COS 426
Computer Graphics
Princeton University

Tianqiang Liu (Tim)

Feb 15, 2012

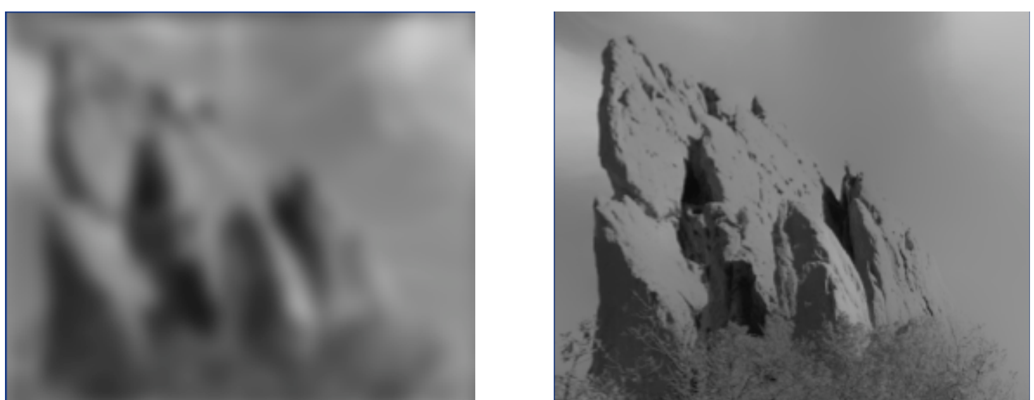
Topics

- **Morphing**

[Beier 1992]

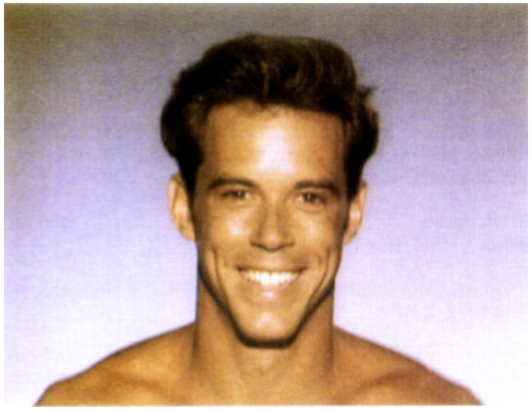
- Bilateral Filtering

[Paris 2008]



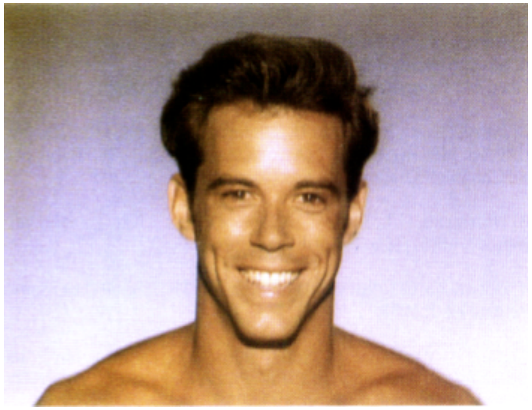
Morphing

- **Beier and Neely, 1992:**
  - Align facial features
  - Blend colors



Morphing

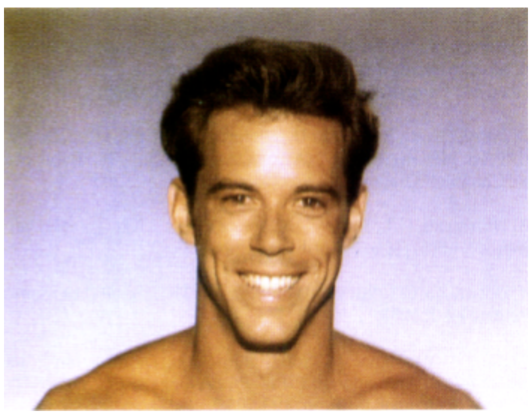
- **Beier and Neely, 1992:**
  - **Align facial features**
  - Blend colors



Why align features?

Morphing

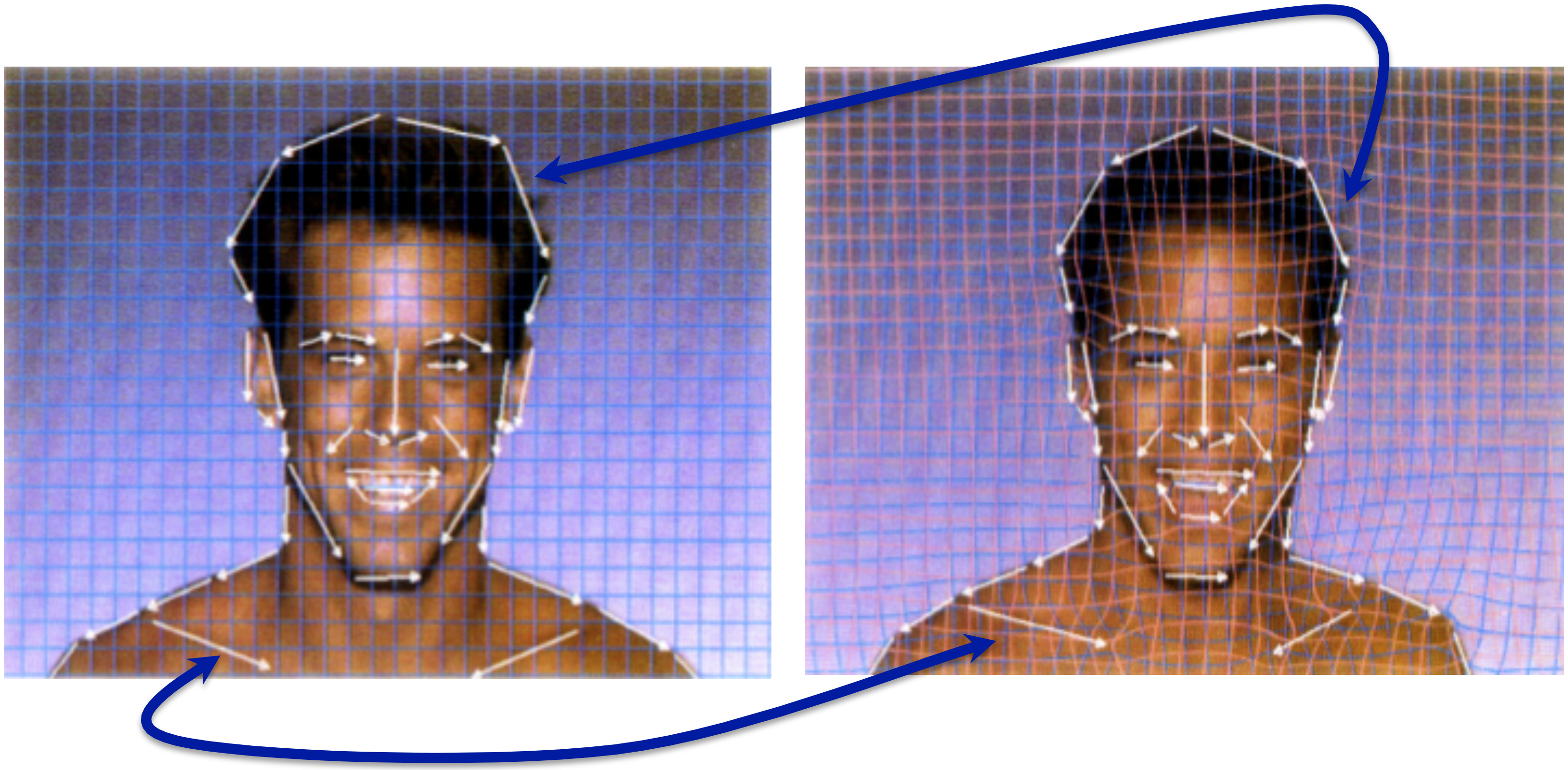
- **Beier and Neely, 1992:**
  - **Align facial features**
  - Blend colors




