



General Computer Science  
Princeton University  
Spring 2015

Douglas Clark

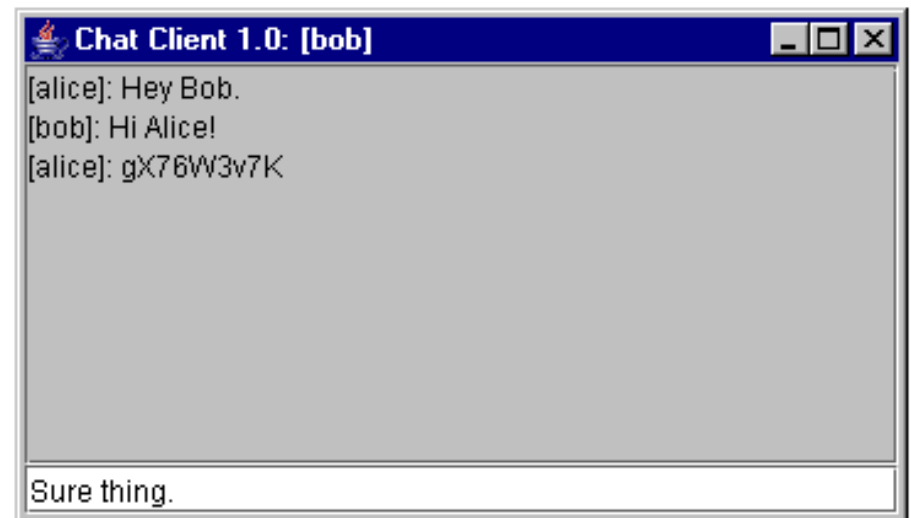
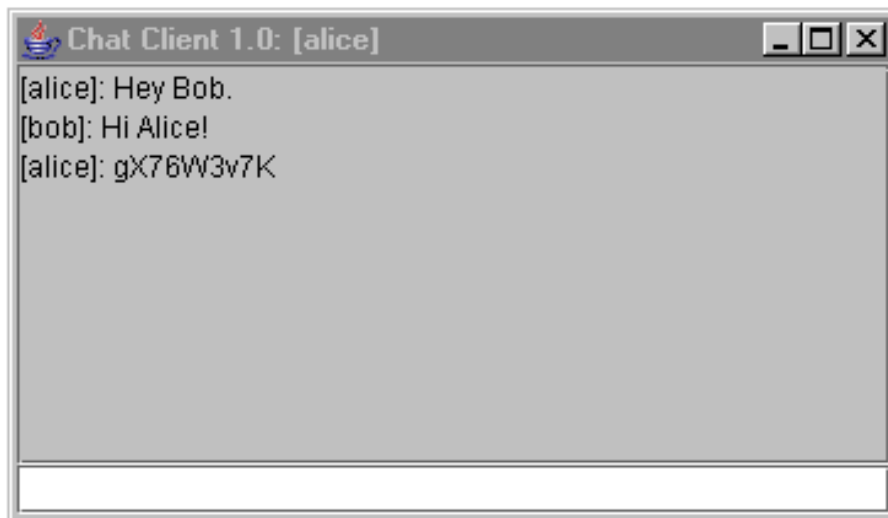
# 0. Prologue: A Simple Machine

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# Secure Chat

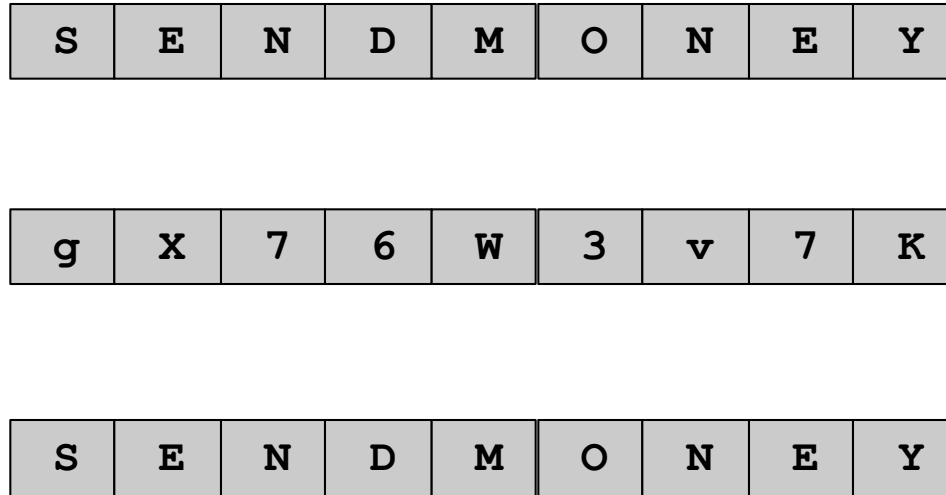
Alice wants to send a secret message to Bob

- Can you read the secret message gX76W3v7K ?
- But Bob can. How?



# Encryption Machine

*Goal.* Design a machine to encrypt and decrypt data.



encrypt



decrypt

## Enigma encryption machine.

- "Unbreakable" German code during WWII.
- Broken by Turing bombe.
- One of first uses of computers.
- Helped win Battle of Atlantic by locating U-boats.



