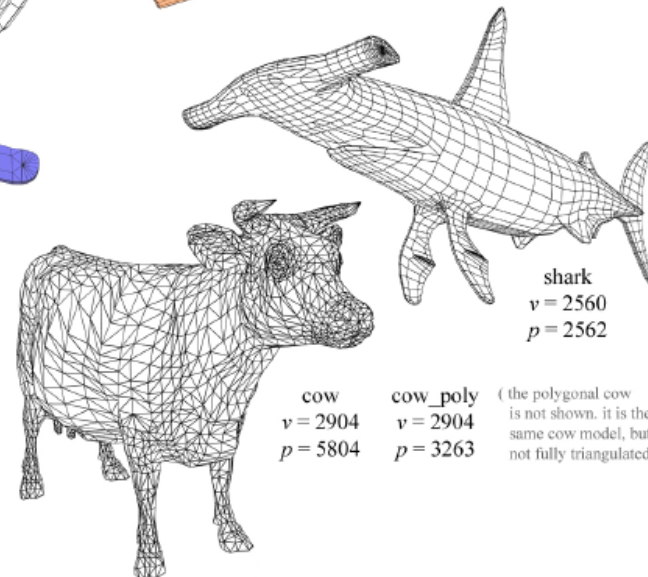
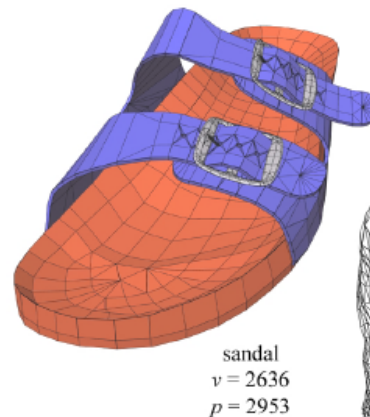
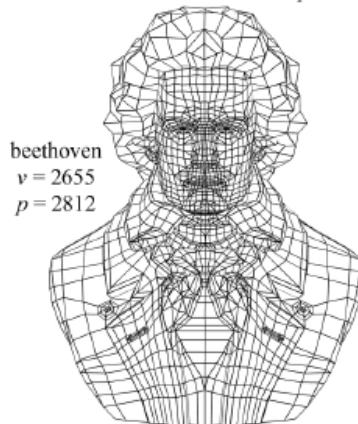
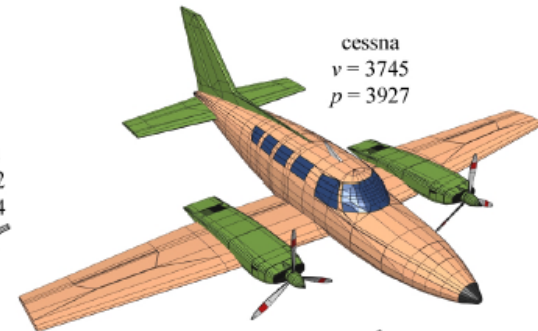
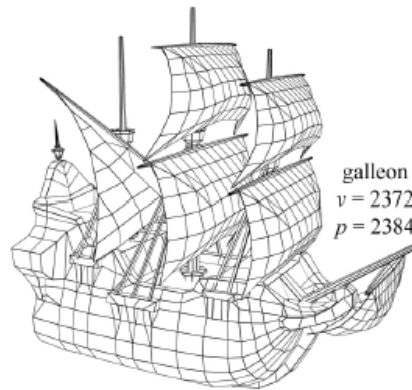
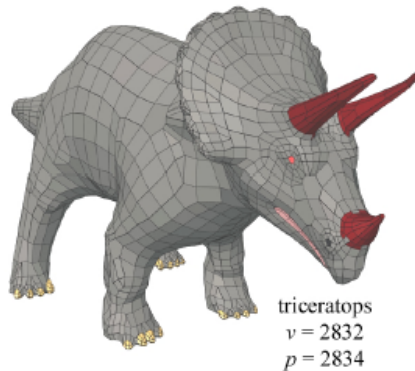
Set of polygons representing a 2D surface embedded in 3D





# 3D Polygonal Mesh

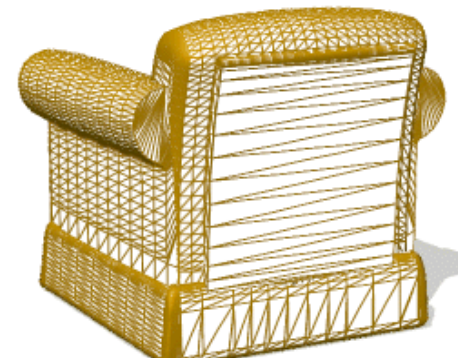
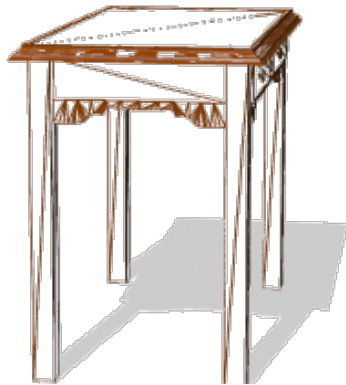
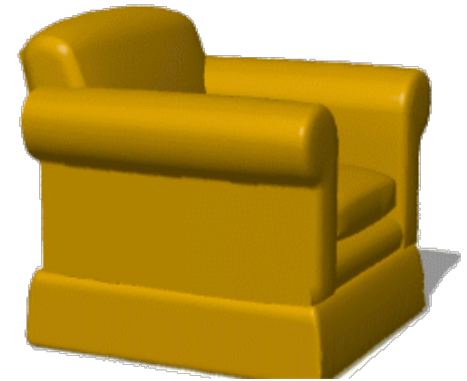
Set of polygons representing a 2D surface embedded in 3D



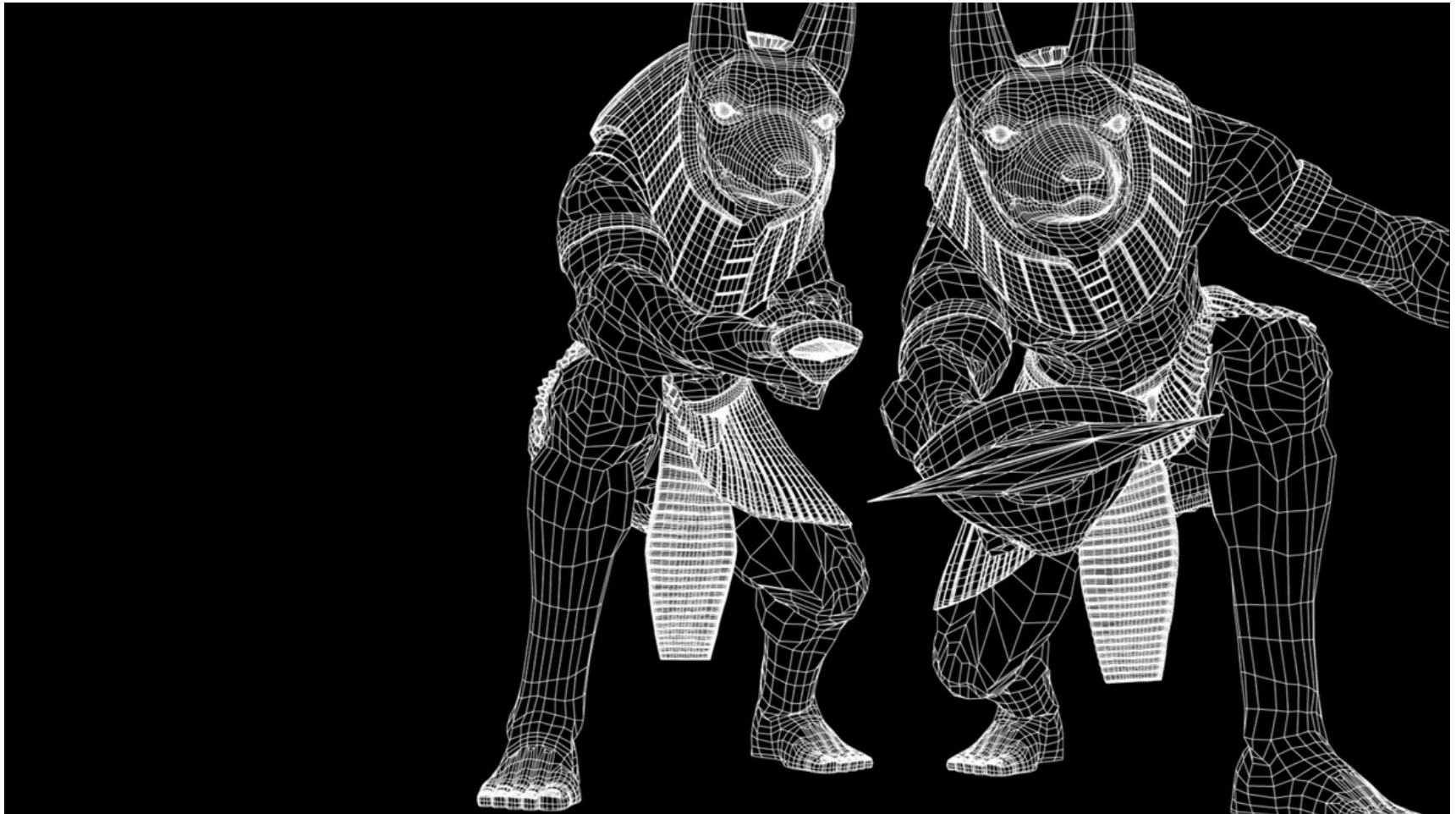
# 3D Polygonal Meshes

Why are they of interest?

- Simple, common representation
- Rendering with hardware support
- Output of many acquisition tools
- Input to many simulation/analysis tools



# 3D Polygonal Meshes



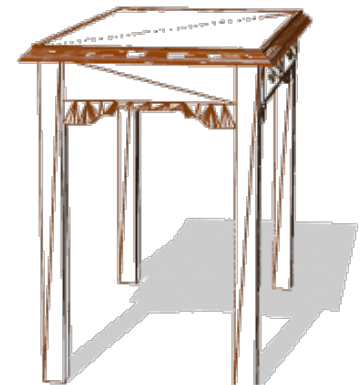
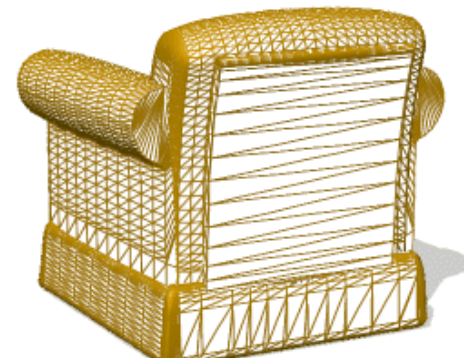
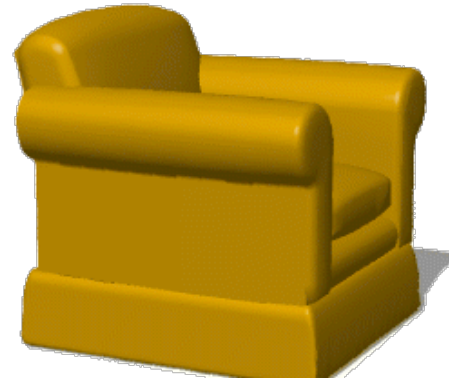
[http://www.fxguide.com/featured/Comic\\_Horrors\\_Rocks\\_Statures\\_and\\_VanDyke/](http://www.fxguide.com/featured/Comic_Horrors_Rocks_Statures_and_VanDyke/)

# 3D Polygonal Meshes



## Properties

- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.