Why align features?

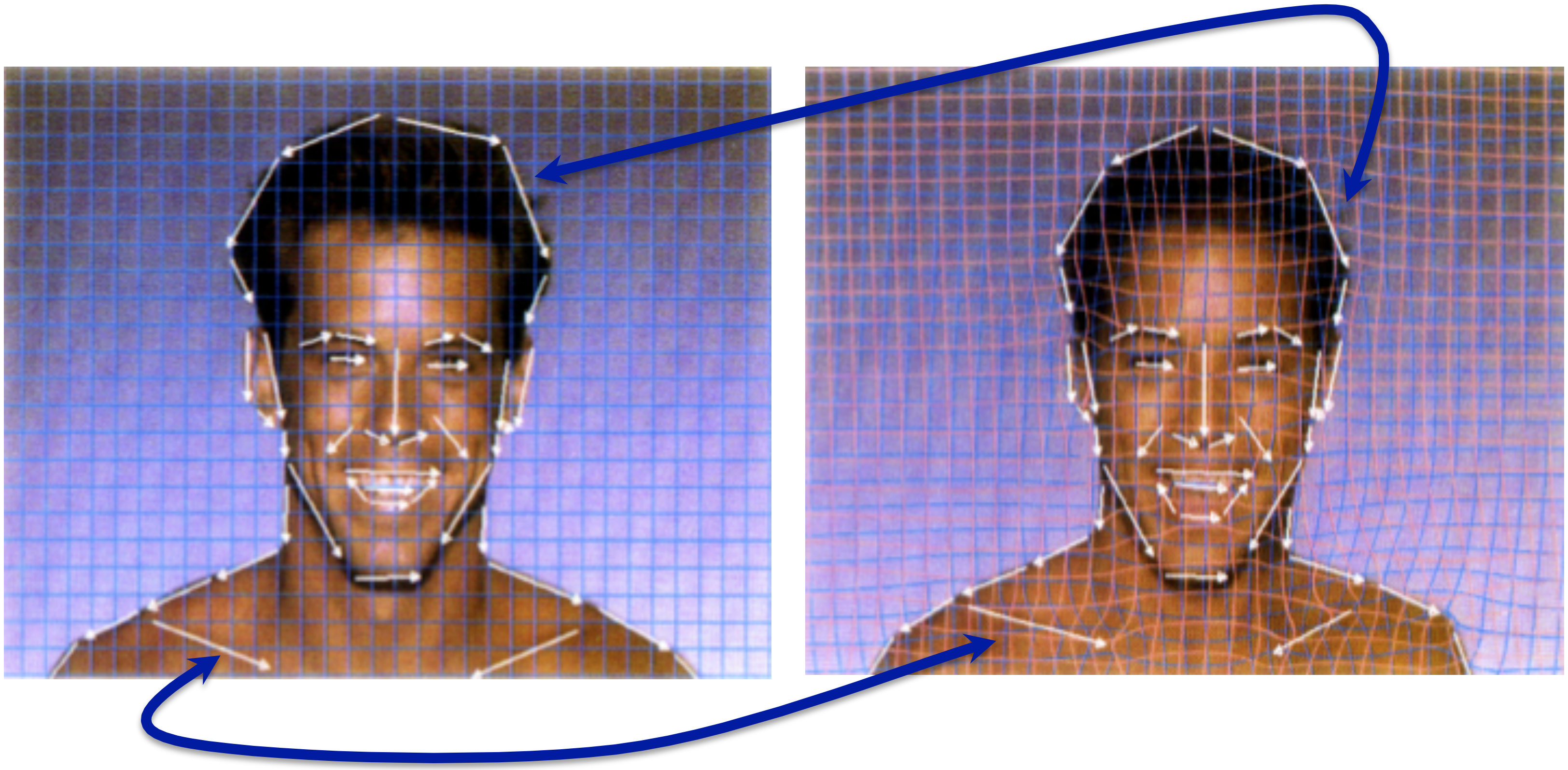
Morphing: Align Features

- Associate primitives: e.g. lines



Morphing: Align Features

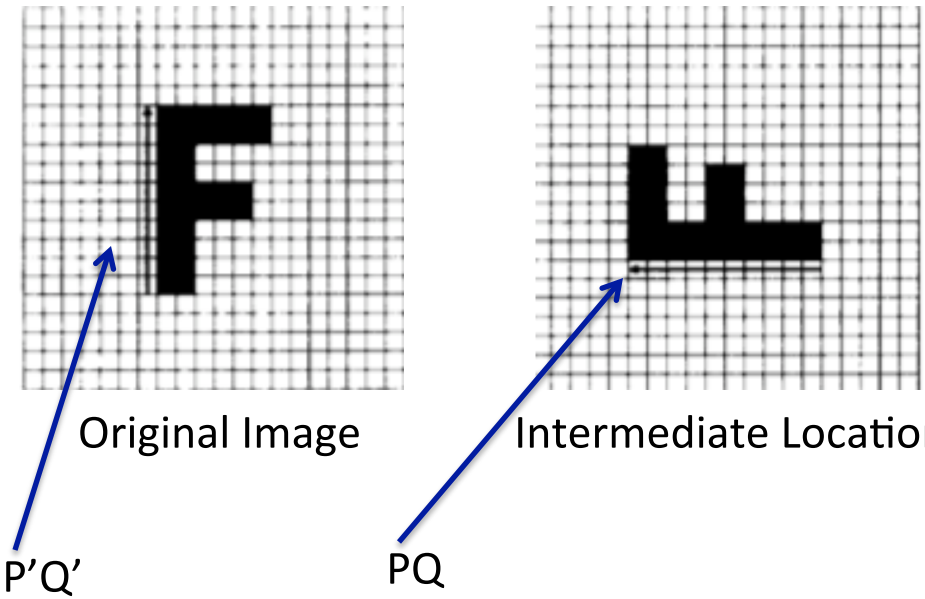
- Associate primitives: e.g. lines



- Move primitives so that they align (at some intermediate location) -> warp accordingly