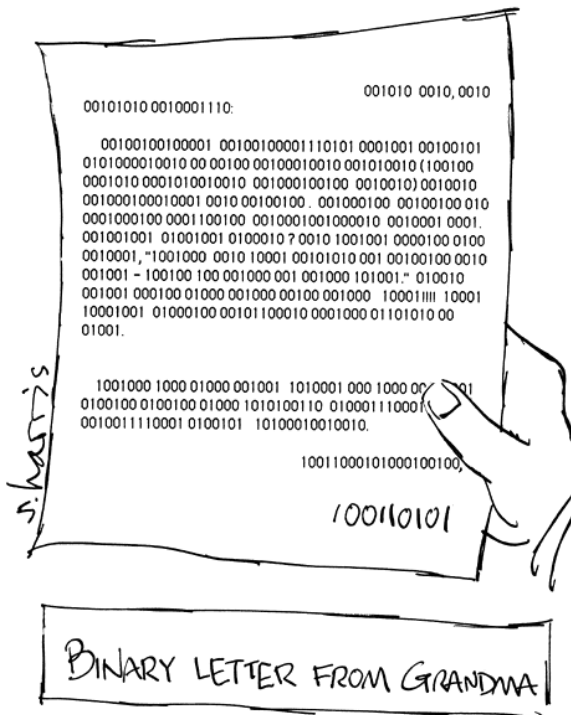
# A Digital World

Data is a sequence of bits. [bit = 0 or 1] ←

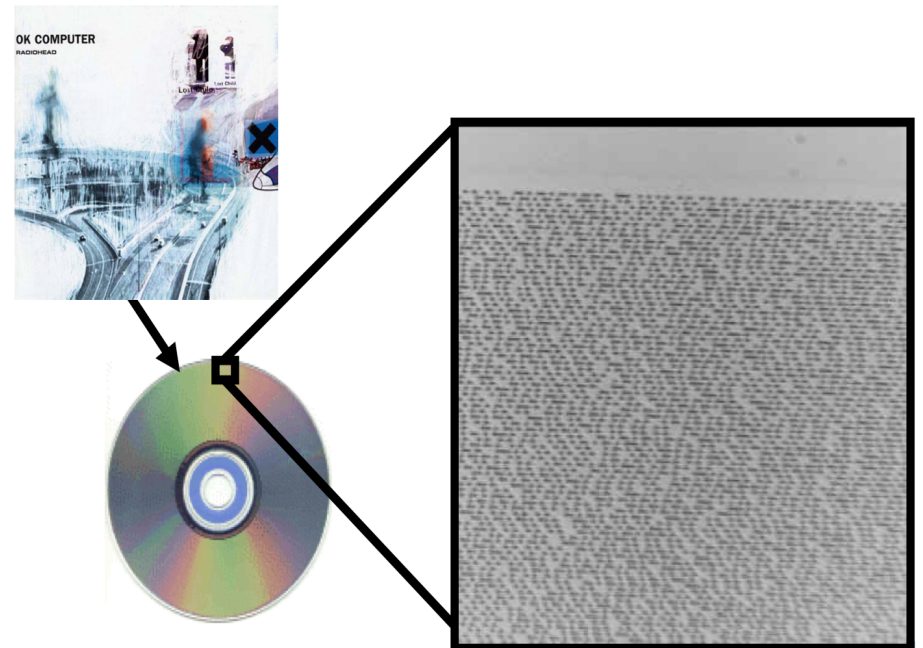
can use decimal digits, letters, or some other system, but bits are more easily encoded physically ("on-off", "up-down", "hot-cold", ...)

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

thousands of bits



billions of bits



# A Digital World

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

Ex. Base64 encoding of text.

- Simple method for representing **A-Z**, **a-z**, **0-9**, **+**, **/**
- 6 bits to represent each symbol (64 symbols)

000000 <b>A</b>	001000 <b>I</b>	010000 <b>Q</b>	011000 <b>Y</b>	100000 <b>g</b>	101000 <b>o</b>	110000 <b>w</b>	111000 <b>4</b>
000001 <b>B</b>	001001 <b>J</b>	010001 <b>R</b>	011001 <b>Z</b>	100001 <b>h</b>	101001 <b>p</b>	110001 <b>x</b>	111001 <b>5</b>
000010 <b>C</b>	001010 <b>K</b>	010010 <b>S</b>	011010 <b>a</b>	100010 <b>i</b>	101010 <b>q</b>	110010 <b>y</b>	111010 <b>6</b>
000011 <b>D</b>	001011 <b>L</b>	010011 <b>T</b>	011011 <b>b</b>	100011 <b>j</b>	101011 <b>r</b>	110011 <b>z</b>	111011 <b>7</b>
000100 <b>E</b>	001100 <b>M</b>	010100 <b>U</b>	011100 <b>c</b>	100100 <b>k</b>	101100 <b>s</b>	110100 <b>0</b>	111100 <b>8</b>
000101 <b>F</b>	001101 <b>N</b>	010101 <b>V</b>	011101 <b>d</b>	100101 <b>l</b>	101101 <b>t</b>	110101 <b>1</b>	111101 <b>9</b>
000110 <b>G</b>	001110 <b>O</b>	010110 <b>W</b>	011110 <b>e</b>	100110 <b>m</b>	101110 <b>u</b>	110110 <b>2</b>	111110 <b>+</b>
000111 <b>H</b>	001111 <b>P</b>	010111 <b>X</b>	011111 <b>f</b>	100111 <b>n</b>	101111 <b>v</b>	110111 <b>3</b>	111111 <b>/</b>

# One-Time Pad Encryption

## Encryption.

- Convert text message to N **bits**. [0 or 1]

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64

# One-Time Pad Encryption

## Encryption.

- Convert text message to N bits.
- Generate N random bits (**one-time pad**).

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits



# One-Time Pad Encryption

## Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	$x \wedge y$
0	0	0
0	1	1
1	0	1
1	1	0

sum corresponding pair of bits: 1 if sum is odd, 0 if even  
 alternatively: 1 if bits are different, 0 if the same

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR

$0 \wedge 1 = 1$

# One-Time Pad Encryption

## Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

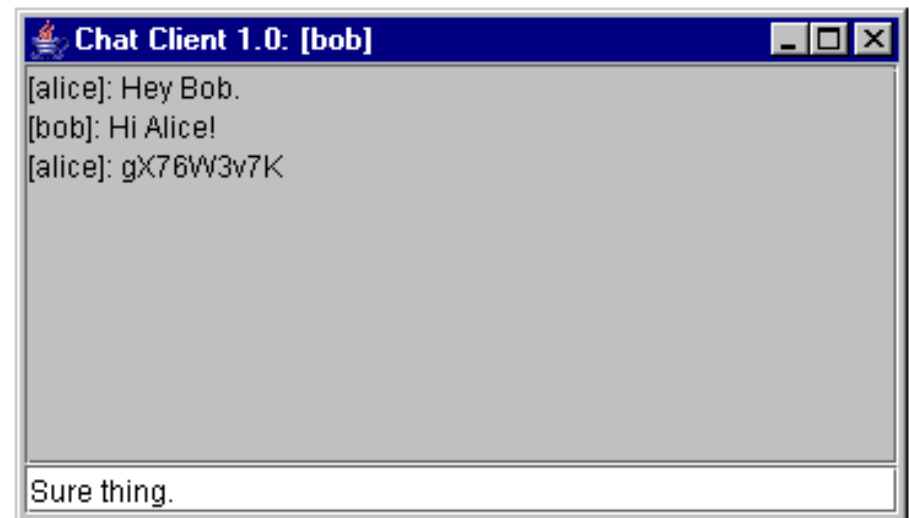
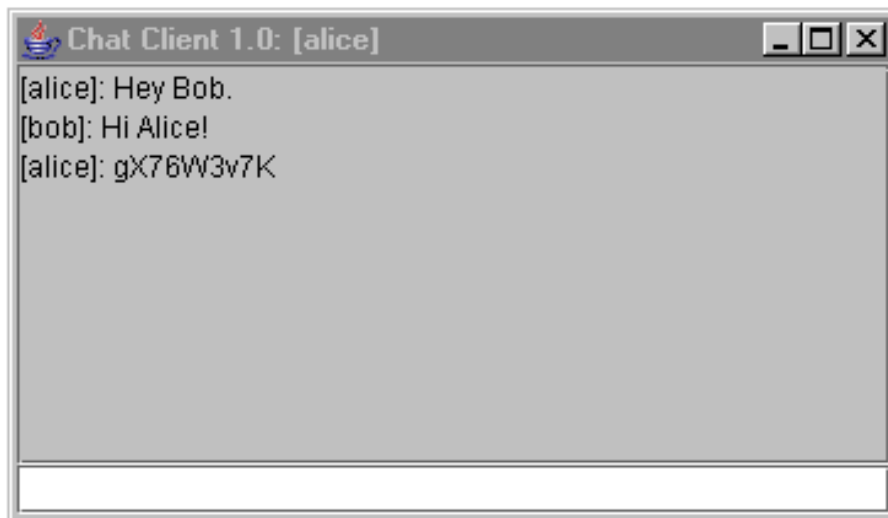
char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	W	3	v	7	K	encrypted