# Outline



Acquisition ←

Processing

Representation



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

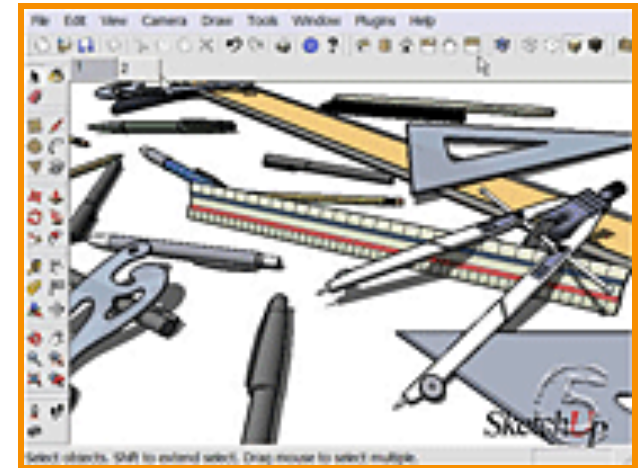
- Laser range scanners
- Geological survey

## Procedural generation

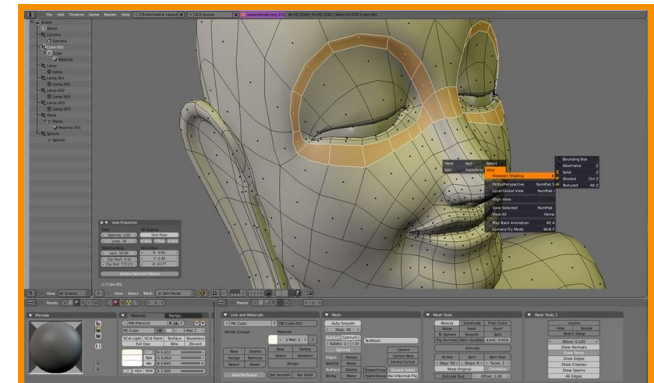
- Surface of revolution
- Sweep

## Simulations

- Physical processes



Sketchup



Blender

# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats



## Scanners

- Laser range scanners
- Geological survey



## Procedural generation

- Surface of revolution
- Sweep



## Simulations

- Physical processes

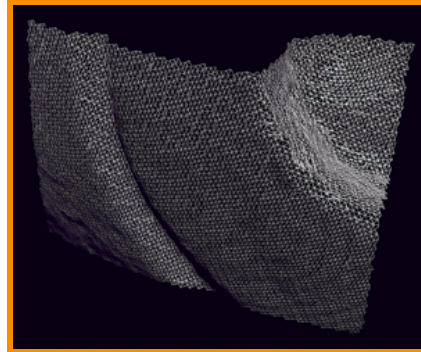




# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats



## Scanners

- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



Digital Michelangelo Project  
Stanford

# Polygonal Mesh Acquisition



## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

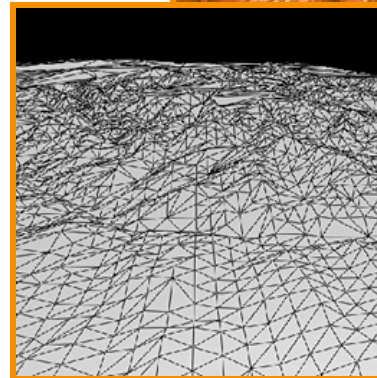
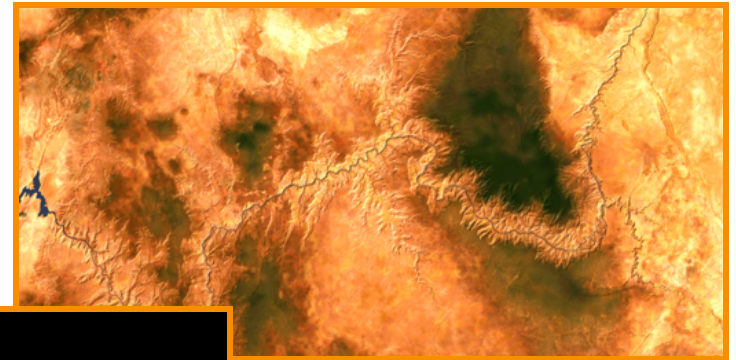
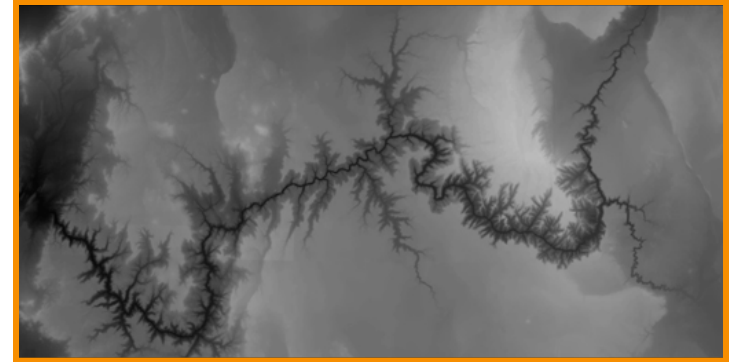
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



Large Geometric  
Model Repository  
Georgia Tech



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

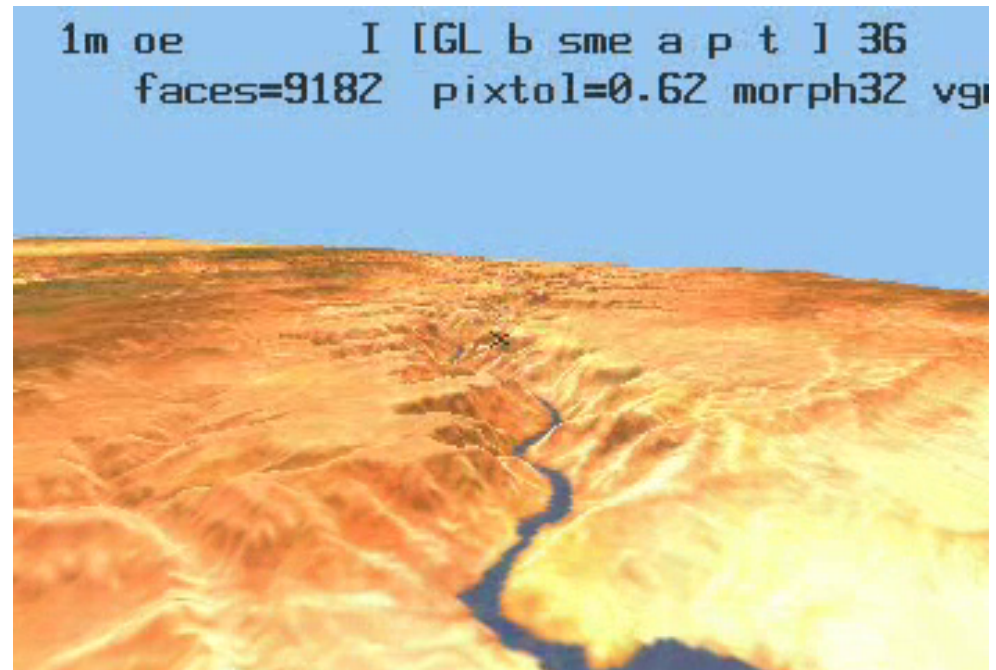
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

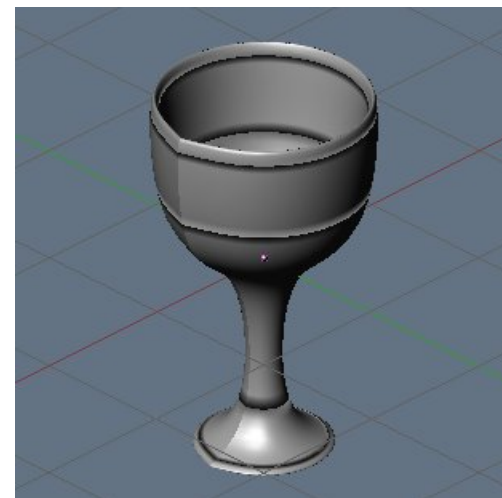
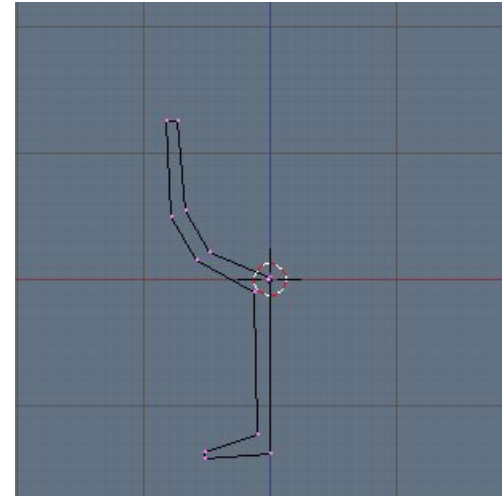
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes







# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

- Laser range scanners
- Geological survey

## Procedural generation

- **Surface of revolution**
- Sweep

## Simulations

- Physical processes



MakeAGIF.com



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

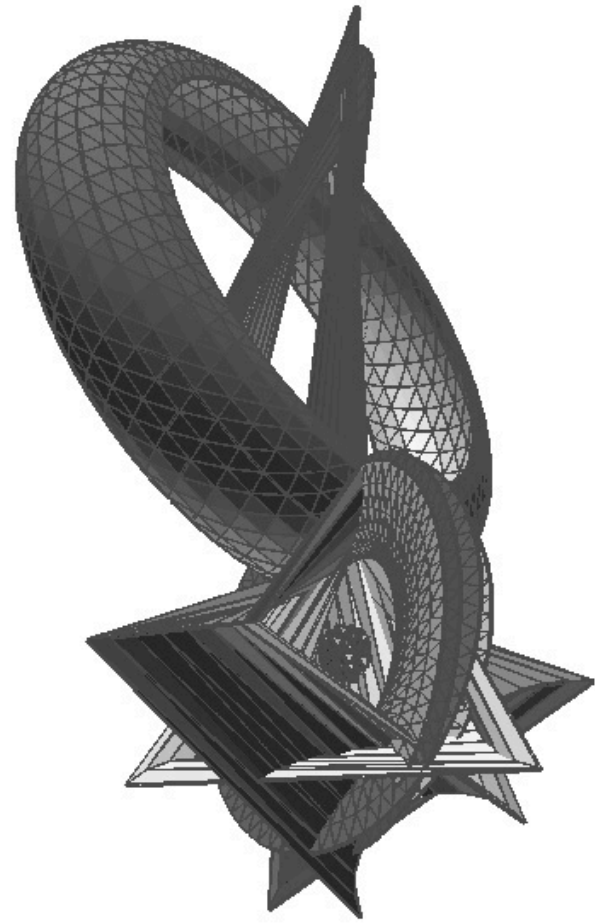
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

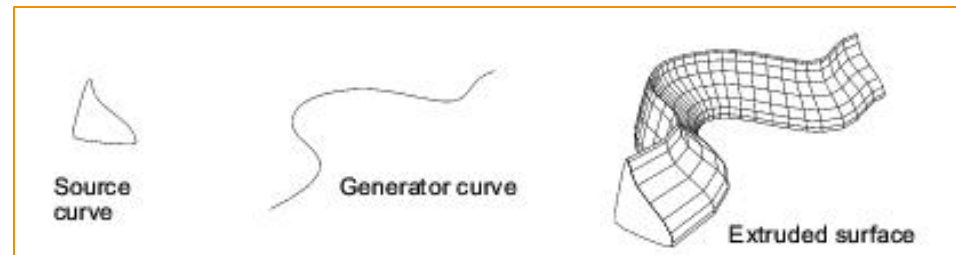
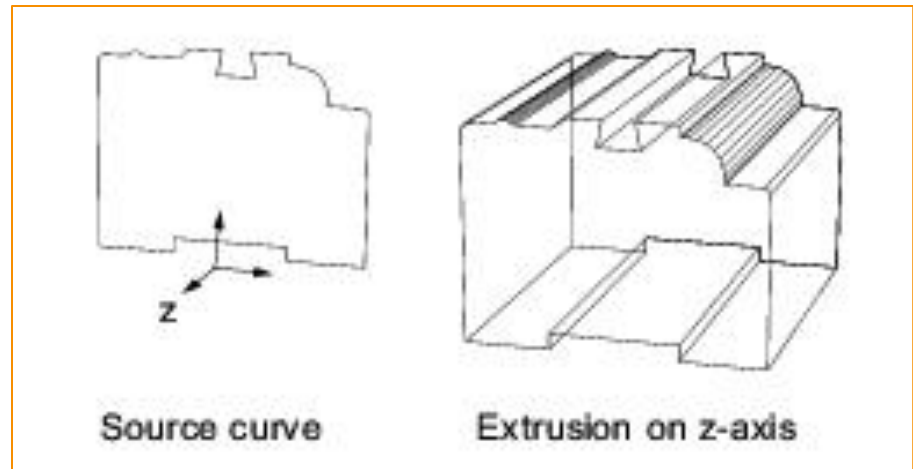
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

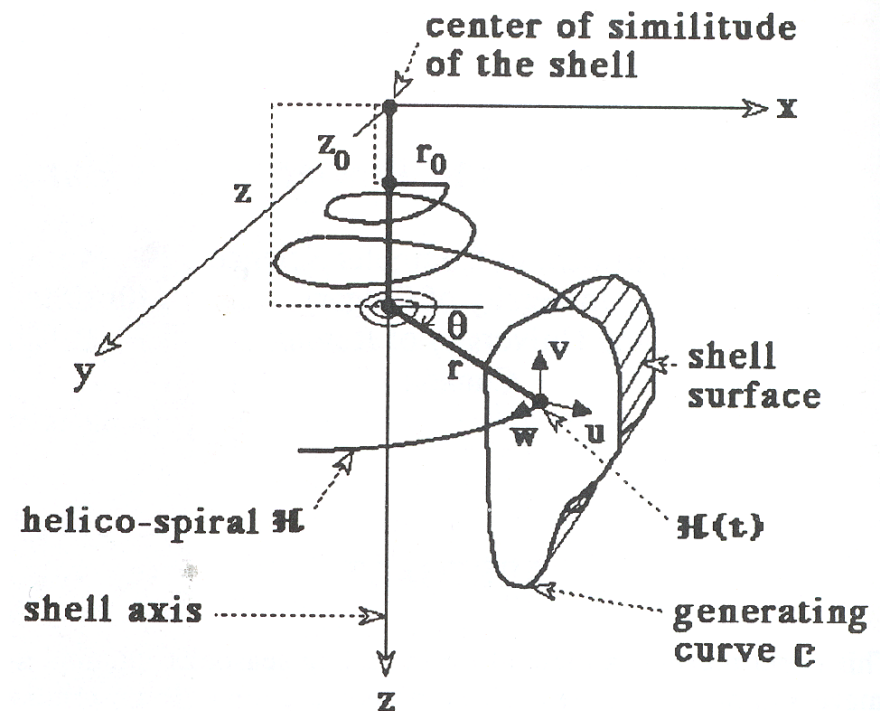
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes





# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

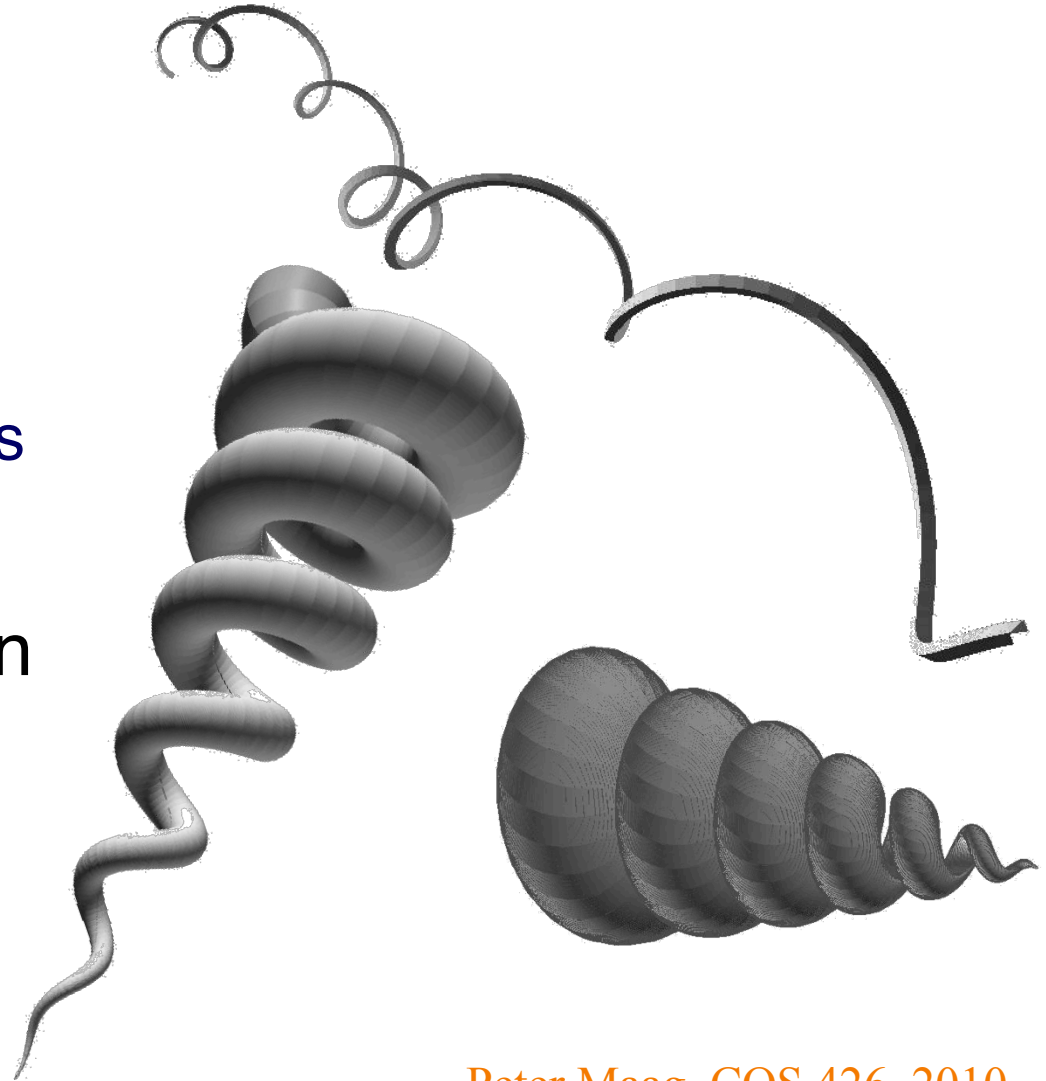
- Laser range scanners
- Geological survey

## Procedural generation

- Surface of revolution
- Sweep

## Simulations

- Physical processes



# Polygonal Mesh Acquisition

## Interactive modeling

- Polygon editors
- Interchange formats

## Scanners

- Laser range scanners
- Geological survey

## Procedural generation

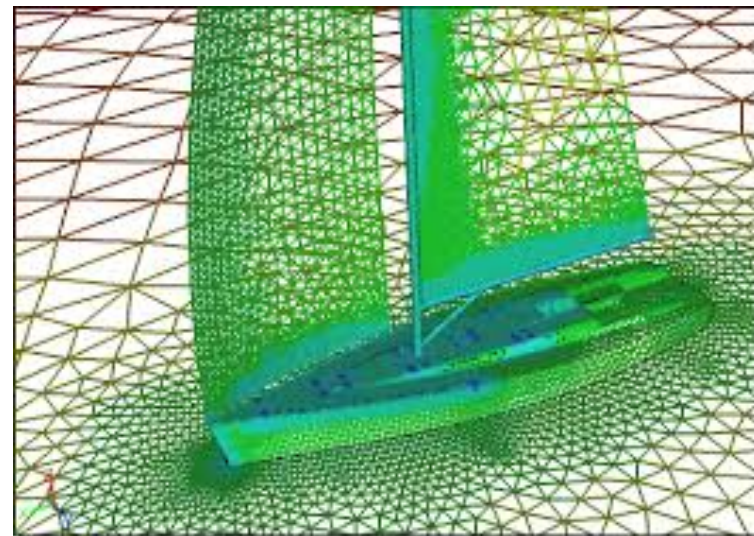
- Surface of revolution
- Sweep

## Simulations

- Physical processes



SGI



sym scape

# Outline



Acquisition

**Processing** ←

Representation



# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel

# Polygonal Mesh Processing



## Analysis ←

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel

# Polygonal Mesh Processing



## Analysis

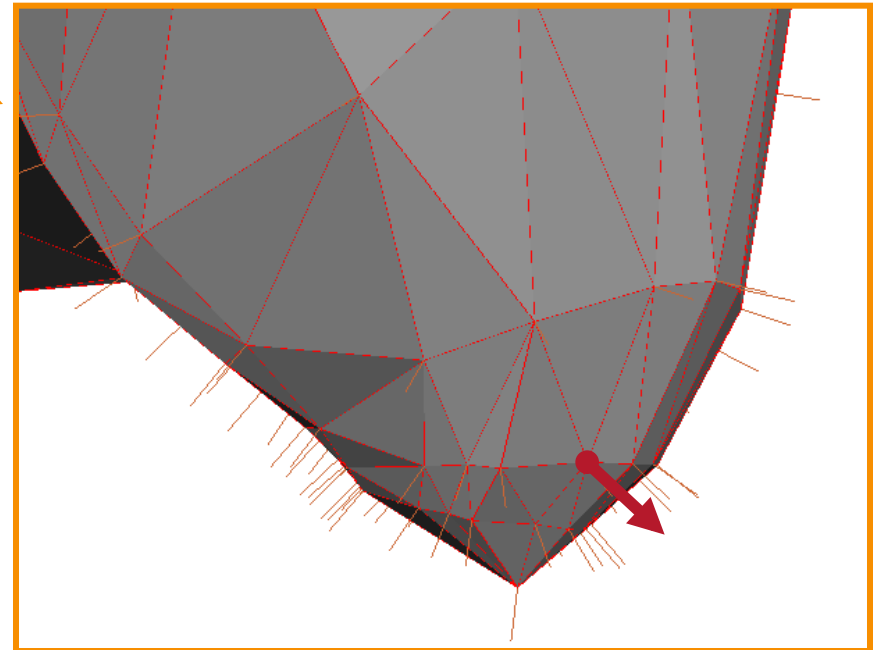
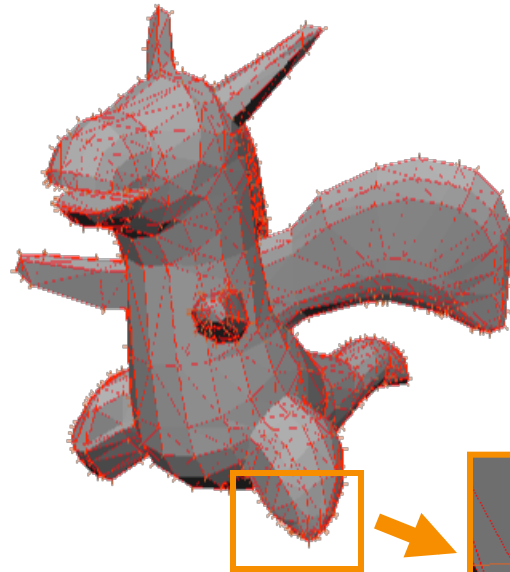
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature

## Warps

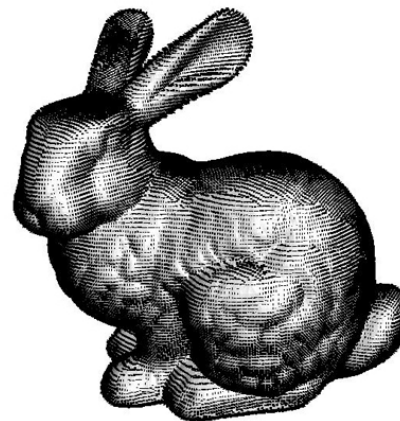
- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel

*NORMAL VERTEX*

presents



*The Next Dual*

“The bunny with normal vertices shown.  
Reminded me of an album cover so I made it into one.”

# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature



## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel

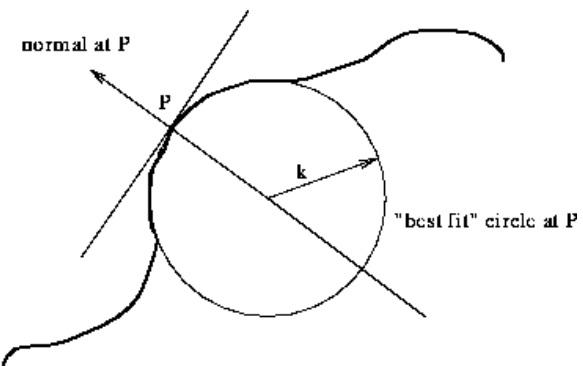


Figure 32: curvature of curve at  $P$  is  $1/k$

# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature

## Warps ←

- Rotate
- Deform

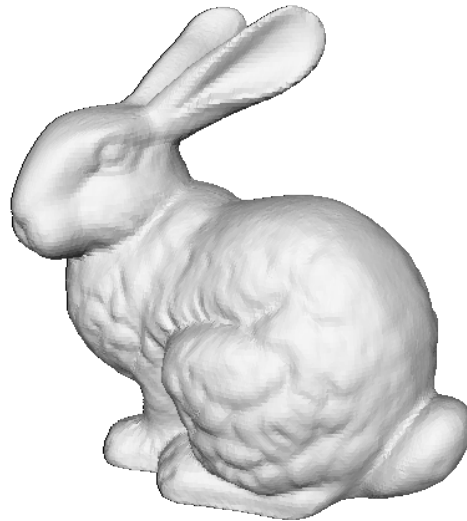
## Filters

- Smooth
- Sharpen
- Truncate
- Bevel

# Polygonal Mesh Processing

## Analysis

- Normals
- Curvature

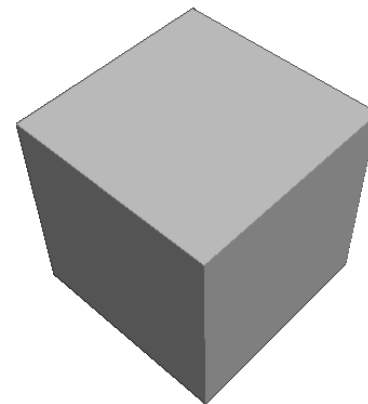
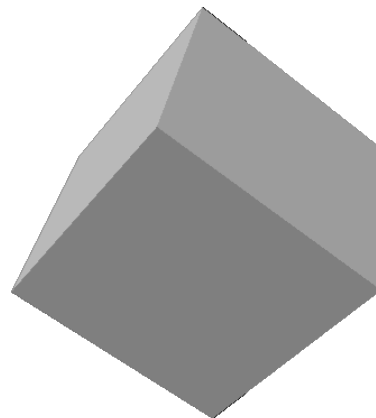


## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Analysis

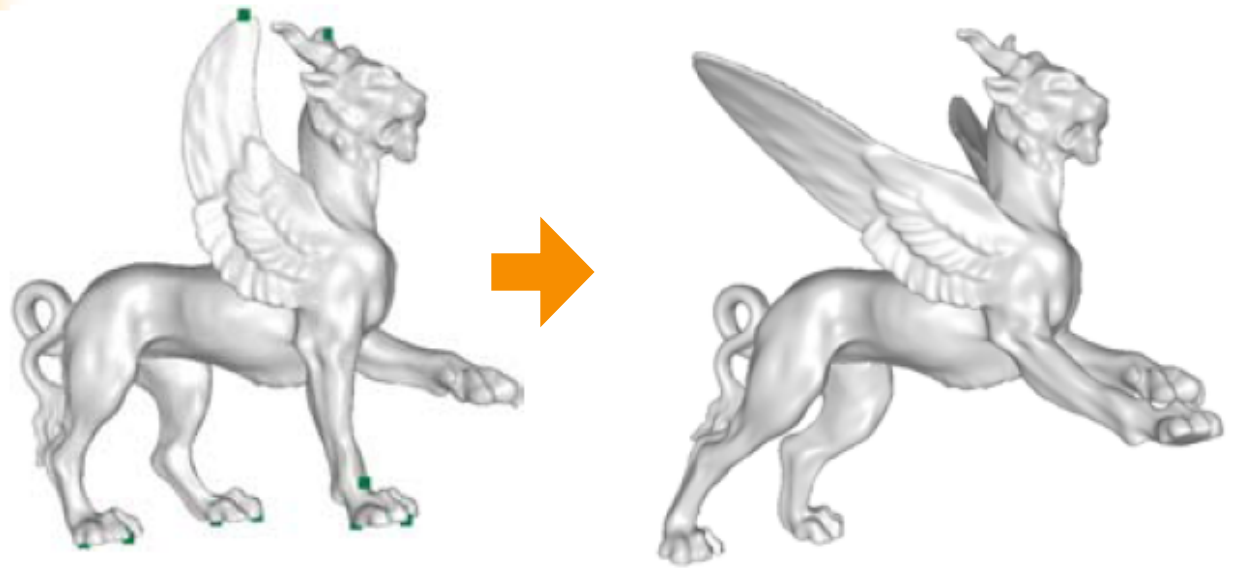
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel





# Polygonal Mesh Processing

## Analysis

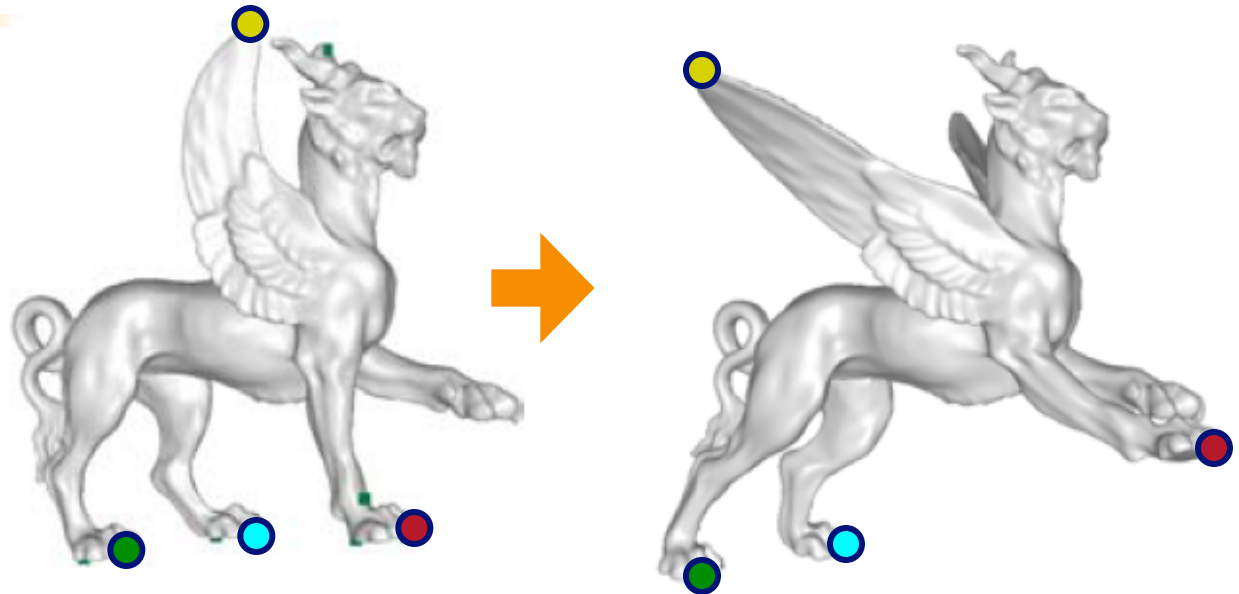
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Analysis

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters ←

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing

## Analysis

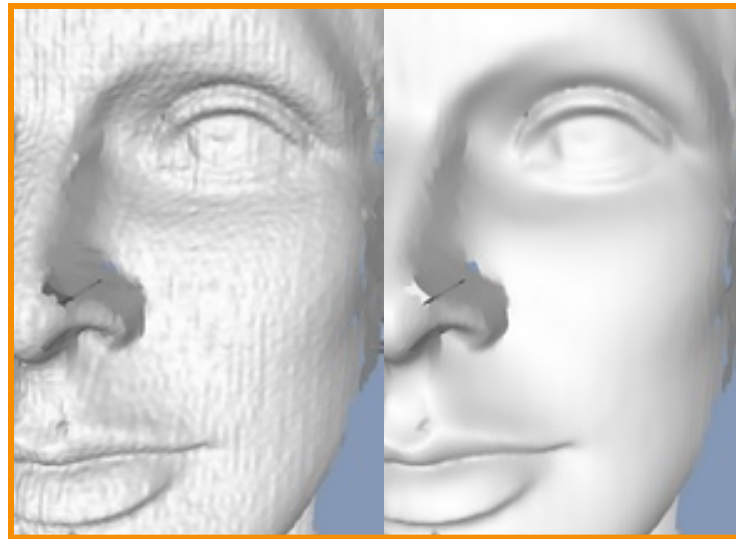
- Normals
- Curvature

## Warps

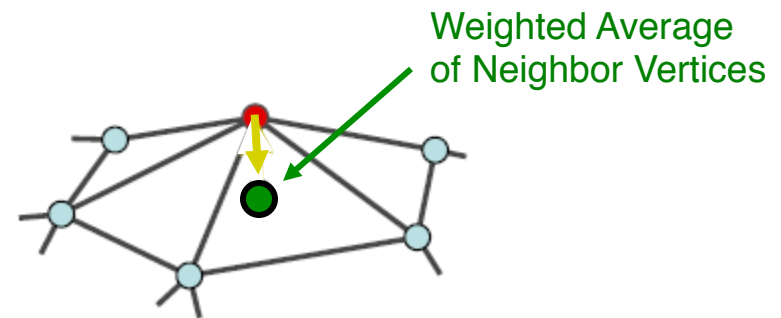
- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



Thouis “Ray” Jones



Olga Sorkine

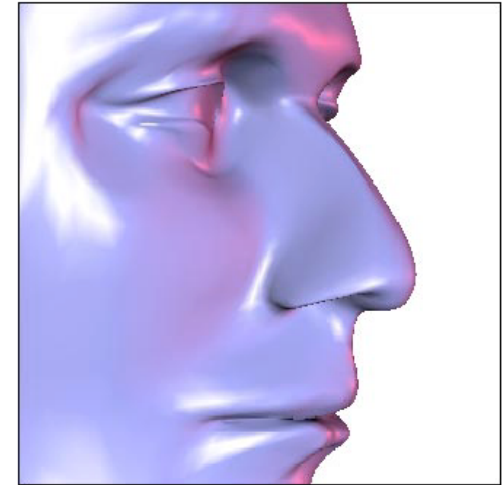
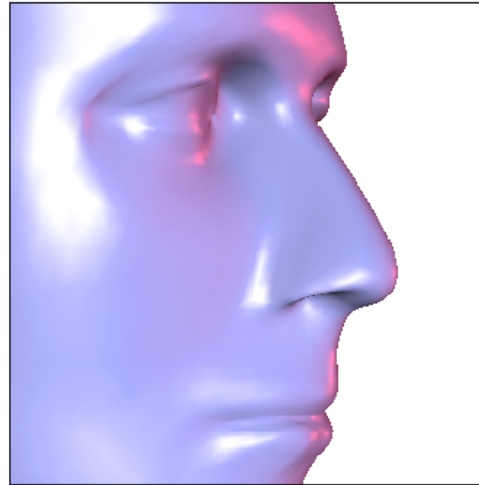
# Polygonal Mesh Processing

## Analysis

- Normals
- Curvature

## Warps

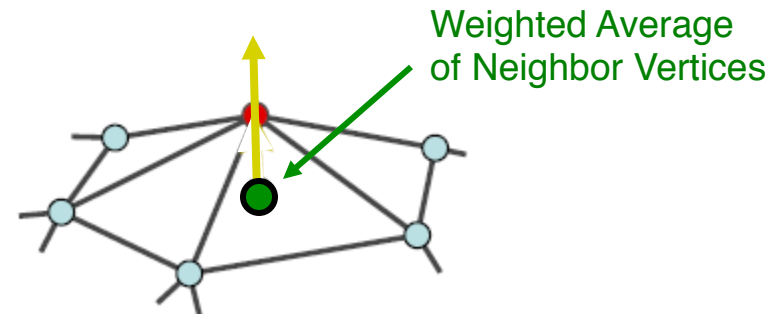
- Rotate
- Deform



Desbrun

## Filters

- Smooth
- **Sharpen**
- Truncate
- Bevel



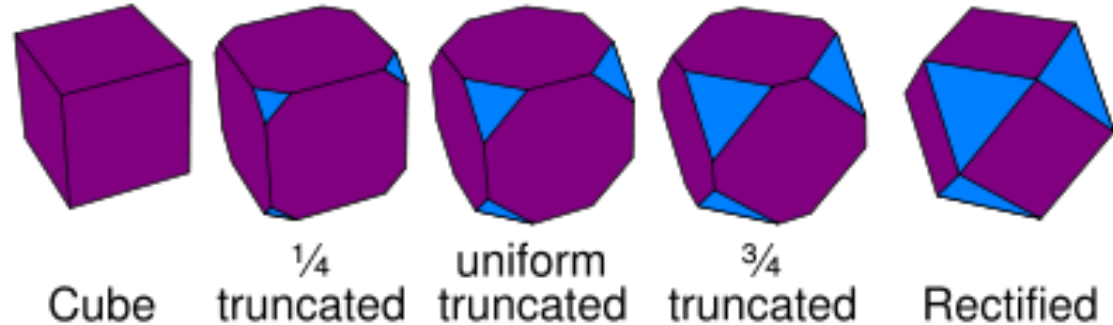
Olga Sorkine



# Polygonal Mesh Processing

## Analysis

- Normals
- Curvature

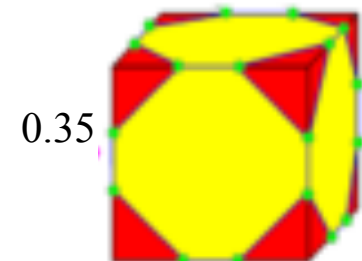
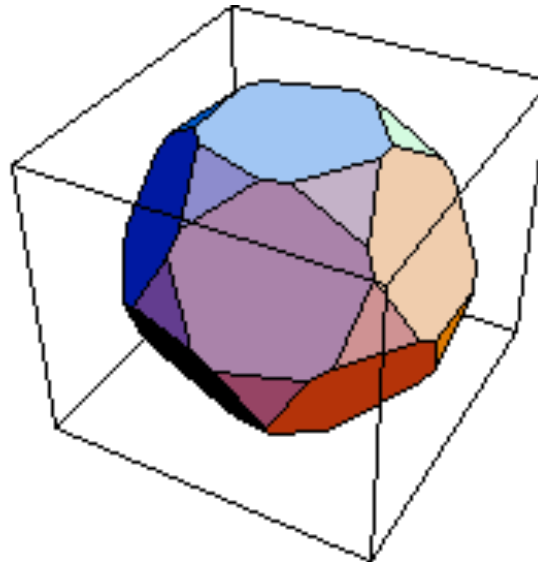


## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- **Truncate**
- Bevel



Conway



# Polygonal Mesh Processing

## Analysis

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- **Truncate**
- Bevel





# Polygonal Mesh Processing

## Analysis

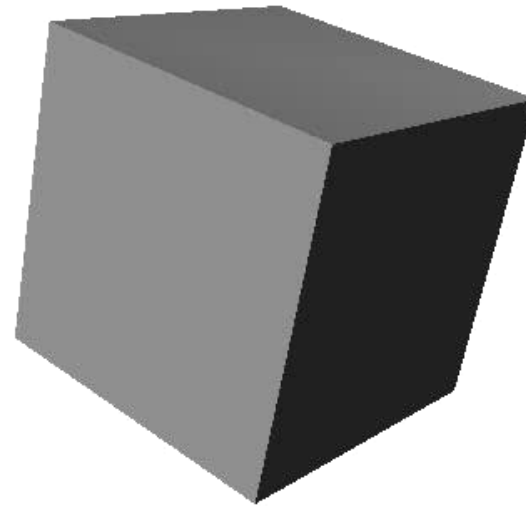
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- **Truncate**
- Bevel







# Polygonal Mesh Processing

## Analysis












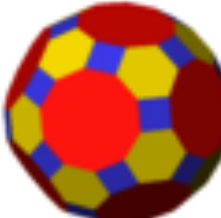
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- **Truncate**
- Bevel

 {3,3}	 (3.6.6)	 (3.3.3.3)	 (4.6.6)
 {4,3}	 (3.8.8)	 (3.4.3.4)	 (4.6.8)
 {5,3}	 (3.10.10)	 (3.5.3.5)	 (4.6.10)



# Polygonal Mesh Processing

## Analysis

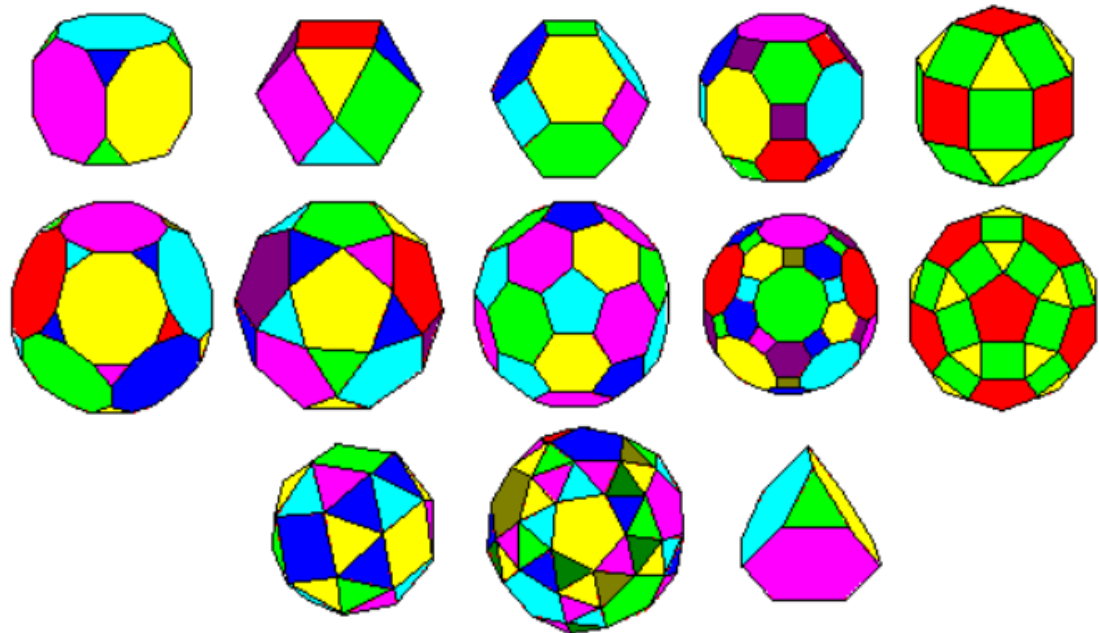
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- **Truncate**
- Bevel