Morphing: Align Features

- A simple case: 1 image, 1 primitive:

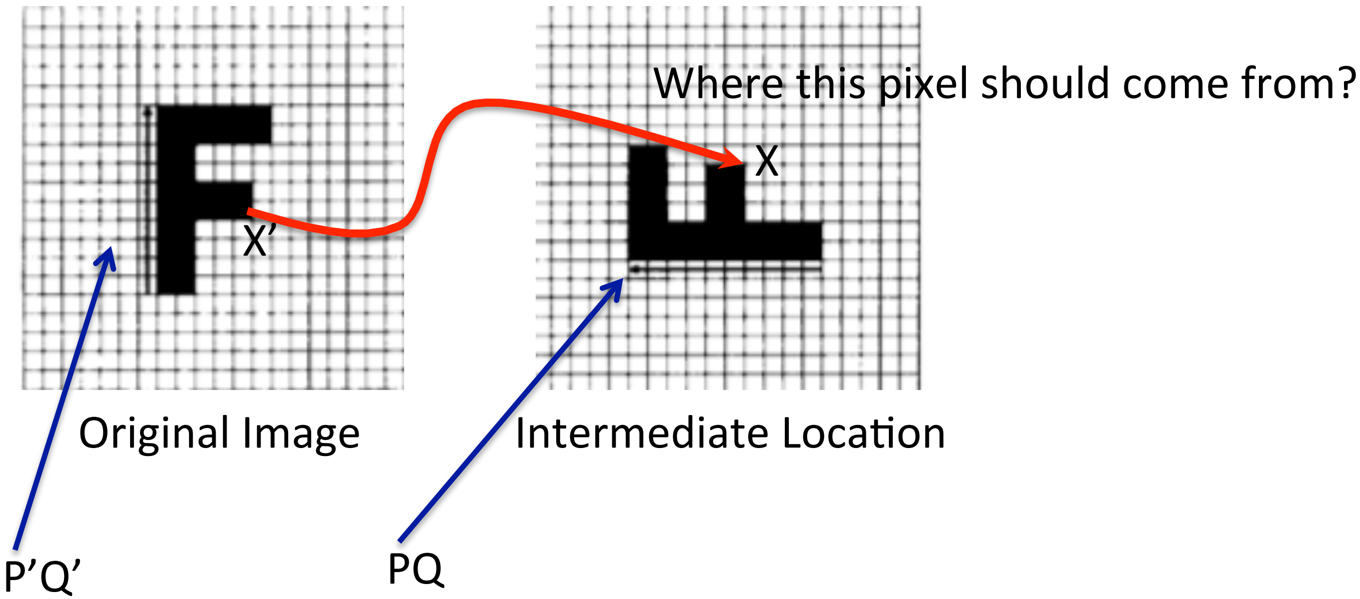


Original Image

Intermediate Location

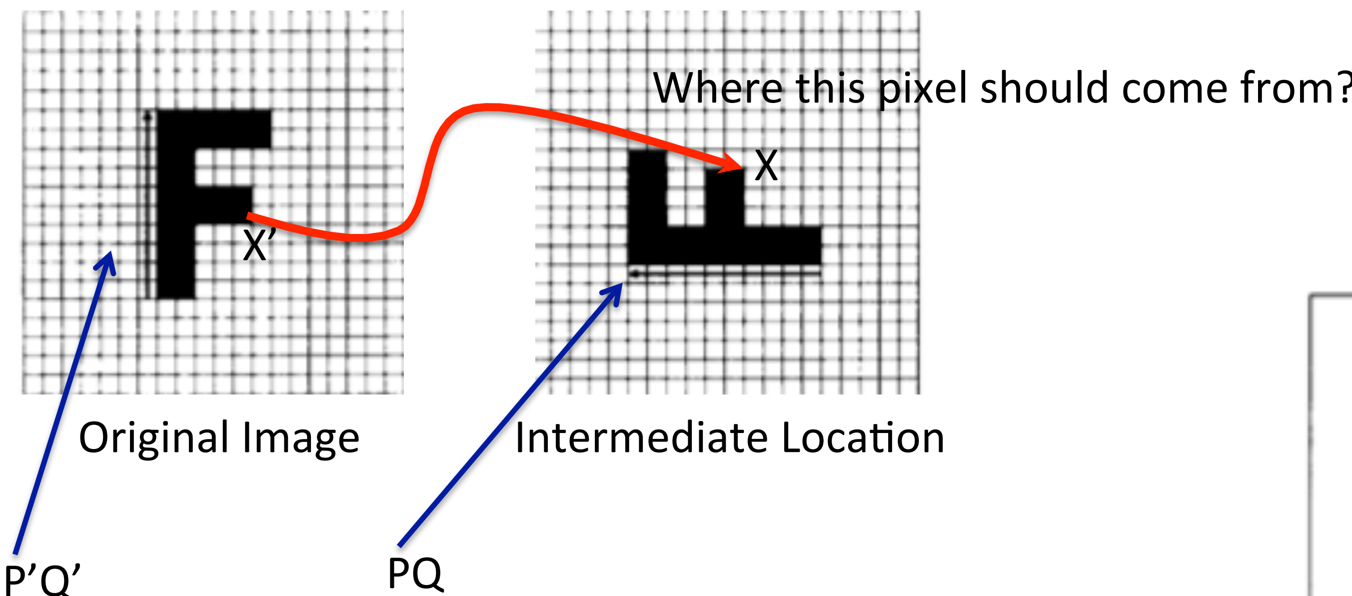
Morphing: Align Features

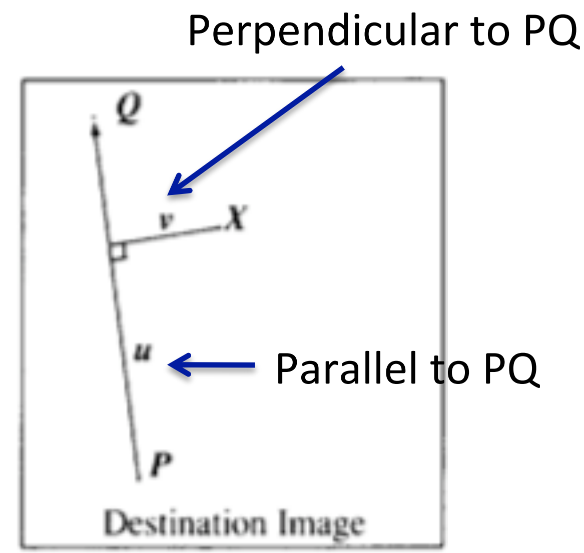
- A simple case: 1 image, 1 primitive:



Morphing: Align Features

- A simple case: 1 image, 1 primitive:

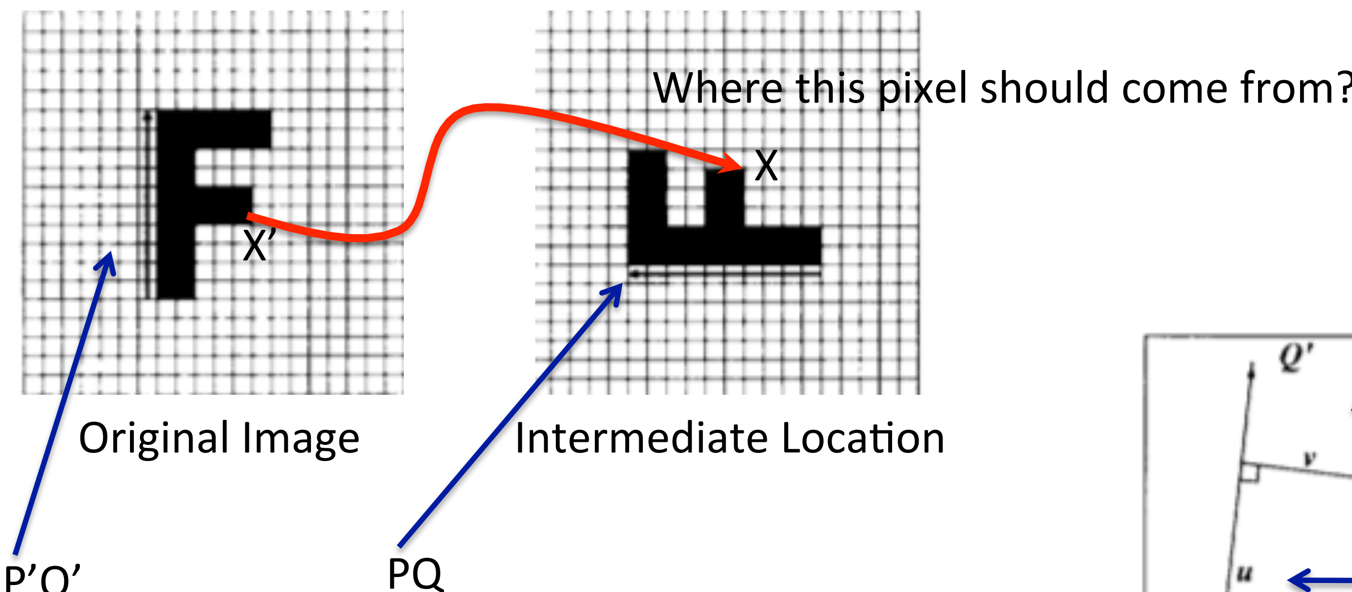


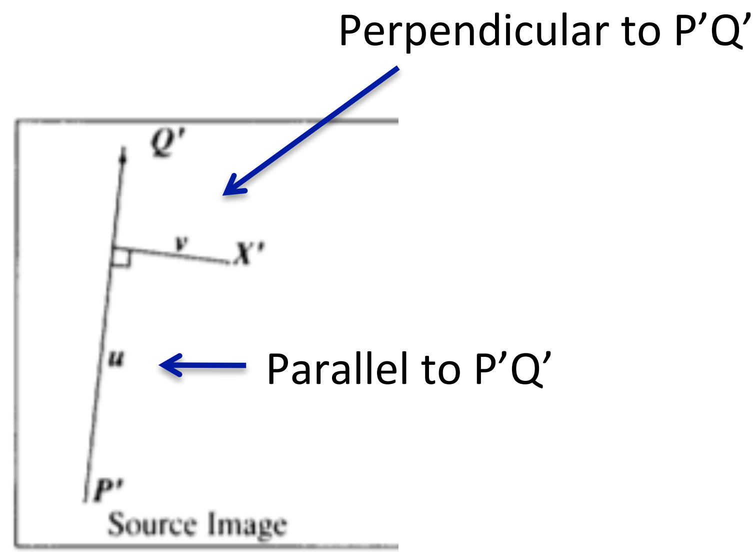


1. Find local coordinates: u,v
   $X = u \cdot PQ + v \cdot PQ^{\perp}$

Morphing: Align Features

- A simple case: 1 image, 1 primitive:





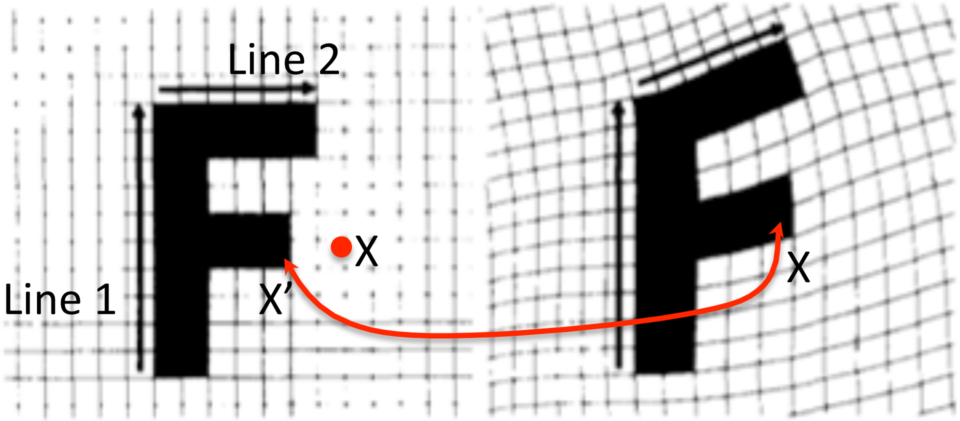
1. Find local coordinates: u,v

   $X = P + u \bullet PQ + v \bullet PQ^{\perp}$

2. Location in original image: $X' = P' + u \bullet P'Q' + v \bullet P'Q'^{\perp}$

Morphing: Align Features

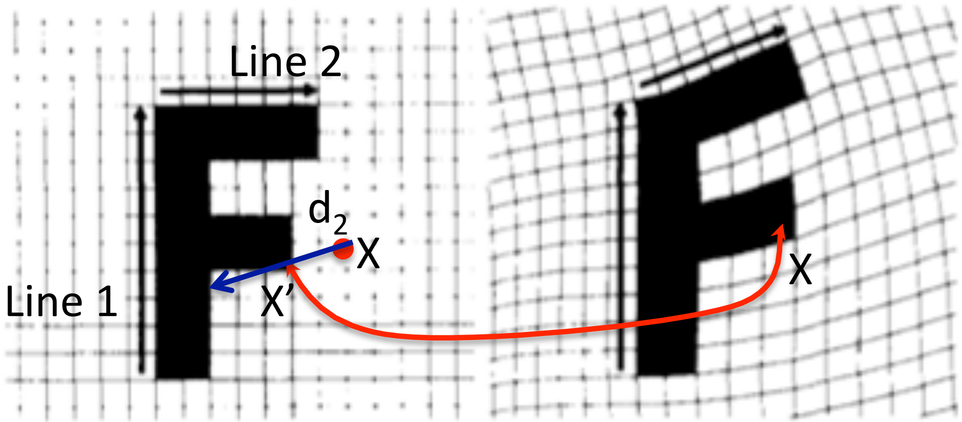
- Multiple lines?



