## Computer Science 426 Midterm <br> 3/16/06, 1:30PM-2:50PM

This test is 5 questions. Do all of your work on these pages (use the back for scratch space), giving the answer in the space provided. This is a closed-book exam -- you may use one-page of notes with writing on both sides during the exam. Please write out and sign the Honor Code pledge before turning in the test.
"I pledge my honor that I have not violated the Honor Code during this examination."

| Question | Score |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| Total |  |

## Q1: Image Processing (10 Points)

a) For each of three sampling techniques (Point, Bilinear, and Gaussian), put a dot on the grid samples below that will be used to determine the color of the image at the point marked with a dot. Assume that image samples are positioned at the junctions of the grid lines. Also, for this part of the question, assume that the Gaussian filter width is 3 pixels.

Point

Bilinear

Gaussian
b) Consider downsampling (scaling) an $8 \times 8$ pixel image down to a 1 x 1 pixel image (this might be needed when drawing a textured polygon that covers just a single pixel). What will be the color of the single pixel in the output image using each of the three following filters (express your answer in terms of the pixels in the input image)?

Point sampling:

Bilinear interpolation:

Gaussian filtering:
c) Describe a method that computer graphics rendering systems use to accelerate this downsampling operation when rendering with texture maps. What preprocessing do they do to make a suitable image representation for each texture and how is that representation used to accelerate downsampling?

## Q2: Color Models (20 Points)

The figure below shows a representation of the CIE chromaticity diagram.
a) What is the important difference between points inside the colored, tongue-shaped region versus ones outside that region (one sentence)?
b) What does the point labeled "Illum C "represent (a phrase or sentence)?
c) Please write a mathematical formula for $\mathrm{C}_{3}$ in terms of $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ (one formula).
$\mathrm{C}_{3}=$
d) What does the triangular region outlined in gray represent (one phrase or sentence)?
e) Please explain why the colors printed on this page cannot possibly be an accurate representation of all colors in the CIE chromaticity diagram (one sentence).
f) This figure represents a 2D slice of the XYZ color space. Please explain how this slice was chosen and how colors vary across the dimension not shown (two short sentences).
g) Please estimate the dominant wavelength of $\mathrm{C}_{2}$ (one number).
h) Please estimate the dominant wavelength of $\mathrm{C}_{1}$ (one number).
i) What are the units of the numbers along the boundary of the colored region (one word)?


## Q3: Transformations (20 Points)

Describe the transformation that takes the 2D shape shown on the left to the 2 D shape shown on the right (e.g., $\mathrm{P} \rightarrow \mathrm{P}^{\prime}$, for every point P ). For each transformation, tell whether it is linear, affine, or projective, and provide a matrix M that applies the transformation (i.e., $\mathrm{P}^{\prime}=\mathrm{MP}$ ). You may express your matrices as a product of other matrices witho ut multiplying them together, and you may represent matrices executing translations by $T(t x$, ty), scales by $S(s x, s y)$, and counterclockwise rotations by $\mathrm{R}(\Theta)$.

b)


## Q3: Transformations (cont)





## Q4: Lighting and Global Illumination (20 Points)

The equation to compute the form factor $\mathrm{F}_{\mathrm{ij}}$ between two polygons i and j is as follows:

$$
F_{i j}=\frac{1}{A_{i}} \iint_{A_{i} A_{j}} \frac{V_{i j} \cos \Theta_{i}^{\prime} \cos \Theta_{o}}{\pi r^{2}} d A_{j} d A_{i} \quad \underbrace{\Theta_{i}^{\prime}}_{A_{i}}
$$

a) What does a form factor $\mathrm{F}_{\mathrm{ij}}$ measure (one precise sentence)?
b) Please provide an intuitive explanation for the two cosine terms in the numerator (one sentence and possibly a drawing).
c) Please provide an intuitive explanation of the visibility term $\left(\mathrm{V}_{\mathrm{ij}}\right)$ in the numerator (one sentence and possibly a drawing).
d) Please provide an intuitive explanation for the $\mathrm{r}^{2}$ term in the denominator (one sentence and possibly a drawing).
e) Which part of the form factor equation is most expensive to compute $\left(\cos \left(\Theta_{\mathrm{i}}{ }^{\prime}\right), \cos \left(\Theta_{\mathrm{o}}\right), \mathrm{V}_{\mathrm{ij}}, \mathrm{r}\right.$, or $\mathrm{A}_{\mathrm{i}}$ )? Please explain your answer with one or two sentences.