# Secure Chat

Alice wants to send a secret message to Bob

- Can you read the secret message gX76W3v7K ?
- But Bob can. How?



# One-Time Pad Decryption

## Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	K
---	---	---	---	---	---	---	---	---

encrypted

# One-Time Pad Decryption

## Decryption.

- Convert encrypted message to binary.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
W	22	010110
...	...	...

g	x	7	6	W	3	v	7	K
100000	010111	111011	111010	010110	110111	101111	111011	001010

encrypted

base64

# One-Time Pad Decryption

## Decryption.

- Convert encrypted message to binary.
- Use **same** N random bits (one-time pad).
- **Key point:** Bob and Alice agreed on the one-time pad **beforehand**

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

# One-Time Pad Decryption

## Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

g	x	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR

$1 \wedge 1 = 0$

# One-Time Pad Decryption

## Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
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...	...	...
M	12	001100
...	...	...

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message



magic?



# Why Does It Work?

Crucial property. Decrypted message = original message.

Notation	Meaning
<code>a</code>	original message bit
<code>b</code>	one-time pad bit
<code>^</code>	XOR operator
<code>a ^ b</code>	encrypted message bit
<code>(a ^ b) ^ b</code>	decrypted message bit

Why is crucial property true?

- Use properties of XOR.
- $(a \oplus b) \oplus b = a \oplus (b \oplus b) = a \oplus 0 = a$

associativity of  $\oplus$       always 0      identity

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

# One-Time Pad Decryption (with the wrong pad)

## Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	K
---	---	---	---	---	---	---	---	---

encrypted

# One-Time Pad Decryption (with the wrong pad)

## Decryption.

- Convert encrypted message to binary.

<b>g</b>	<b>x</b>	<b>7</b>	<b>6</b>	<b>w</b>	<b>3</b>	<b>v</b>	<b>7</b>	<b>K</b>	<b>encrypted</b>
<b>100000</b>	<b>010111</b>	<b>111011</b>	<b>111010</b>	<b>010110</b>	<b>110111</b>	<b>101111</b>	<b>111011</b>	<b>001010</b>	<b>base64</b>

# One-Time Pad Decryption (with the wrong pad)

## Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits

# One-Time Pad Decryption (with the wrong pad)

## Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

# One-Time Pad Decryption (with the wrong pad)

## Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text: **Oops**.

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR
I	L	O	V	E	O	K	R	A	wrong message

# Eve's Problem (one-time pads)

Key point: Without the pad, Eve cannot understand the message.



But Eve has a computer. Why not try all possible pads?

One problem: it might take a long time [stay tuned].

Worse problem: she would see all possible messages!

- 54 bits
- $2^{54}$  possible messages, all different.
- $2^{54}$  possible **encoded** messages, all different.
- No way for Eve to distinguish real message from any other message.

One-time pad is "provably secure".

→ **IF** pad is random and used only once

AAAAAAAAAA	gX76W3v7K
AAAAAAAAAB	gX76W3v7L
AAAAAAAAAC	gX76W3v7I
...	
oc1tS5lqK	ILOVEOKRA
...	
qwDgbDuav	Kn4aN0Bh1
...	
tTtpWk+1E	NEWTATTOO
...	
yT25a5i/S	SENDMONEY
...	
/////////+	fo7FpIQE0
/////////	fo7FpIQE1

# Goods and Bads of One-Time Pads

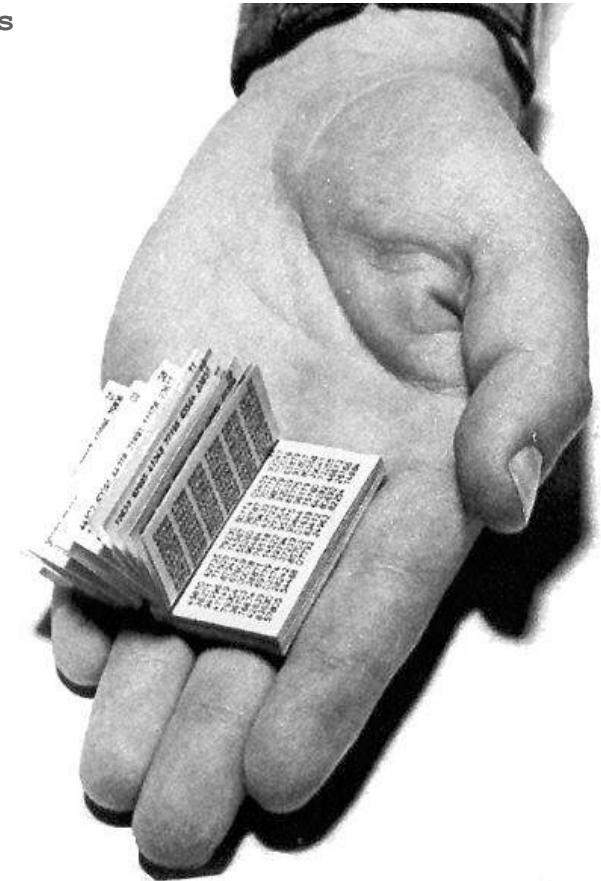
## Good.

- Easily computed by hand.
- Very simple encryption/decryption processes.
- Provably unbreakable if bits are truly random. [Shannon, 1940s]

eavesdropper Eve sees only random bits

## Bad.

- (After a short break . . .)



a Russian one-time pad



# COS 126 Overview

What is **COS 126**? Broad, but technical, introduction to **computer science**.

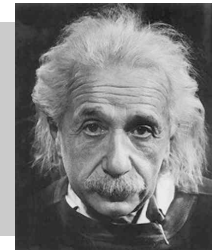
## Goals.

- Demystify computer systems.
- Empower you to exploit available technology.
- Build awareness of substantial intellectual underpinnings.

## Topics.

- **Programming** in Java.
- Machine architecture.
- Theory of computation.
- **Applications** to science, engineering, and commercial computing.