  - Find $X' = X + w_1 \bullet d_1 + w_2 \bullet d_2$

Morphing: Align Features

- Multiple lines?



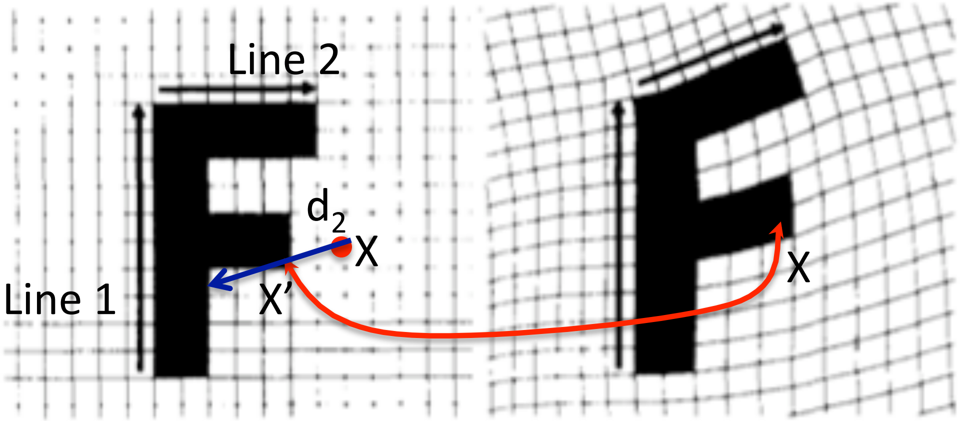
  – Find $X' = X + w_1 \bullet \underline{d_1} + w_2 \bullet \underline{d_2}$

=0 ≠0

Line 1 did not move

Morphing: Align Features

- Multiple lines?

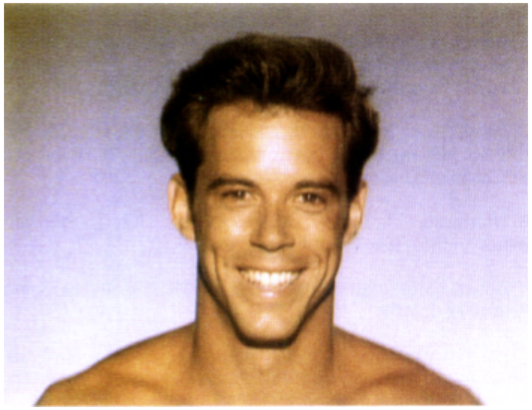


  - Find $X' = X + \underset{\approx .6}{\underline{w_1}} \bullet \underset{=0}{\underline{d_1}} + \underset{\approx .4}{\underline{w_2}} \bullet \underset{\neq 0}{\underline{d_2}}$

Line 1 is longer and closer

Morphing

- **Beier and Neely, 1992:**
  - Align facial features
  - **Blend colors**



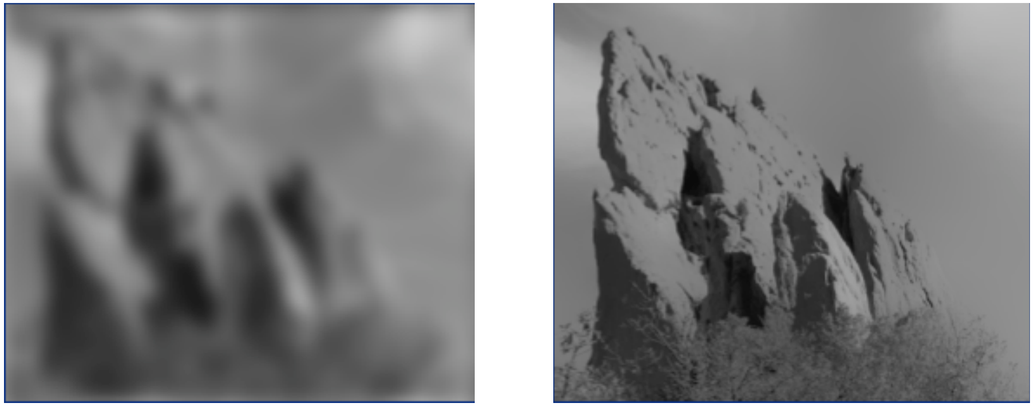
Topics

- Morphing

[Beier 1992]

- **Bilateral Filtering**

[Paris 2008]



Bilateral Filtering

Input

Taken from
SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Gaussian Blur

Bilateral Filtering

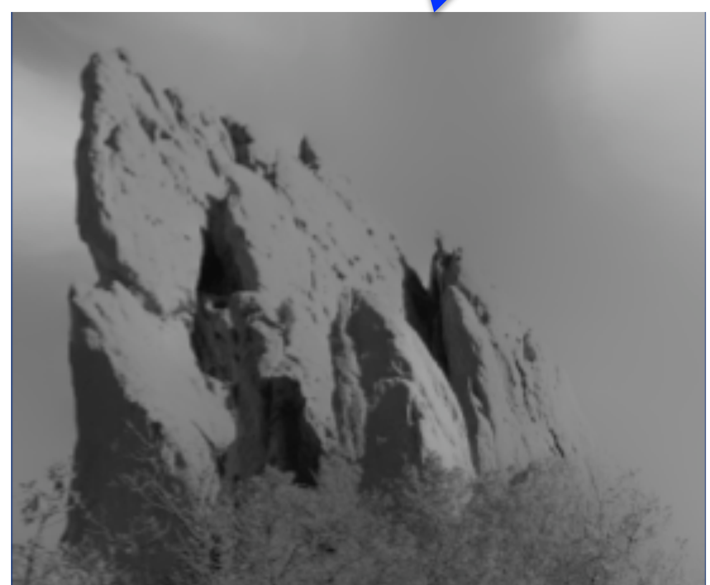
Bilateral Filtering

Input

Taken from
SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Edge-preserving

Gaussian Blur



Bilateral Filtering

Bilateral Filtering

- How?