## Q4: Lighting and Global Illumination (cont)

f) What are the input parameters to a BRDF? (phrases)
g) Why do light waves scatter diffusely from rough surfaces? (one sentence and a picture)
h) Caustics occur when light is focused onto diffuse surfaces after specular reflections and/or refractions on other surfaces. Is your ray tracer as implemented in assignment \#2 (with rays traced from the camera viewpoint) able to produce images with caustics of this type? Please explain your answer with one or two sentences.

## Q5: Visibility (30 Points)

Imagine that our goal is to develop an algorithm that can determine the subset of polygons in a scene $S$ that are visible to a given point $P$ - i.e., every visible polygon has at least one point on its surface unoccluded from the perspective of P . The ing uts to our algorithm are a point P and a set of polygons $S$, and the output is a set of polygons $V$ that contains all polygons of $S$ that are at least partially visible to P . This problem is different than hidden surface removal in that we do not need to know exactly which portion of every polygon is visible to P , nor do we need to produce an image from the perspective of P , but rather we only need to determine for each polygon whether it is at least partially visible to P - i.e., whether it is in the visible set V. For example, in the 2 D scene on the right, $\mathrm{S}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}\}$ and $\mathrm{V}=\{\mathrm{a}, \mathrm{b}, \mathrm{d}\}$.

## THE ITEM BUFFER ALGORITHM:



The z-buffer algorithm for hidden surface removal can be adapted to address this visibility problem. The general strategy is to render the polygons of the scene into an image with: 1) a perspective projection having P as the center of projection, 2) a $z$-buffer to eliminate hidden surfaces, 3) flat shading to fill polygons with a unique color (r,g,b) for each polygon encoding an ID for the polygon (e.g., if the frame buffer has 8 bits for each of the red, green, and blue channels, then $\mathrm{r}=(\mathrm{ID} \gg 16) \& 255, \mathrm{~g}=(\mathrm{ID} \gg 8) \& 255, \mathrm{~b}=\mathrm{ID} \& 255)$. Then, the image can be read from the frame buffer, the ( $\mathrm{r}, \mathrm{g}, \mathrm{b}$ ) of every non-background pixel can be converted to a polygon ID (e.g., ID $=\mathrm{r} \ll 16+\mathrm{g}<8+\mathrm{b}$ ), and those polygon IDs read from the frame buffer can be added uniquely to the visible set.
a) What is the computational complexity of this item buffer algorithm for $n$ polygons and $p$ pixels? Please give your answer in big-O notation (e.g., $\mathrm{O}\left(\mathrm{n}^{5} \mathrm{p}^{6}\right)$ ) and explain how you got your answer.
b) Can this item buffer algorithm miss polygons that are actually visible to P? If not, explain how you can be sure. If so, describe a situation in which a visible polygon will not be added to the set.
c) Can this item buffer algorithm include polygons that are actually not visible to P? If not, explain how you can be sure. If so, describe a situation in which an occluded polygon will be added to the set.

## Q5: Visibility (cont)

d) Select any other hidden surface removal algorithm discussed in class or in the book (not z buffer), and describe how it could be used or adapted to solve the visibility problem described on the previous page (i.e., compute the set of polygons at least partially visible to a point P). Your description should be about as detailed as the ITEM BUFFER description on the previous page, and you may want to include a drawing to support your description.

## Q5: Visibility (cont)

e) What is the computational complexity of your algorithm for $n$ polygons and $p$ pixels? Please give your answer in big-O notation (e.g., $O\left(n^{5} p^{6}\right)$ ) and explain how you got your answer.
f) Can your algorithm miss polygons that are actually visible to P? If not, explain how you can be sure. If so, describe a situation in which a visible polygon will not be added to the set.
g) Can your algorithm include polygons that are actually not visible to P? If not, explain how you can be sure. If so, describe a situation in which an occluded polygon will be added to the set.
h) Describe at least one advantage of your algorithm over the item buffer algorithm described on the page before last.
i) Describe at least one advantage of the item buffer algorithm over your algorithm.