*“ Computers are incredibly fast, accurate, and stupid; humans are incredibly slow, inaccurate, and brilliant; together they are powerful beyond imagination. ” – Albert Einstein*



# The Basics

Lectures. [Clark]

Precepts and grading. [Gabai · Ginsburg · Leyzberg · Ararat · Cook · Frankle · Goldfeder · Gossels · Grover · Hasdemir · Jones · Kaplan · Li · Petersen · Song · Ulus · Yang]

- Tips on assignments, worked examples, clarify lecture material.
- Informal and interactive.

Friend 016/017 lab.

- Undergraduate lab assistants.
- Help with systems and debugging.

Piazza. [online discussion]

- Best chance for quick response to a question.
- Post to class or via private post to staff.

# Grades

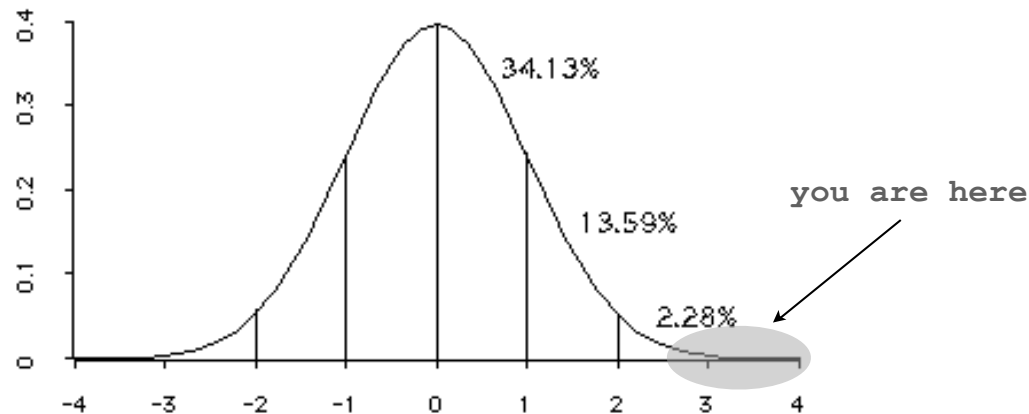
Course grades. No preset "curve" or quota.

9 programming assignments. 40%.

2 written exams (in lecture, midterm week & last week). 35%.

2 programming exams (evenings). 15%.

Final programming project (due Dean's date - 1). 10%.



# Course Materials

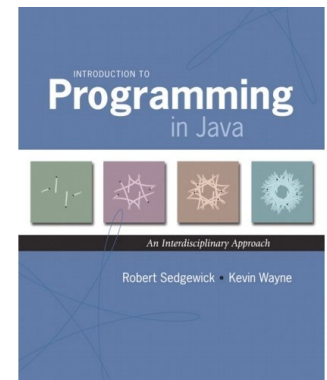
Course website. [[www.princeton.edu/~cos126](http://www.princeton.edu/~cos126)]

- Submit assignments.
- Programming assignments.
- Lecture slides ← (print before lecture)  
annotate during lecture
- "Booksite".
  - Summary of course content.
  - Code, exercises, examples.
  - Supplementary material.
  - NOT the same as Text
  - for use while online

Course text. [Sedgewick and Wayne]

- Full introduction to course material
- Developed for this course
- For use while learning and studying

Recommended reading (lectures 18-19). [Harel]



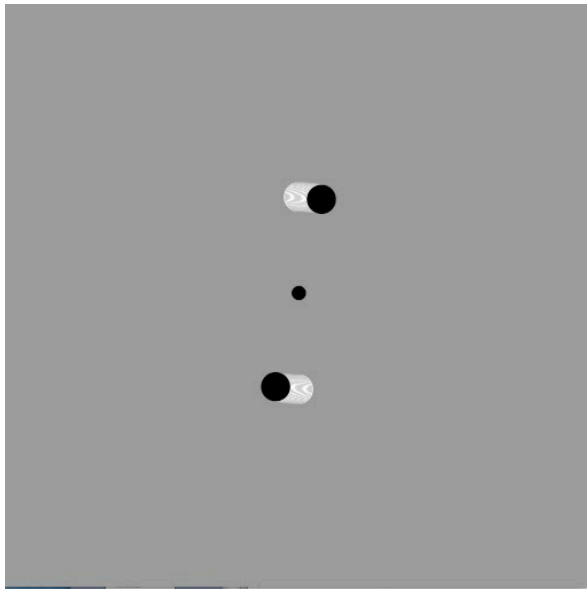
skim before lecture;  
read thoroughly  
afterwards



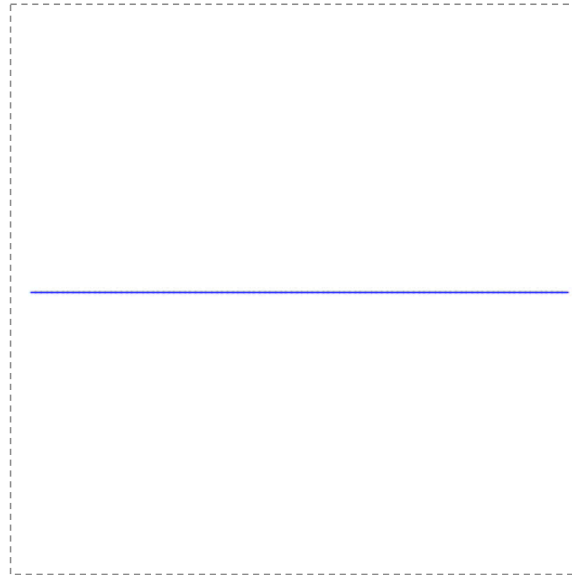
# Programming Assignments

## Desiderata.

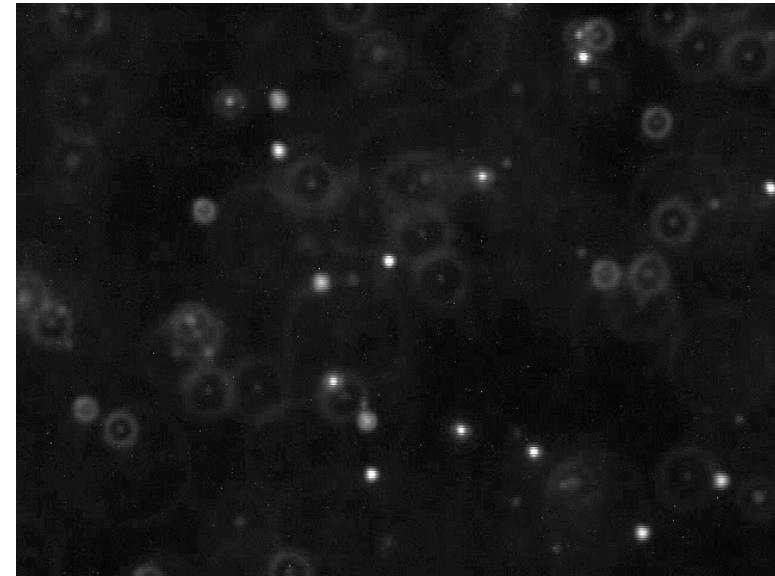
- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch on your own computer !