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value
at pixel x

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

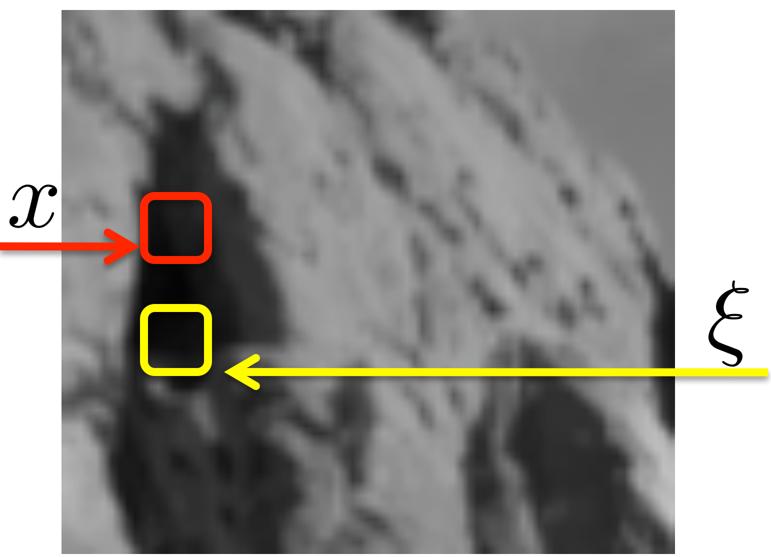
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

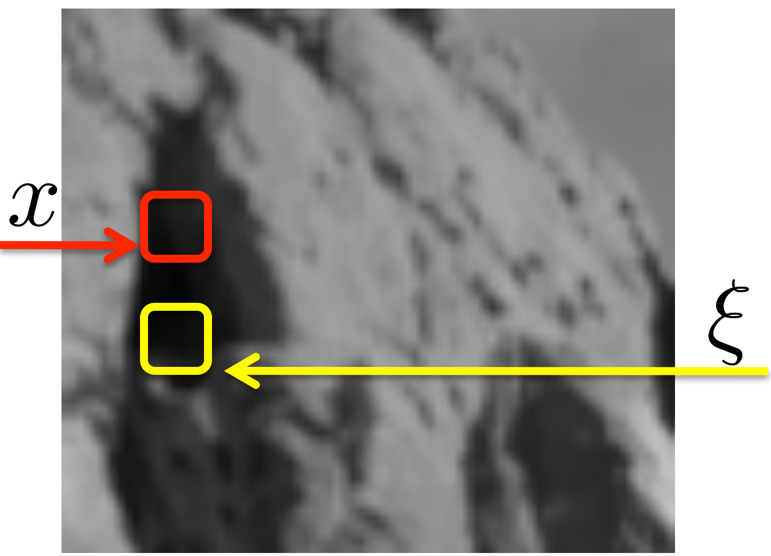
Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?

???



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

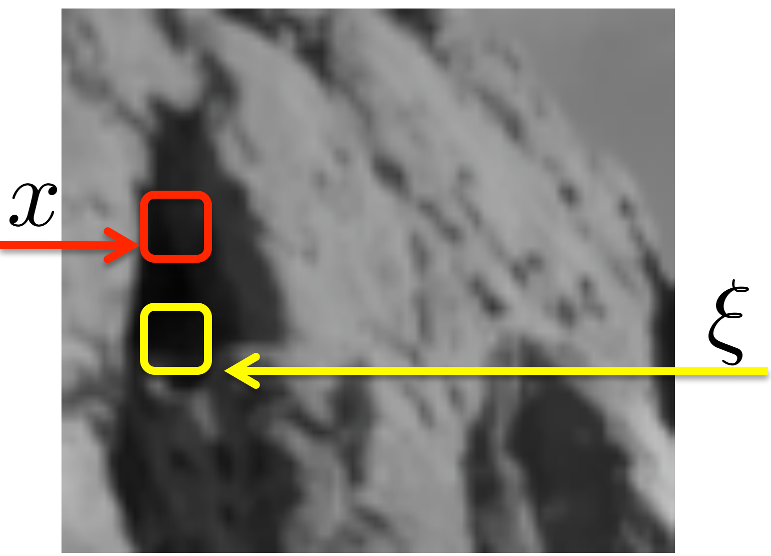
Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?

0

???



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

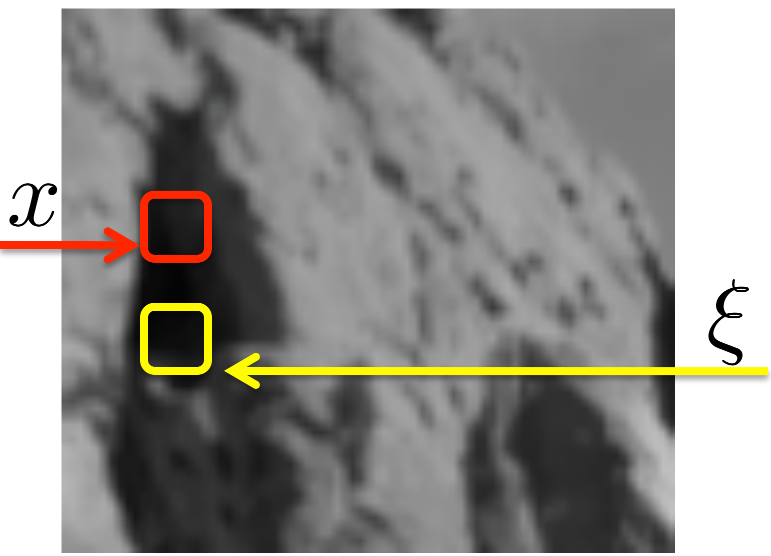
Is pixel close to x ?

Is pixel similar to x ?

0

≈1

???



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

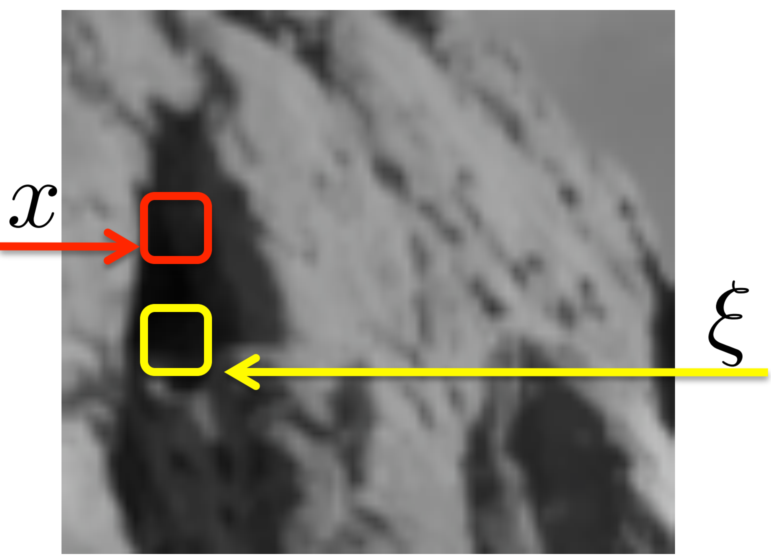
(k_d^{-1} crossed out: Later)

- $h(x)$: Filtered value at pixel x
- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$: Go over every pixel ξ in image
- $f(\xi)$: Value at a pixel
- $c(\xi - x)$: Is pixel close to x?
- $s(\xi - x)$: Is pixel similar to x?