Archimedean Polyhedra

<http://www.uwgb.edu/dutchs/symmetry/archpol.htm>



# Polygonal Mesh Processing

## Analysis

- No
- C

## Warp

- R
- D

## Filter

- S
- S
- T

- Bevel



Carlo Séquin

# Polygonal Mesh Processing



Wikipedia

## Analysis

- Normals
- Curvature



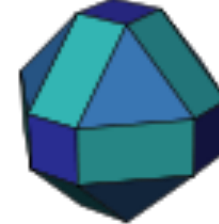
(regular polyhedron)  
Cube



$\frac{1}{2}$  cantellated  
(beveled cube)



Uniform cantellation  
Rhombicuboctahedron



$\frac{3}{4}$  cantellated  
(beveled octahedron)



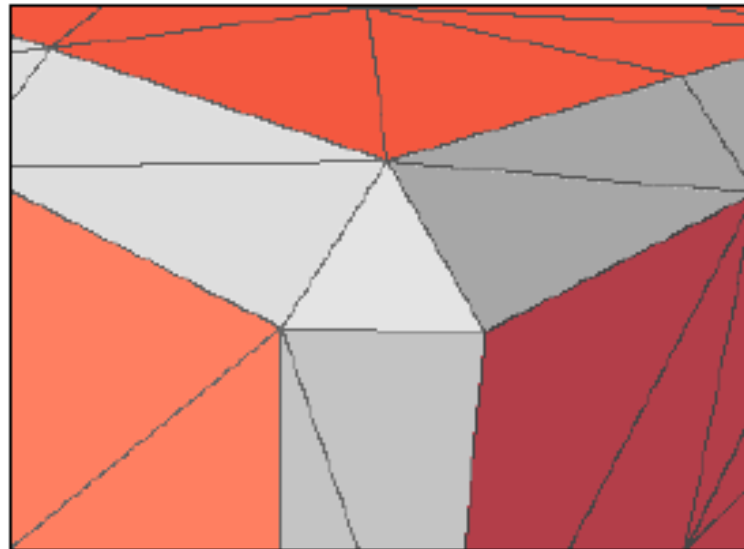
(regular dual)  
Octahedron

## Warps

- Rotate
- Deform

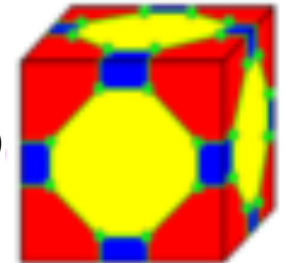
## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



Jarek Rossignac

0.40



Conway

# Polygonal Mesh Processing

## Analysis

- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Analysis

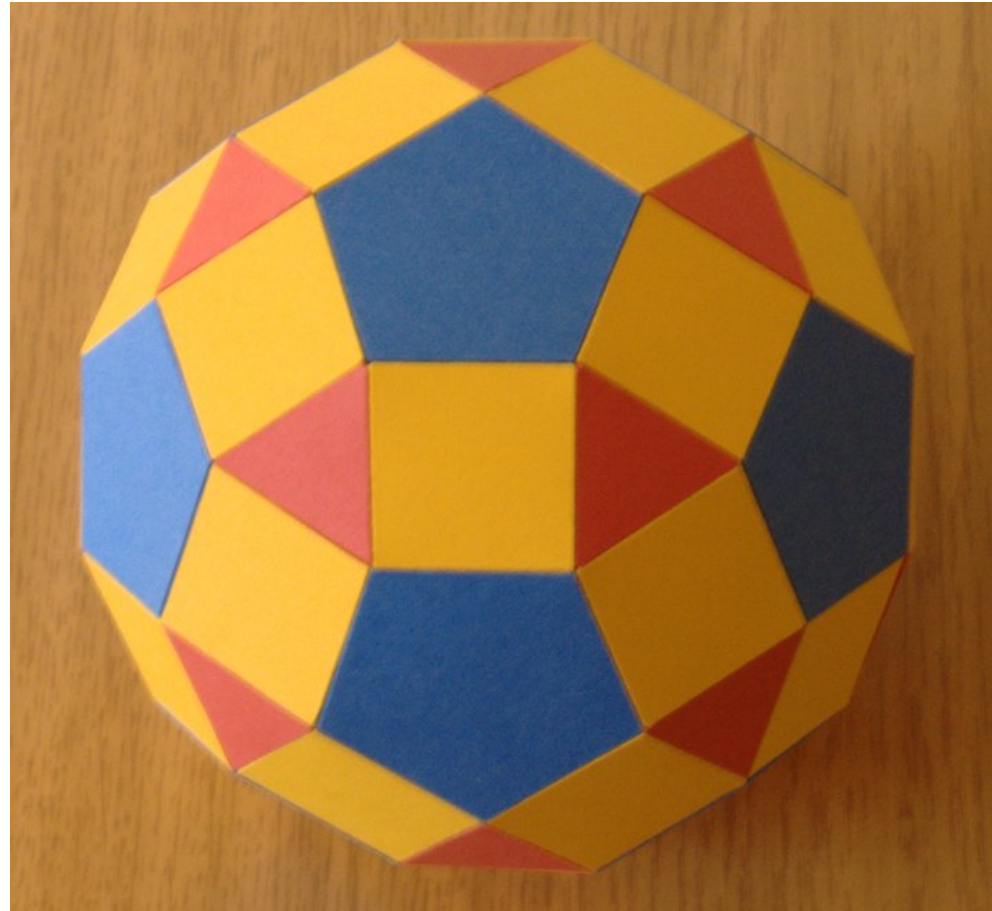
- Normals
- Curvature

## Warps

- Rotate
- Deform

## Filters

- Smooth
- Sharpen
- Truncate
- Bevel



# Polygonal Mesh Processing



## Remeshing

- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract

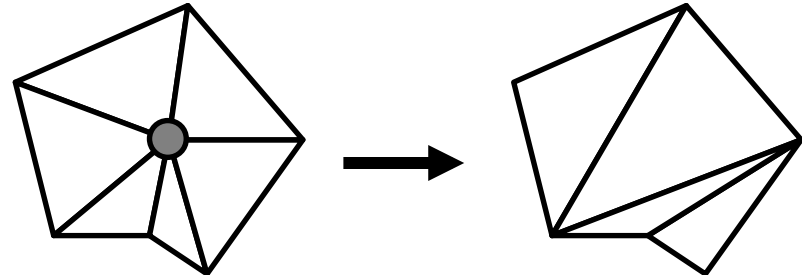


# Polygonal Mesh Processing

## Remeshing



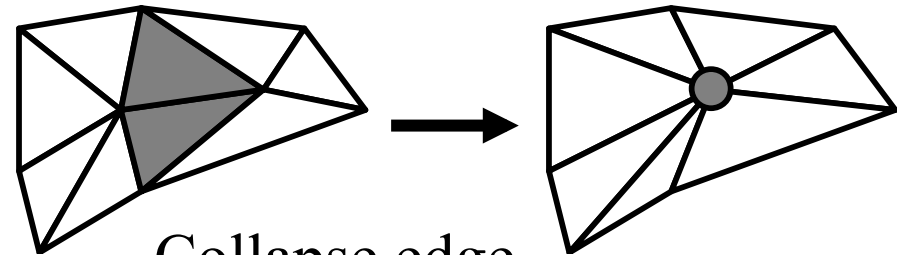
- Subdivide
- Resample
- Simplify



Remove Vertex

## Topological fixup

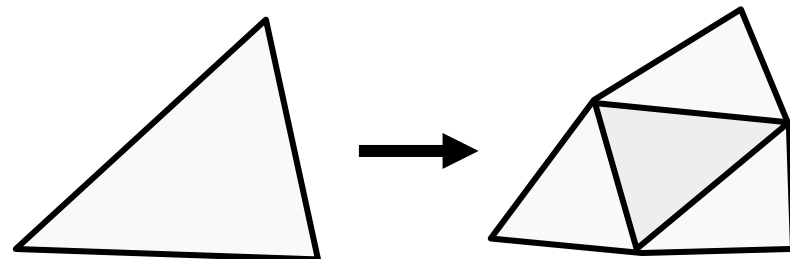
- Fill holes
- Fix self-intersections



Collapse edge

## Boolean operations

- Crop
- Subtract



Subdivide face

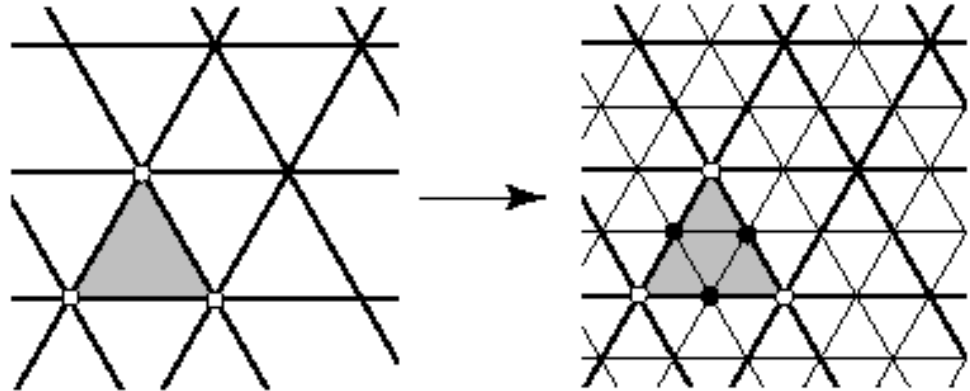


# Polygonal Mesh Processing



## Remeshing

- **Subdivide**
- Resample
- Simplify

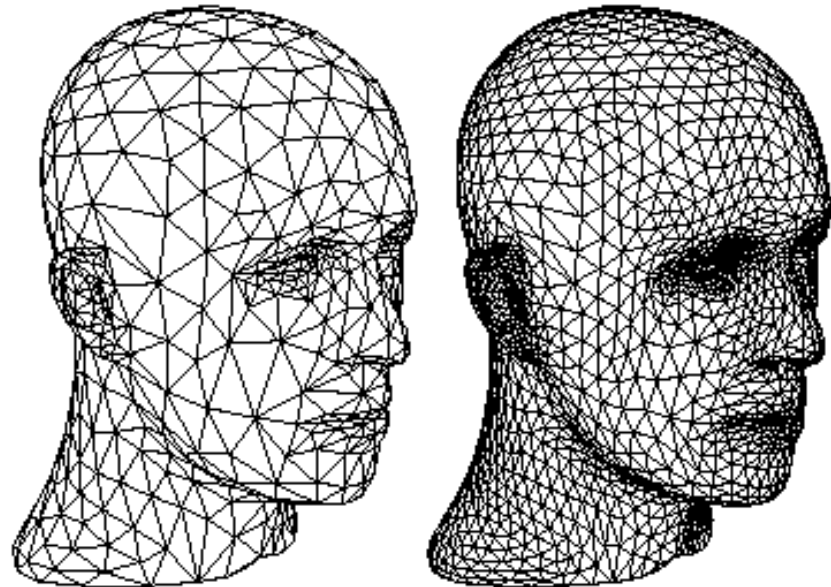


## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract





# Polygonal Mesh Processing

## Remeshing

- **Subdivide**
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



# Polygonal Mesh Processing

## Remeshing

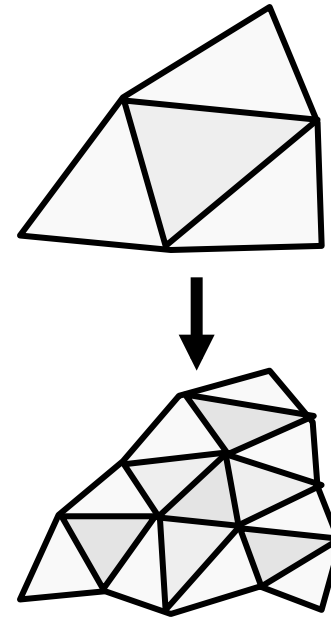
- **Subdivide**
- Resample
- Simplify

## Topological fixup

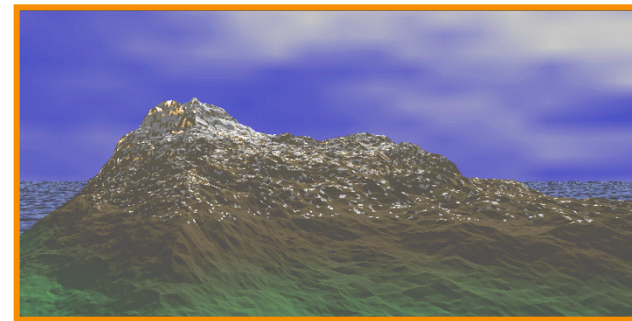
- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