N-body simulation



pluck a guitar string



estimate Avogadro's number

# Programming Assignments

## Desiderata.

- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch on your own computer!

**Due.** Mondays midnight via Web submission.

## Computing equipment.

- Your laptop. [OS X, Windows, Linux, iPhone, ... ]
- OIT desktop. [Friend 016 and 017 labs]

## Advice.

- Start early; plan multiple sessions.
- Seek help when needed. (Our job is to help you!)
- Use the **Piazza** online forum for Q&A about assignments, course material.


# What's Ahead?

Lecture 2. Intro to Java.

Precept 1. Meets today/tomorrow.

Not registered? Go to any precept now; officially register ASAP.

Need to Change precepts? Use TigerHub.

  
see Colleen Kenny-McGinley in CS 210 (ckenny@cs.princeton.edu) only  
if the only precept time you can attend is closed

Assignment 0.

- Due Monday midnight.
- Read Sections 1.1 and 1.2 in textbook.
- Install Java programming environment (find directions in Assignment 0)
- Lots of help available, don't be bashful.

END OF ADMINISTRATIVE STUFF

# Goods and Bads of One-Time Pads

## Good.

- Easily computed by hand.
- Very simple encryption/decryption processes.
- Provably unbreakable if bits are truly random. [Shannon, 1940s]

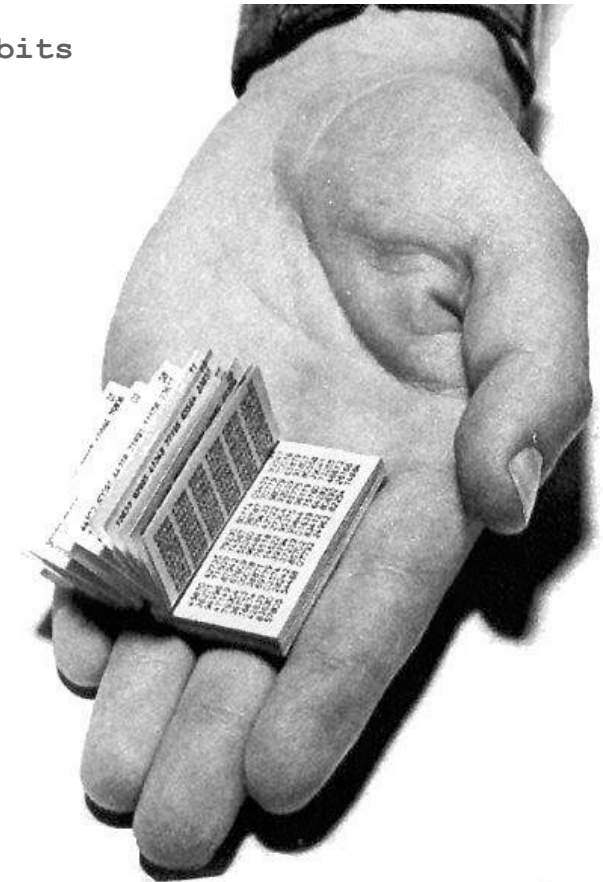
eavesdropper Eve sees only random bits

"one time" means one time only!

## Bad.

- Easily breakable if pad is re-used.
- **Pad must be as long as the message.**
- Truly random bits are very hard to come by.
- Pad must be distributed securely.

impractical for Web commerce




a Russian one-time pad



# Pseudo-Random Bit Generator

## Practical middle-ground.

- Make a "random" bit generator gadget.
- Alice and Bob each get identical small gadgets. 
- also, matching initial values, or "**seeds**," for their gadgets

instead of identical  
large one-time pads

**Goal.** Small gadget that produces a long sequence of bits.

# Pseudo-Random Bit Generator

Small **deterministic** gadgets that produce long sequences of **pseudo-random** bits:

- Enigma
- **Linear feedback shift register.**
- Linear congruential generator.
- Blum-Blum-Shub generator.
- [many others have been invented]

**Pseudo-random?** Bits are not really random:

- Bob's and Alice's gadgets must produce the same bits from the same seed.
- Bits must have as many properties of random bits as possible (to foil Eve).

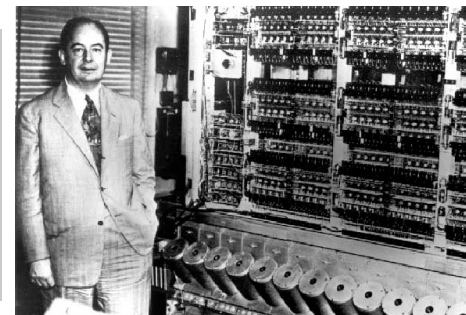
Ex 1. approximately  $\frac{1}{2}$  0s and  $\frac{1}{2}$  1s

Ex 2. approximately  $\frac{1}{4}$  each of 00, 01, 10, 11

*“ Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin. ”*

– John von Neumann (left)

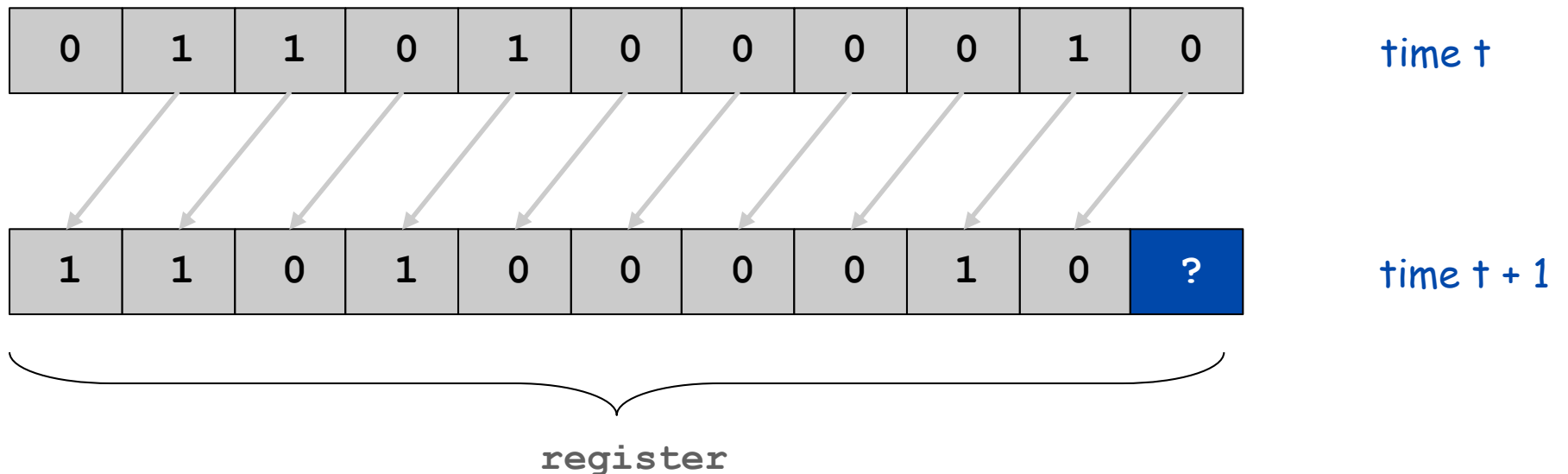
– ENIAC (right)