0

≈1

≈1



Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

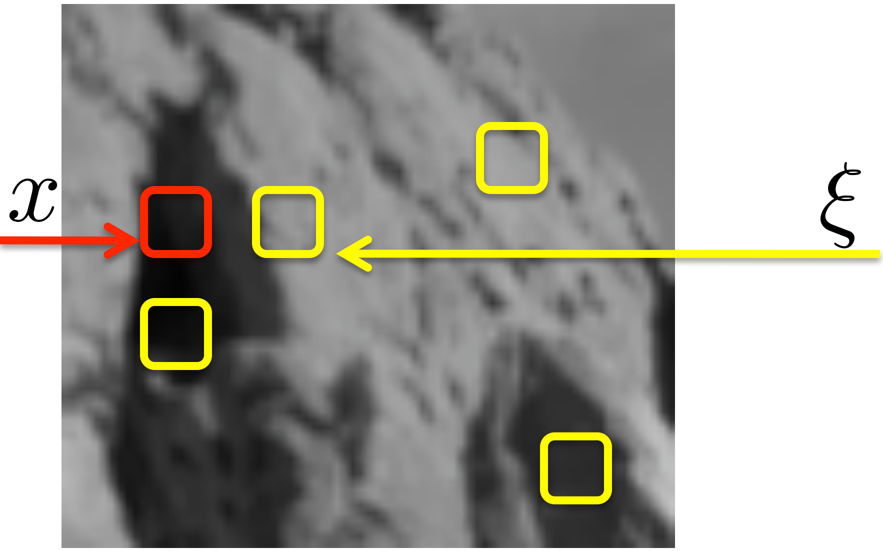
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x?

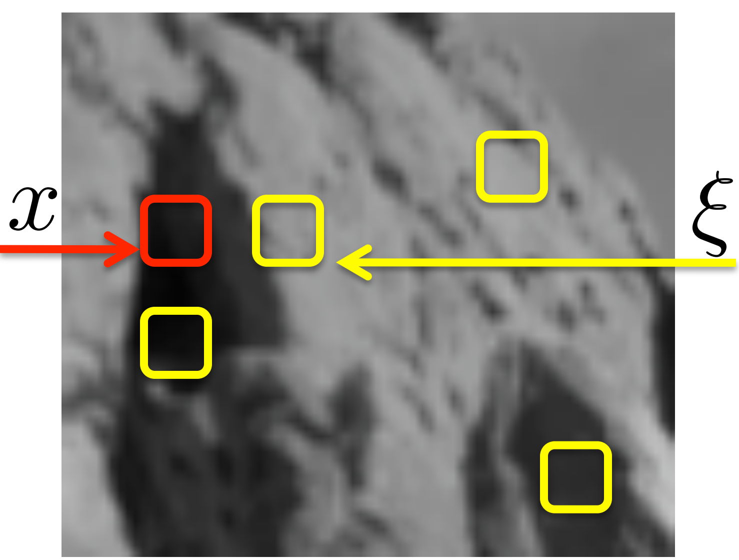
Is pixel similar to x?

≈1

≈1

≈0

EDGE!



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

(k_d^{-1} is crossed out on the slide.)

Filtered value at pixel x

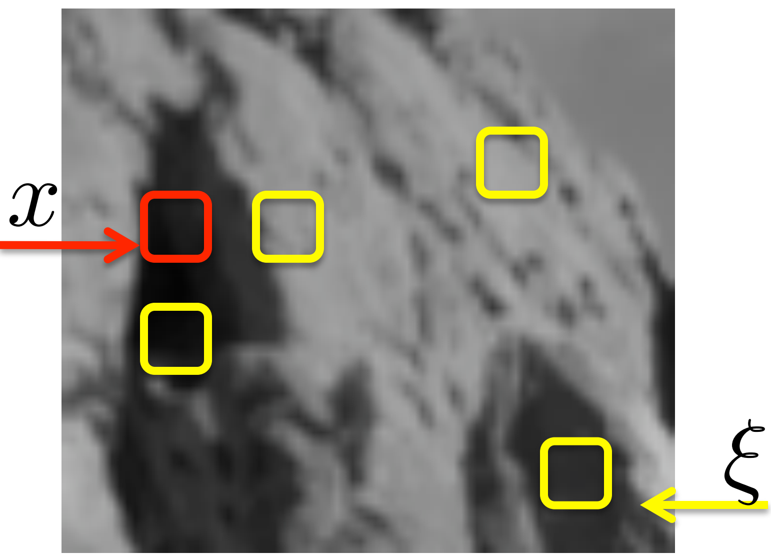
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

(k_d^{-1} crossed out: Later)

Filtered value at pixel x

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x?

Is pixel similar to x?

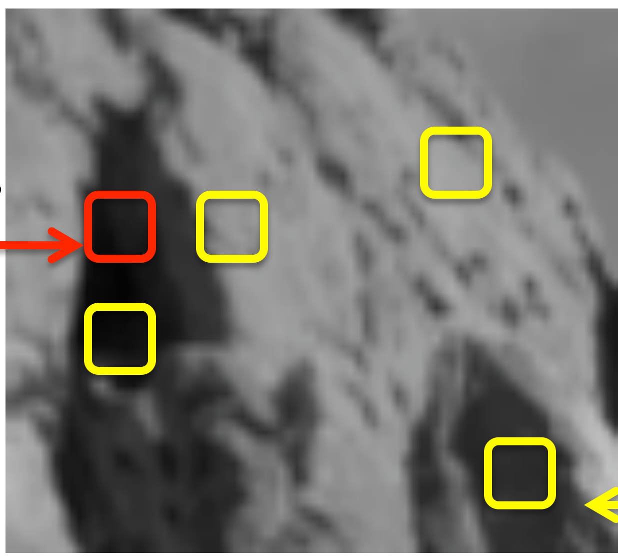
≈0

≈0

FAR!