Fractal Landscape



*Dirk Balfanz, Igor Guskov,  
Sanjeev Kumar, & Rudro Samanta,*

# Polygonal Mesh Processing



## Remeshing

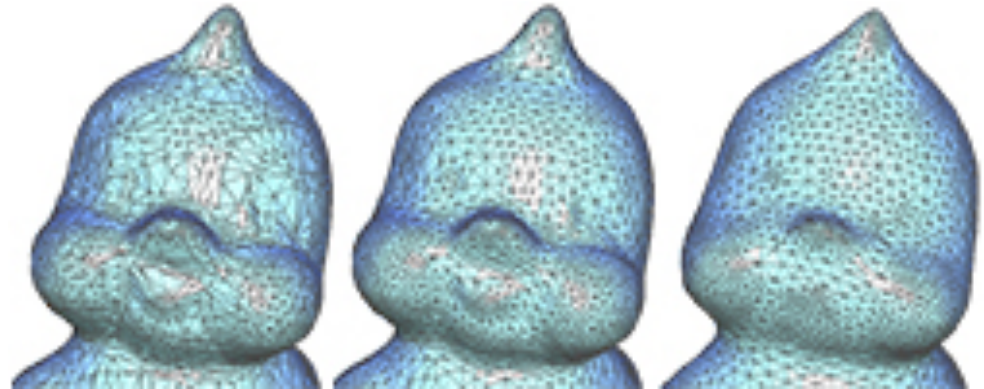
- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



Original

Resampled

- more uniform distribution
- triangles with nicer aspect

# Polygonal Mesh Processing



## Remeshing

- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



# Polygonal Mesh Processing



## Remeshing

- Subdivide
- Resample
- Simplify

## Topological fixup ←

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract

# Polygonal Mesh Processing

## Remeshing

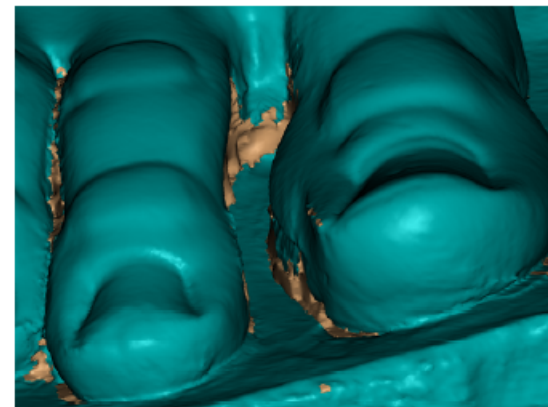
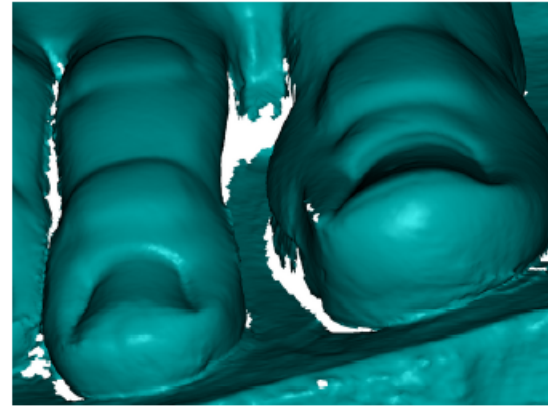
- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



# Polygonal Mesh Processing

## Remeshing

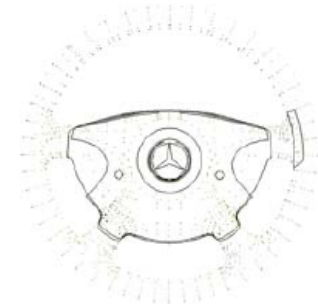
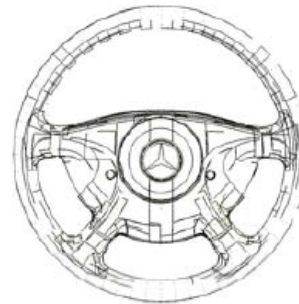
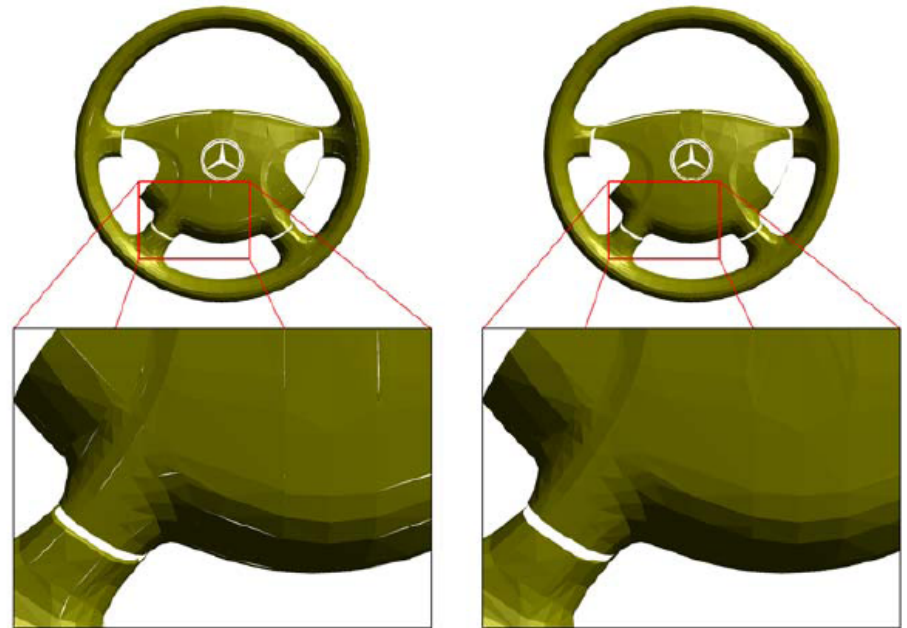
- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- **Fix self-intersections**

## Boolean operations

- Crop
- Subtract







# Polygonal Mesh Processing

## Remeshing

- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

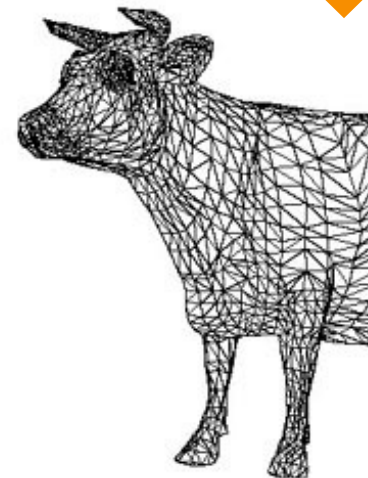
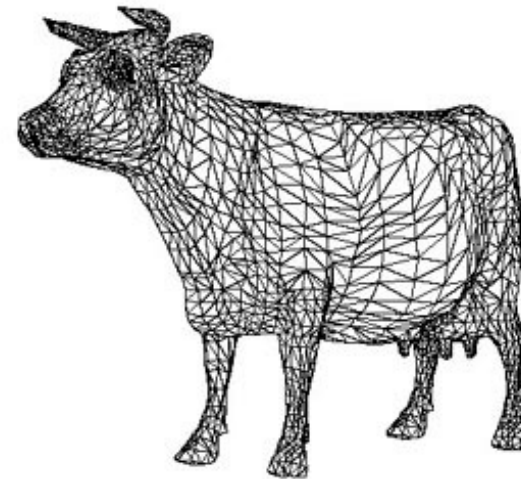
## Boolean operations ←

- Crop
- Subtract

# Polygonal Mesh Processing

## Remeshing

- Subdivide
- Resample
- Simplify



## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract

# Polygonal Mesh Processing

## Remeshing

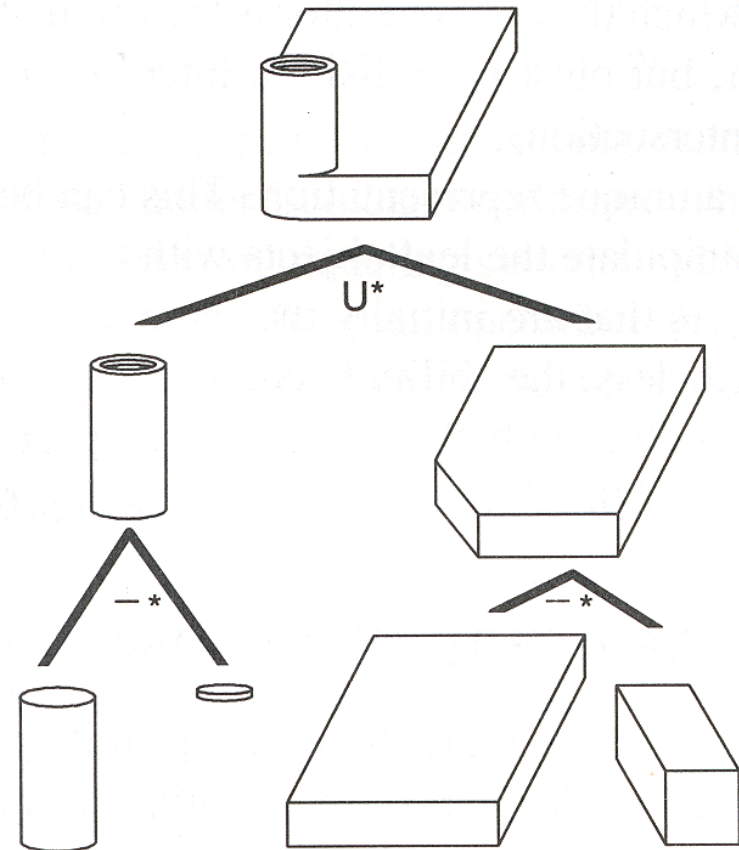
- Subdivide
- Resample
- Simplify

## Topological fixup

- Fill holes
- Fix self-intersections

## Boolean operations

- Crop
- Subtract



# Outline



Acquisition

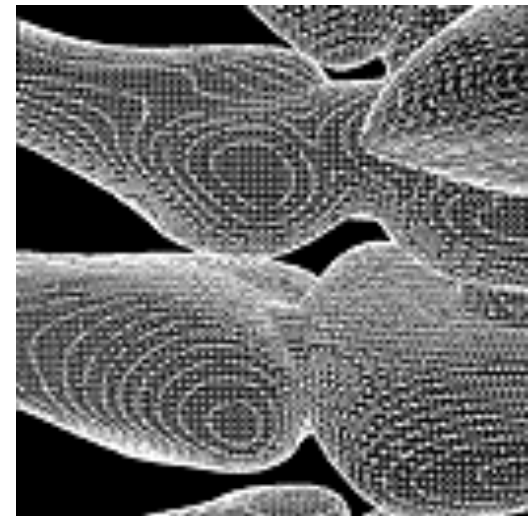
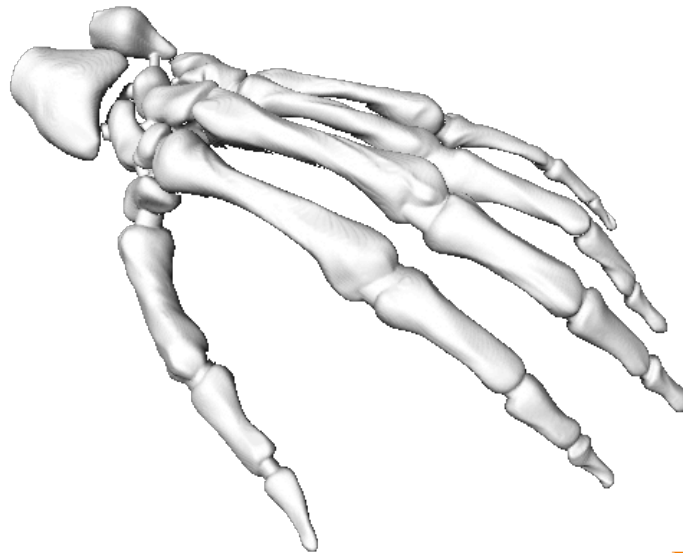
Processing

Representation ←

# Polygon Mesh Representation



Important properties of mesh representation?



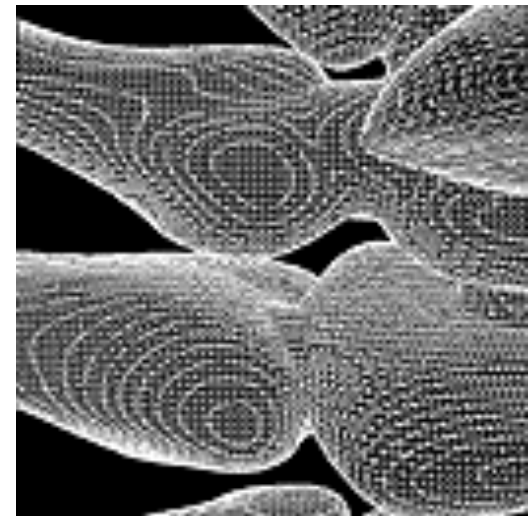
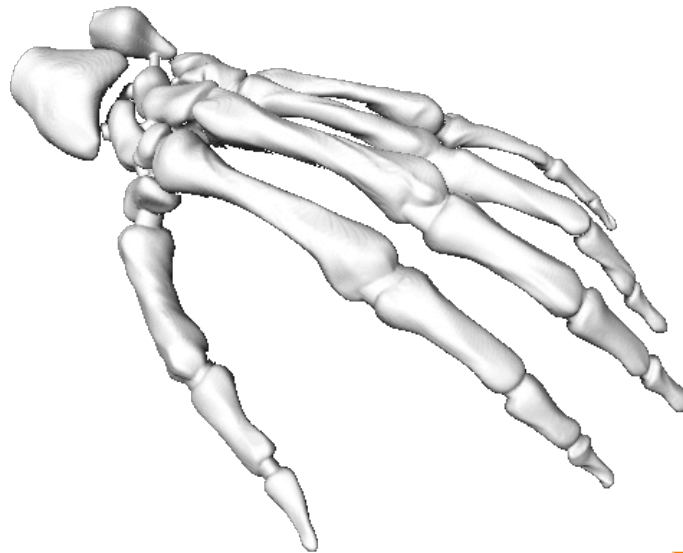
Large Geometric Model Repository  
Georgia Tech

# Polygon Mesh Representation



Important properties of mesh representation?