# Shift Register

## Shift register terminology.

- Bit: 0 or 1.
- Cell: storage element that holds one bit.
- Register: sequence of cells.
- Seed: initial sequence of bits.
- Shift register: when clock ticks, bits propagate one position to left.



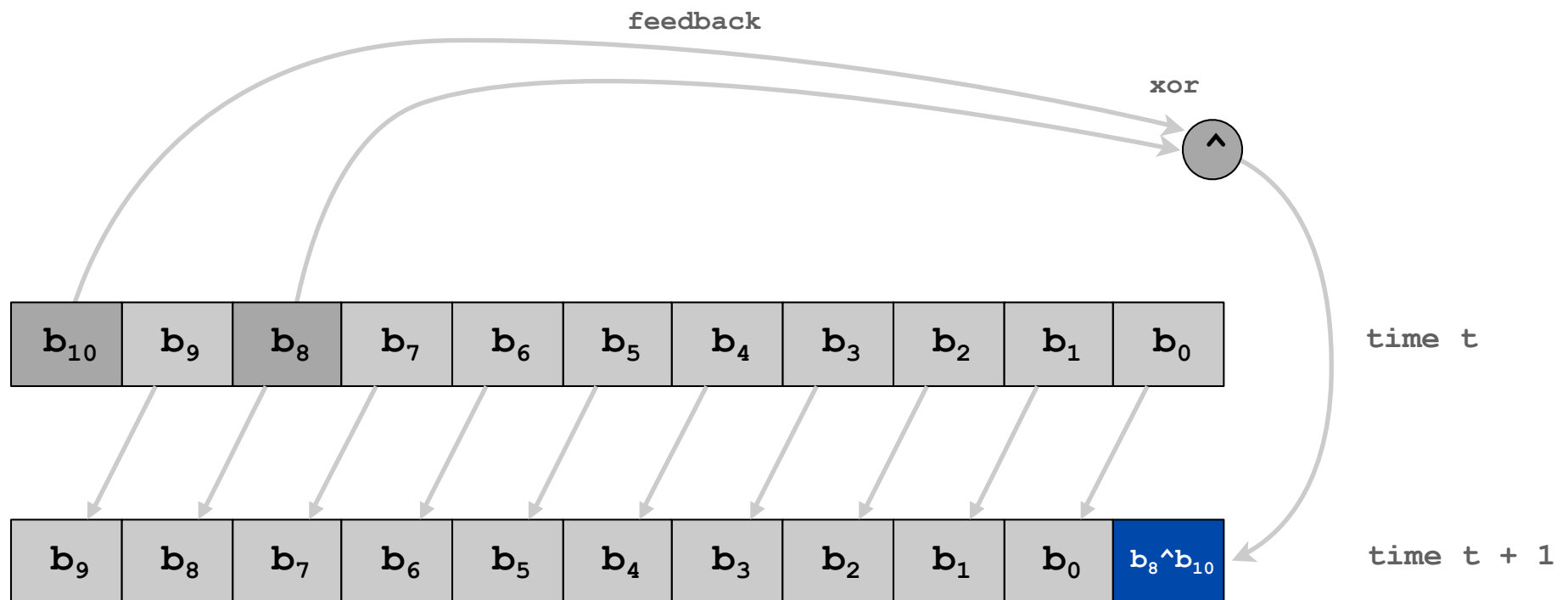
# Linear Feedback Shift Register (LFSR)

{8, 10} linear feedback shift register.

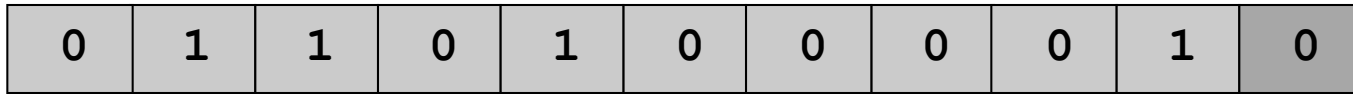
- Shift register with 11 cells.
- Bit  $b_0$  is XOR of previous bits  $b_8$  and  $b_{10}$ .
- Pseudo-random bit = (new)  $b_0$ .