- Efficient traversal of topology
- Efficient use of memory
- Efficient updates

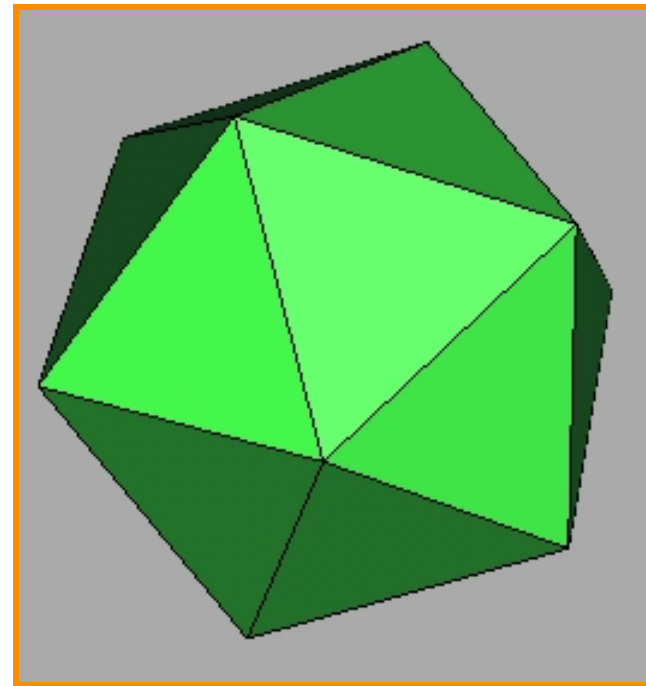


Large Geometric Model Repository  
Georgia Tech

# Polygon Mesh Representation

## Possible data structures

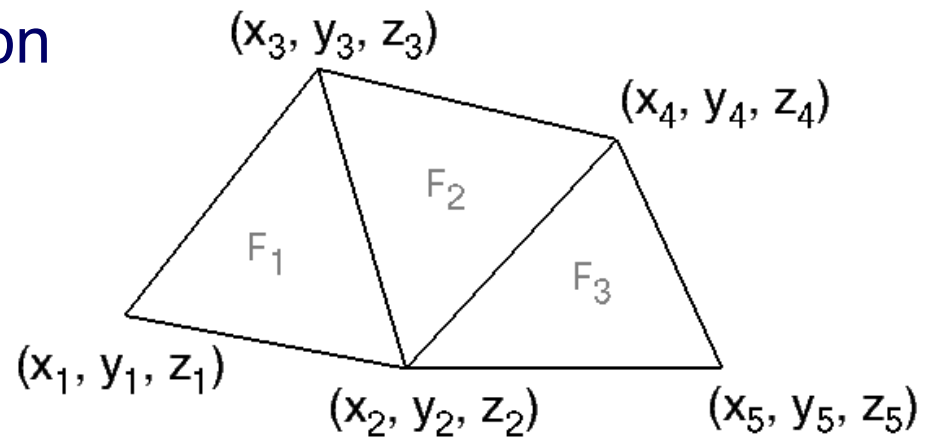
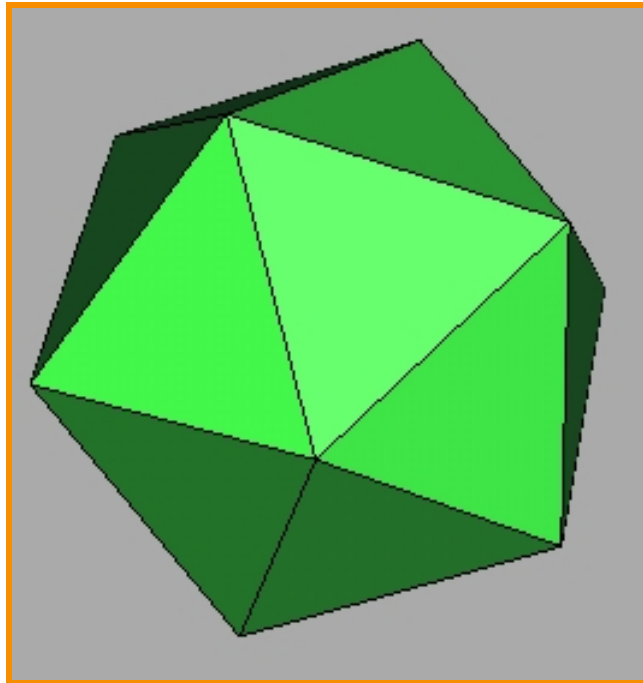
- List of independent faces
- Vertex and face tables
- Adjacency lists
- Winged edge
- Half edge
- etc.



# Independent Faces

Each face lists vertex coordinates

- Redundant vertices
- No adjacency information



FACE TABLE

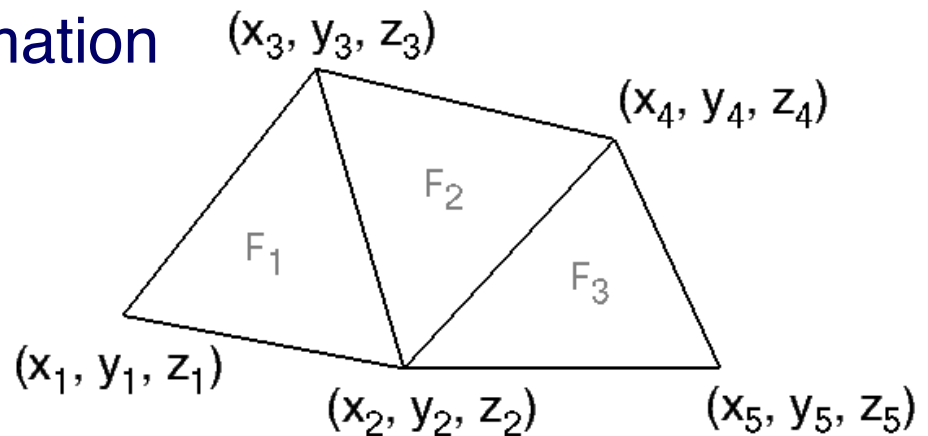
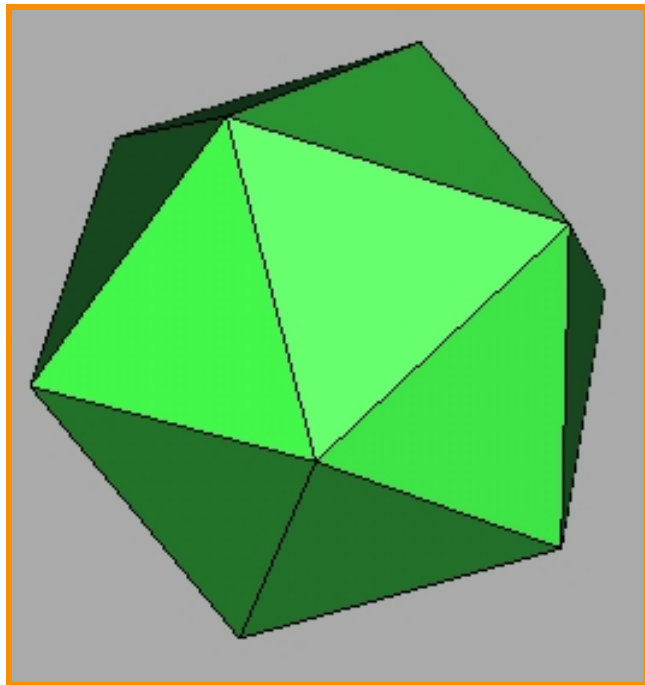
F <sub>1</sub>	(x <sub>1</sub> , y <sub>1</sub> , z <sub>1</sub> ) (x <sub>2</sub> , y <sub>2</sub> , z <sub>2</sub> ) (x <sub>3</sub> , y <sub>3</sub> , z <sub>3</sub> )
F <sub>2</sub>	(x <sub>2</sub> , y <sub>2</sub> , z <sub>2</sub> ) (x <sub>4</sub> , y <sub>4</sub> , z <sub>4</sub> ) (x <sub>3</sub> , y <sub>3</sub> , z <sub>3</sub> )
F <sub>3</sub>	(x <sub>2</sub> , y <sub>2</sub> , z <sub>2</sub> ) (x <sub>5</sub> , y <sub>5</sub> , z <sub>5</sub> ) (x <sub>4</sub> , y <sub>4</sub> , z <sub>4</sub> )



# Vertex and Face Tables

Each face lists vertex references

- Shared vertices
- Still no adjacency information



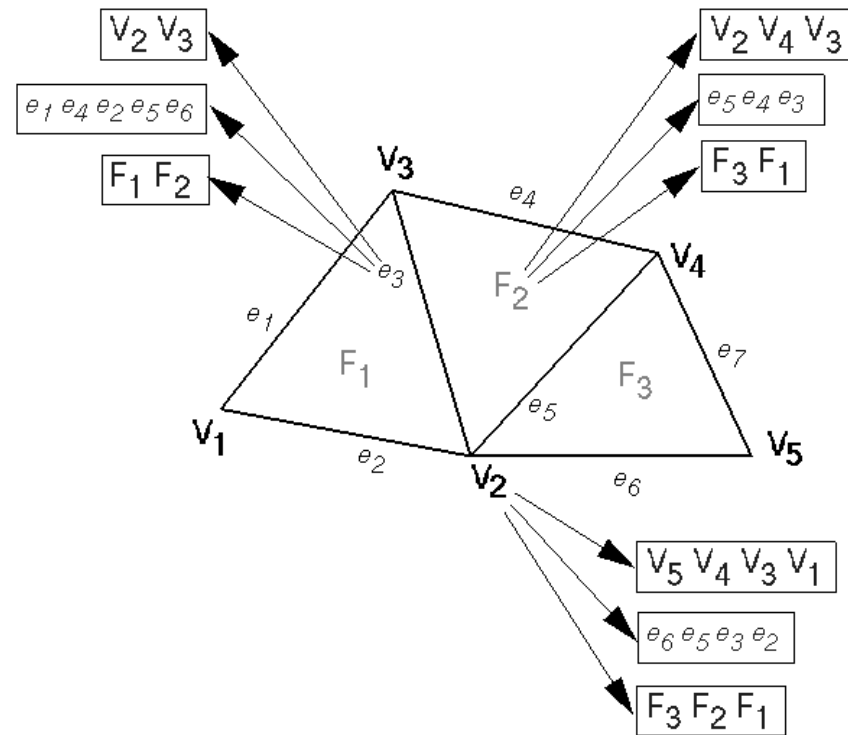
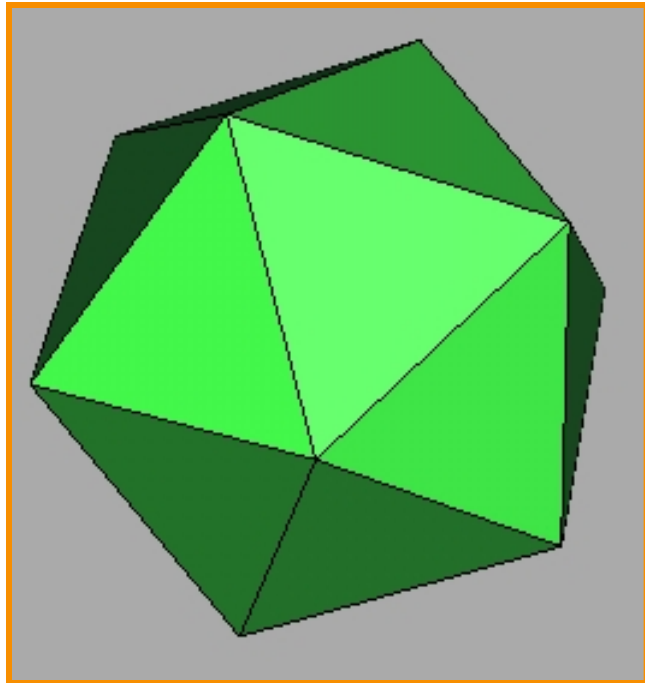
VERTEX TABLE			
V <sub>1</sub>	X <sub>1</sub>	Y <sub>1</sub>	Z <sub>1</sub>
V <sub>2</sub>	X <sub>2</sub>	Y <sub>2</sub>	Z <sub>2</sub>
V <sub>3</sub>	X <sub>3</sub>	Y <sub>3</sub>	Z <sub>3</sub>
V <sub>4</sub>	X <sub>4</sub>	Y <sub>4</sub>	Z <sub>4</sub>
V <sub>5</sub>	X <sub>5</sub>	Y <sub>5</sub>	Z <sub>5</sub>

FACE TABLE			
F <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>
F <sub>2</sub>	V <sub>2</sub>	V <sub>4</sub>	V <sub>3</sub>
F <sub>3</sub>	V <sub>2</sub>	V <sub>5</sub>	V <sub>4</sub>