XOR Truth Table

x	y	$x \wedge y$
0	0	0
0	1	1
1	0	1
1	1	0



# Linear Feedback Shift Register Demo

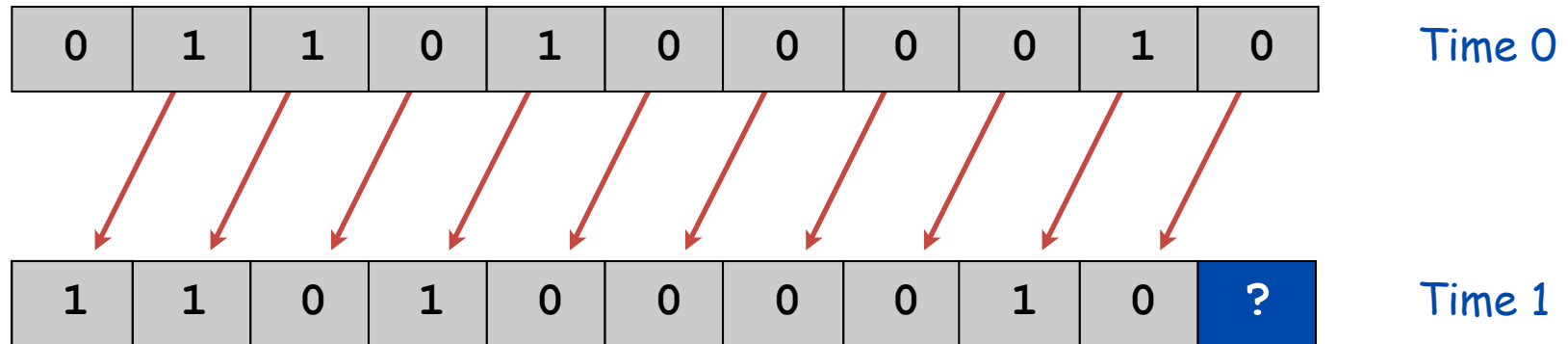


Time 0

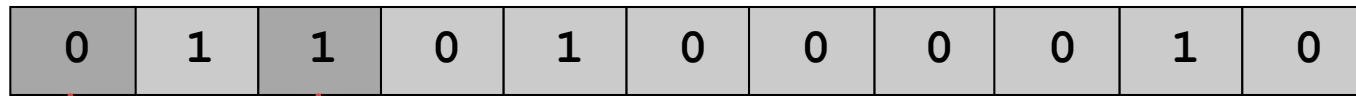


"seed" = initial  
contents

# Linear Feedback Shift Register Demo



# Linear Feedback Shift Register Demo



Time 0

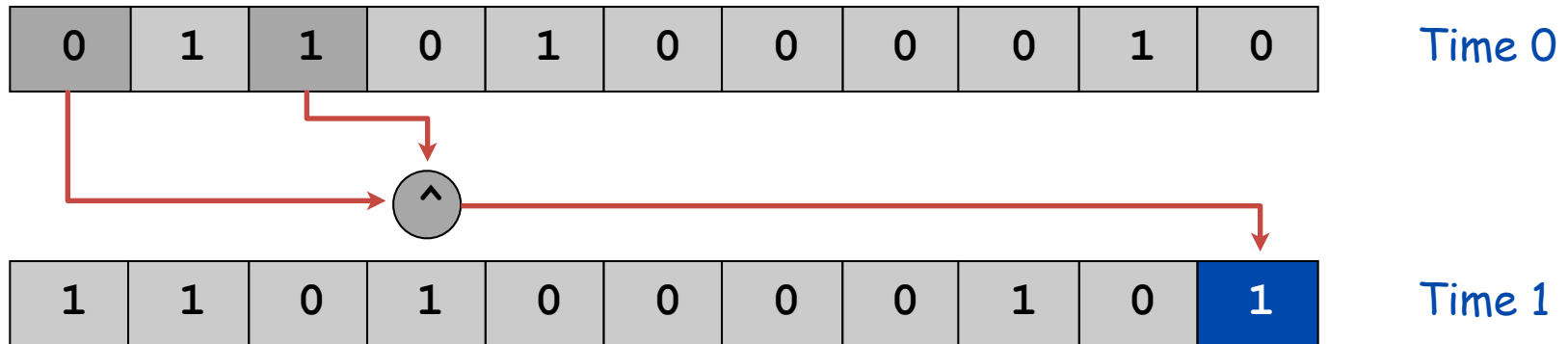


Time 1

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

# Linear Feedback Shift Register Demo



XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0



# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

1	0	0	0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 4

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

1	0	0	0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 4

0	0	0	0	1	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 5

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

1	0	0	0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 4

0	0	0	0	1	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 5

0	0	0	1	0	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 6

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

1	0	0	0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 4

0	0	0	0	1	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 5

0	0	0	1	0	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 6

0	0	1	0	1	1	0	0	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 7

# Linear Feedback Shift Register Demo

0	1	1	0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 0

1	1	0	1	0	0	0	0	1	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 1

1	0	1	0	0	0	0	1	0	1	1
---	---	---	---	---	---	---	---	---	---	---

Time 2

0	1	0	0	0	0	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 3

1	0	0	0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---	---	---	---

Time 4

0	0	0	0	1	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 5

0	0	0	1	0	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---

Time 6

0	0	1	0	1	1	0	0	1	0	0
---	---	---	---	---	---	---	---	---	---	---

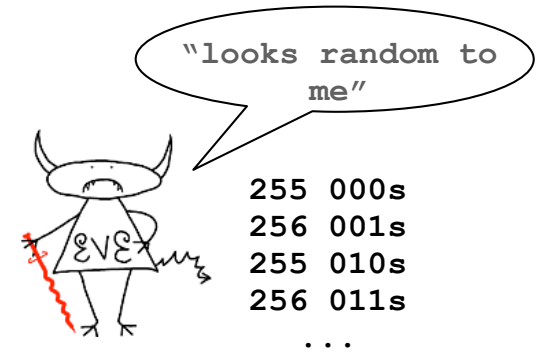
Time 7

0	1	0	1	1	0	0	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

Time 8

# Random Numbers

Q. Are these 2000 numbers random?  
If not, what is the pattern?



```
1100100100111101101110010110101110011000101111110100100001001101001011110011001001111111011100
000101011000100001110101001101000011110010011001110111111010100000100001000101001010100011000
0010111100010010011010110111100011010011011100111101011110010001001110101011101000001010010001
000110101010111000000010110000010011100010111011010010101100110000111111001100000111111000110
000110111100111010011110100111001001110111011101010101010000000001000000001010000001000100001
0101010010000000110100000111001000110111010111010100010100001010001001000101011010100001100001
0011110010111001110010111101110010010101110110000101011100100001011101001001010011011000111101
1101100101010111100000010011000010111110010010001110110101101011000110001110111101101010010110
0001100111001111110111100001010011001000111111010110000100011100101011011100001101011001110001
111101101100010110111010011010100111100001110011001101111111101000000010010000010110100010011
0010101111110000100001100101001111100011100011011011011101101101010110110000011011100011101011
011010001101100101110111100101010011100000111011000110101110111000101011011000000110010000111
1101001100010011111010111000100010110101010011000000111110000110001100111101111110010100001110
0010011011010111101100010010111010110010100011110001011001101001111110011100001111011001100101
111111100100000011101000011010010011100110111011110101010001000000101010000100000100101000101
100010100111010001110100101101001100110011111111110000000001100000001111000001100110001111111
10110000001011100001001011001011001111001111001111001111001101100111110111110001010001101000
1011100101001011100011001011011111001101000111110010110001110011101101111010110100100011001101
0111111100010000011010100011100001011011001001101111011110100101001001100011011111011101000101
0100101000001100010001111010101100100000111101000110010010111110110010001011110101001001000011
0110100111011001110101111101000100010010101010110000000011100000011011000011101110011010101111
10000010001100010101111010
```

A. No. This is output of {8, 10} LFSR with seed 01101000010 !



# LFSR Encryption

## Encryption.

- Convert text message to N bits.
- Initialize LFSR with given seed
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	<b>LFSR bits</b>
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	W	3	v	7	K	encrypted

# LFSR Decryption

## Decryption.

- Convert encrypted message to binary.
- Initialize identical LFSR with **same** seed
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

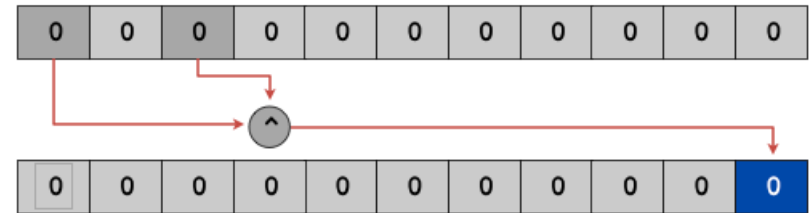
char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

# Key properties of LFSRs

**Property 1:** A zero fill (all 0s) produces all 0s.

- So don't use all 0s as a seed!
- Fill of all 0s will not otherwise occur.