# Adjacency Lists

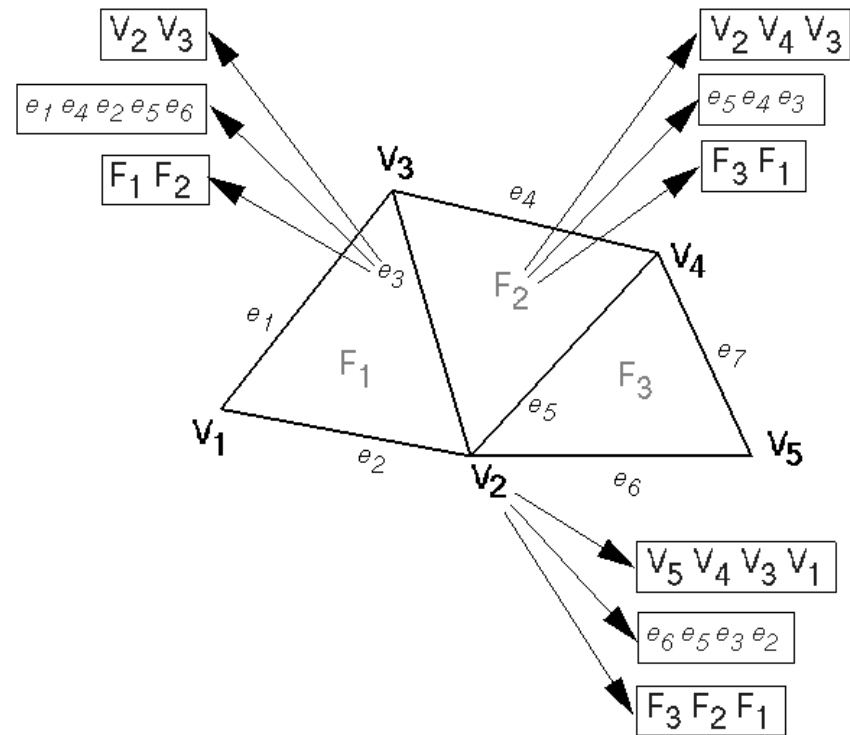
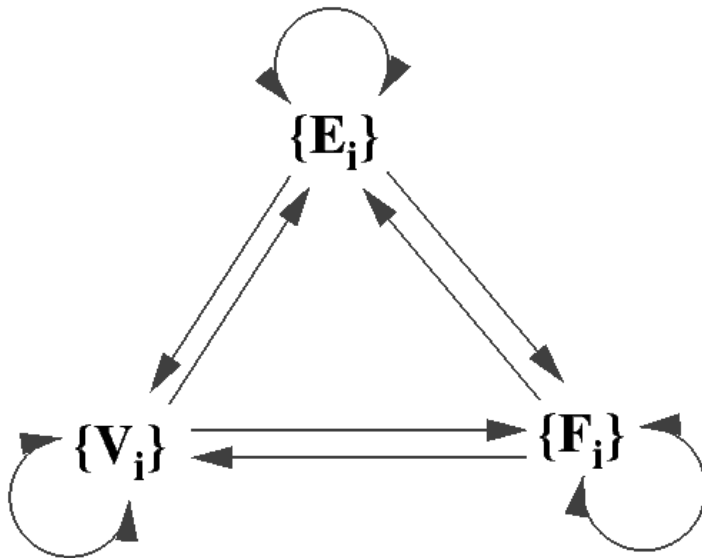
Store all vertex, edge, and face adjacencies

- Efficient adjacency traversal
- Extra storage



# Partial Adjacency Lists

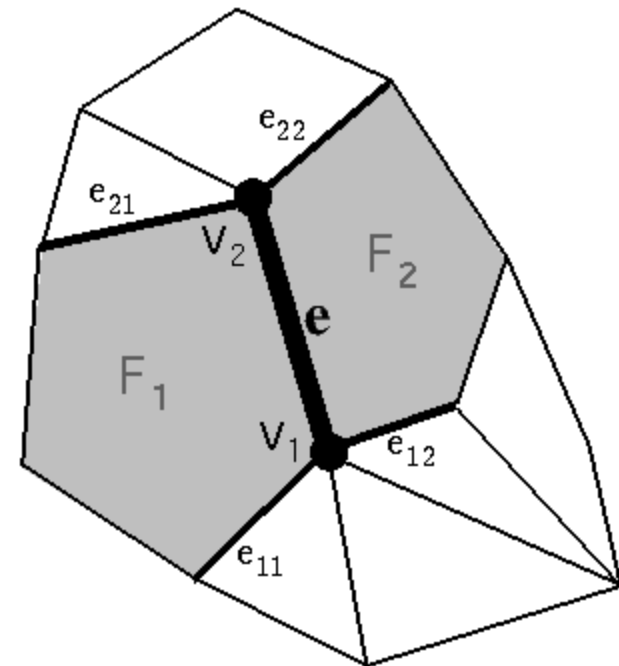
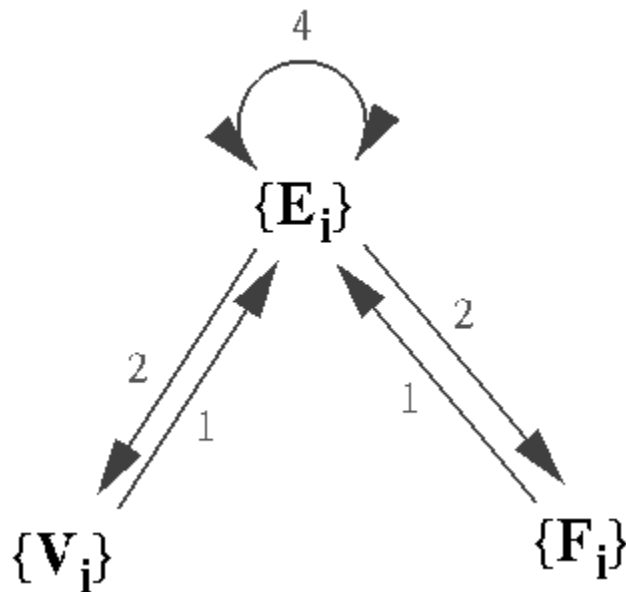
Can we store only some adjacency relationships and derive others?



# Winged Edge

Adjacency encoded in edges

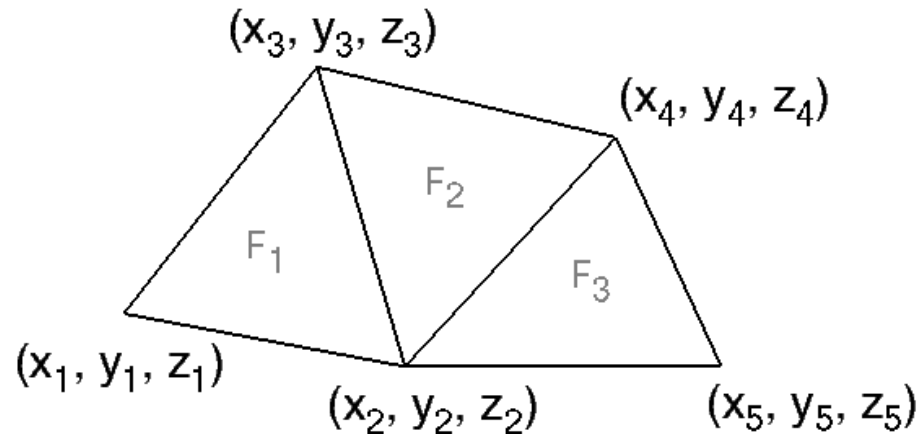
- All adjacencies in  $O(1)$  time
- Little extra storage (fixed records)
- Arbitrary polygons





# Winged Edge

Example:



VERTEX TABLE				
V <sub>1</sub>	X <sub>1</sub>	Y <sub>1</sub>	Z <sub>1</sub>	e <sub>1</sub>
V <sub>2</sub>	X <sub>2</sub>	Y <sub>2</sub>	Z <sub>2</sub>	e <sub>6</sub>
V <sub>3</sub>	X <sub>3</sub>	Y <sub>3</sub>	Z <sub>3</sub>	e <sub>3</sub>
V <sub>4</sub>	X <sub>4</sub>	Y <sub>4</sub>	Z <sub>4</sub>	e <sub>5</sub>
V <sub>5</sub>	X <sub>5</sub>	Y <sub>5</sub>	Z <sub>5</sub>	e <sub>6</sub>

EDGE TABLE					11	12	21	22
e <sub>1</sub>	V <sub>1</sub>	V <sub>3</sub>	F <sub>1</sub>		e <sub>2</sub>	e <sub>2</sub>	e <sub>4</sub>	e <sub>3</sub>
e <sub>2</sub>	V <sub>1</sub>	V <sub>2</sub>	F <sub>1</sub>		e <sub>1</sub>	e <sub>1</sub>	e <sub>3</sub>	e <sub>6</sub>
e <sub>3</sub>	V <sub>2</sub>	V <sub>3</sub>	F <sub>1</sub>	F <sub>2</sub>	e <sub>2</sub>	e <sub>5</sub>	e <sub>1</sub>	e <sub>4</sub>
e <sub>4</sub>	V <sub>3</sub>	V <sub>4</sub>		F <sub>2</sub>	e <sub>1</sub>	e <sub>3</sub>	e <sub>7</sub>	e <sub>5</sub>
e <sub>5</sub>	V <sub>2</sub>	V <sub>4</sub>	F <sub>2</sub>	F <sub>3</sub>	e <sub>3</sub>	e <sub>6</sub>	e <sub>4</sub>	e <sub>7</sub>
e <sub>6</sub>	V <sub>2</sub>	V <sub>5</sub>	F <sub>3</sub>		e <sub>5</sub>	e <sub>2</sub>	e <sub>7</sub>	e <sub>7</sub>
e <sub>7</sub>	V <sub>4</sub>	V <sub>5</sub>		F <sub>3</sub>	e <sub>4</sub>	e <sub>5</sub>	e <sub>6</sub>	e <sub>6</sub>

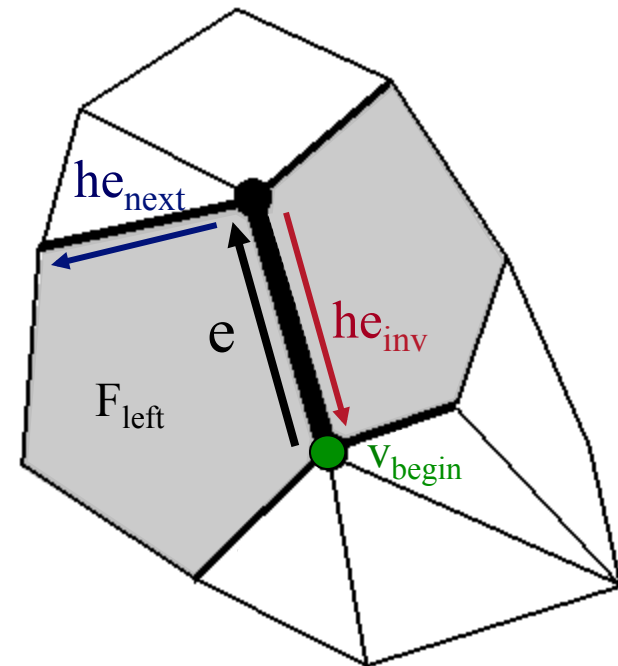
FACE TABLE	
F <sub>1</sub>	e <sub>1</sub>
F <sub>2</sub>	e <sub>3</sub>
F <sub>3</sub>	e <sub>5</sub>

# Half Edge

Adjacency encoded in edges

- All adjacencies in  $O(1)$  time
- Little extra storage (fixed records)
- Arbitrary polygons

Similar to winged-edge,  
except adjacency  
encoded in half-edges



# Summary



## Polygonal meshes

- Most common surface representation
- Fast rendering

## Processing operations

- Must consider irregular vertex sampling
- Must handle/avoid topological degeneracies

## Representation

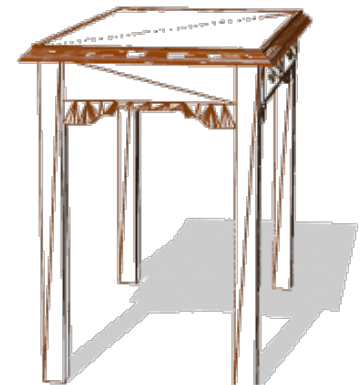
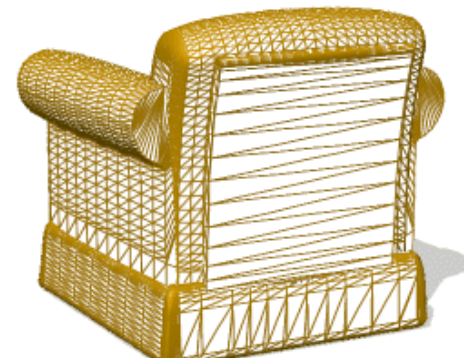
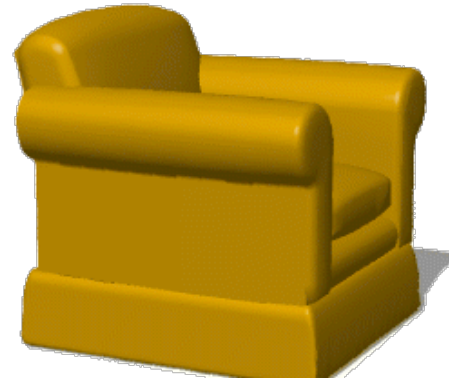
- Which adjacency relationships to store depend on which operations must be efficient

# 3D Polygonal Meshes



## Properties

- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.





# 3D Polygonal Meshes



## Properties

- ☺ Efficient display
- ☺ Easy acquisition
- ☹ Accurate
- ☹ Concise
- ☹ Intuitive editing
- ☹ Efficient editing
- ☹ Efficient intersections
- ☹ Guaranteed validity
- ☹ Guaranteed smoothness