**Property 2:** Bitstream must eventually cycle.

- $2^N - 1$  nonzero fills in an N-bit register.
- Future output completely determined by current fill.

**Property 3:** Cycle length in an N-bit register is at most  $2^N - 1$ .

- Could be smaller; cycle length depends on tap positions.
- Need higher math (theory of finite groups) to know tap positions for given N

Ex: (1, 2) LFSR

001

010

101

011

111

110

100

001  $2^3 - 1 = 7$

**Bottom line:** 11-bit register generates at most 2047 bits before cycling, so use a longer register (say,  $N = 61$ ).

challenge for the bored: what tap positions?

# Eve's Problem (LFSR encryption/decryption)

**Key point:** Without the (short) seed,  
Eve cannot understand the (long) message.



**But Eve has a computer. Why not try all possible seeds?**

- Seeds are short, messages are long.
- All seeds give a tiny fraction of all messages.
- Extremely likely that all but real seed will produce gibberish.

assume Eve has a machine  
(knows LFSR length and taps)

**Bad news (for Eve): Alice and Bob can use a much larger LFSR.**

- For instance: 61-bit register implies  $2^{61}$  possibilities.
- If Eve could check 1 million seeds per second,  
it would take her **730 centuries** to try them all!

Exponential growth dwarfs technological improvements [stay tuned].

- 1000 bits:  $2^{1000}$  possibilities.
- Age of the universe in microseconds:  $2^{70}$

(20,  $2^{20}$ )

# Goods and Bads of LFSRs

## Good.

- Easily computed with simple machine.
- Very simple encryption/decryption processes.
- Bits have many of the same properties as random bits.
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits.  
[ but need theory of finite groups to know where to put taps ]



a commercially available LFSR

## Bad.

- Still need secure, independent way to distribute LFSR seed.
- The bits are not truly random.  
[ bits in our 11-bit LFSR cycle after  $2^{11} - 1 = 2047$  steps ]
- Experts have cracked LFSR encryption.  
[ need more complicated machines ]

# Other LFSR Applications

## What else can we do with a LFSR?

- DVD encryption with CSS.
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems.



DVD Jon  
(Norwegian hacker)

```
/*      efdtt.c      Author: Charles M. Hannum <root@ihack.net>      */
/*      Usage is:  cat title-key scrambled.vob | efdtt >clear.vob      */

#define m(i) (x[i]^s[i+84])<<

        unsigned char x[5]          ,y,s[2048];main(
n){for( read(0,x,5          );read(0,s ,n=2048
        ); write(1 ,s,n)          )if(s
[y=s [13]%8+20] /16%4 ==1 ) {int
i=m( 1)17 ^256 +m(0) 8,k =m(2)
0,j= m(4) 17^ m(3) 9^k* 2-k%8
^8,a =0,c =26;for (s[y] -=16;
--c;j *=2)a= a*2^i& 1,i=i /2^j&1
<<24;for(j= 127; ++j<n;c=c>
        y)
        c

        +=y=i^i/8^i>>4^i>>12,
i=i>>8^y<<17,a^=a>>14,y=a^a*8^a<<6,a=a
>>8^y<<9,k=s[j],k = "7Wo~ 'G_ \216" [k
&7]+2^"cr3sfw6v;*k+>/n." [k>>4]*2^k*257/
8,s[j]=k^(k&k*2&34)*6^c+~y
        ;}}

#l/usr/bin/perl
# 472-byte qrpff
#Keith Winstein and Marc Horowitz
#<sipb-iap-dvd@mit.edu>
# MPEG 2 PS VOB file ->
#descrambled output on stdout.
# usage: perl -l
#<k1>:<k2>:<k3>:<k4>:<k5> qrpff
# where k1..k5 are the title key bytes
#in least to most-significant order
s' '$/=2048;while(<)(G=29;R=142;
if((@a=unqT="C",_) [20]&48)
(D=89;_ =unqb24,qT,@
b=map(ord qb8,unqb8,qT,^$a
[-D])@INC;s/...$/1$&/:Q=unqV,
qb25,;H=73;O=$b[4]<<9
|256;$b[3];Q=Q>>8^(P=(E=255)&
(Q>>12^Q>>4^Q/8^Q))<<17,O=O>>
8^(E&(F=$(S=O>>14&7^O)
^S*8^S<<6))<<9,=
(map(U=_%16orE^=R^=110&
(S=(unqT;"\xb\ntd\xbz\x14d")[_/16%8]);E
^=(72,@z=(64,72,G^=12^(U-?0:S&17)),
H^=_%64?12:0,@z)[_%8])
(16..271)[_]^(D>>=8)+=P+
(~F&E))for@a[128..$#a])
print+qT,@a';s/[D-HO-U_]
\$$&/g;s/q/q/pack+/g;eval
```

<http://www.cs.cmu.edu/~dst/DeCSS/Gallery>

# LFSR and "General Purpose Computer"

## Important properties.

- Built from simple components.
- Scales to handle huge problems.
- Requires a deep understanding to use effectively.

Basic Component	LFSR	Computer
control	start, stop, load	same
clock	regular pulse	2.8 GHz pulse
memory	11 bits	1 GB
input	seed	sequence of bits
computation	shift, XOR	logic, arithmetic, ...
output	pseudo-random bits	Sequence of bits

**Critical difference.** General purpose machine can be programmed to simulate ANY abstract machine.

# A Profound Idea

**Programming.** Can write a Java program to simulate the operations of **any** abstract machine.

- Basis for theoretical understanding of computation. [stay tuned]
- Basis for bootstrapping real machines into existence. [stay tuned]

**Stay tuned.** See Assignment 5.

```
public class LFSR {
    private int seed[];
    private final int tap;
    private final int N;

    public LFSR(String seed, int tap) { ... }

    public int step() { ... }

    public static void main(String[] args) {
        LFSR lfsr = new LFSR("01101000010", 8);
        for (int i = 0; i < 2000; i++)
            StdOut.print(lfsr.step());
    }
}
```

```
% java LFSR
1100100100111101101110010110101
1100110001011111101001000010011
0100101111001100100111...
```