≈1

x



ξ

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

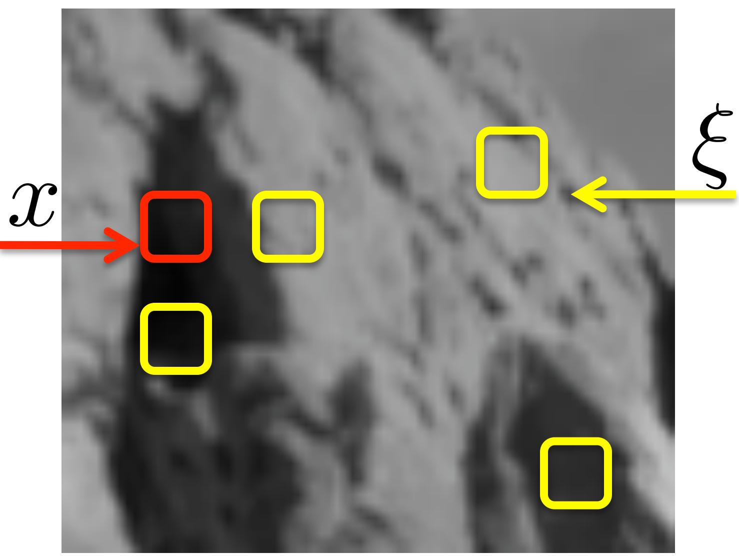
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x ?

Is pixel close to x ?



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

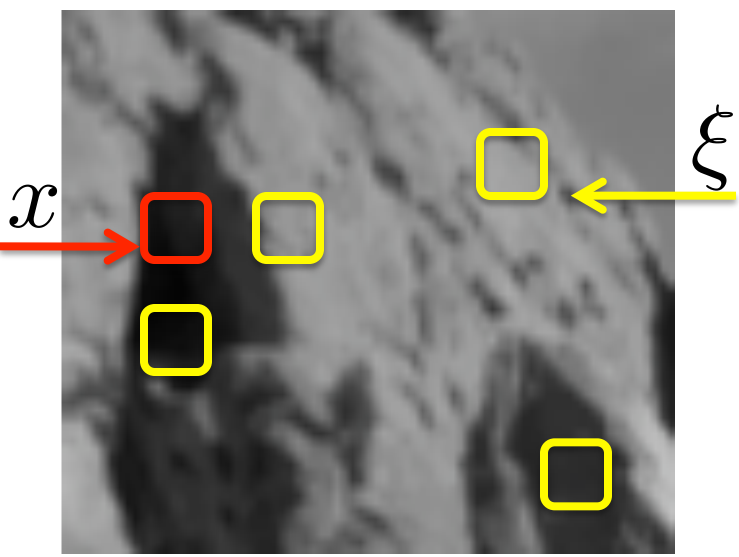
Is pixel close to x?

Is pixel similar to x?

≈1

≈0
FAR!

≈0
EDGE!



Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Is pixel close to x ?

Is pixel similar to x ?

$$c(\xi - x) = e^{-\frac{1}{2}\left(\frac{\| \xi - x \|}{\sigma_d}\right)}$$

$$s(\xi - x) = e^{-\frac{1}{2}\left(\frac{\| f(\xi) - f(x) \|}{\sigma_r}\right)}$$

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x ?

Is pixel close to x ?

Normalization:

$$k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) d\xi$$

Bilateral Filtering

- In Practice?

???

???

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Normalization

Go over every pixel ξ in image

Value at a pixel

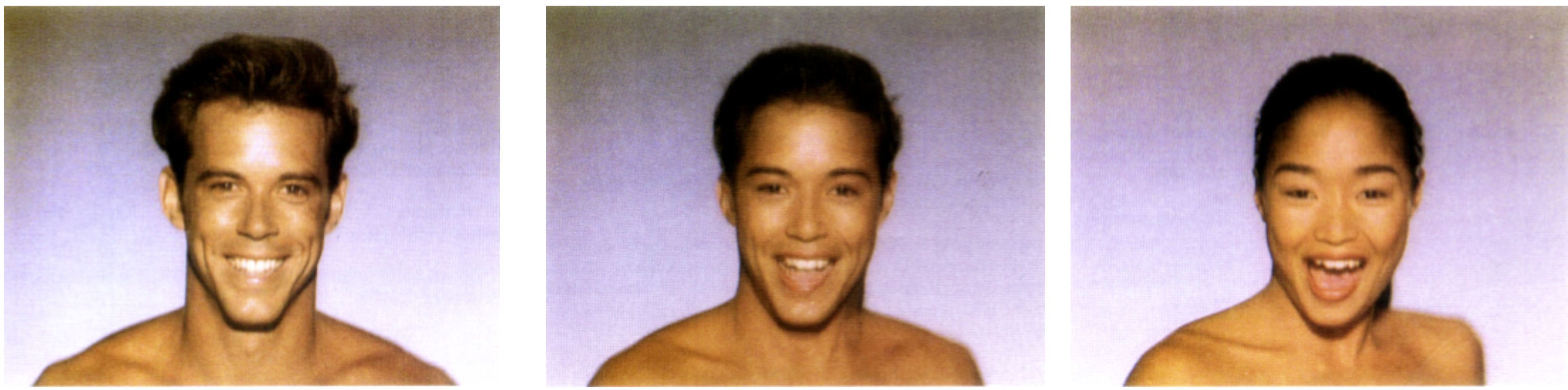
Is pixel close to x ?

Is pixel similar to x ?

  – Not going to infinity, but using a window
  – Using 'for' loops instead of integral
  – Similar for $k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) d\xi$

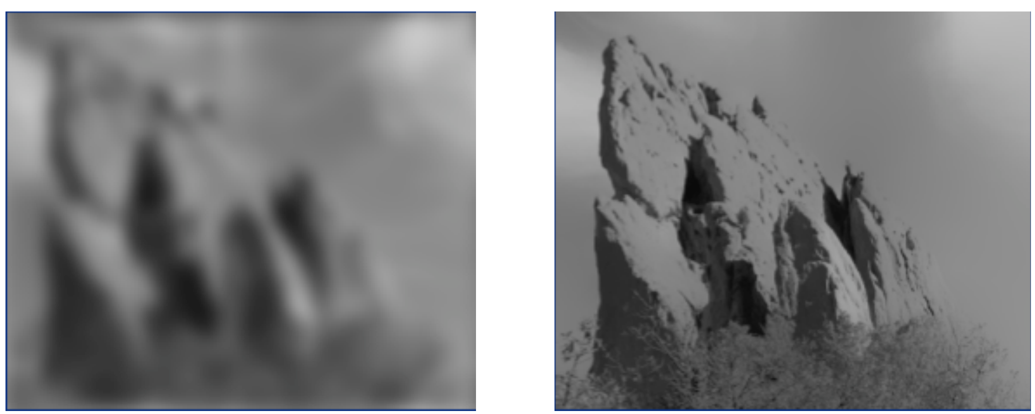
Questions?

- Morphing



[Beier 1992]

- Bilateral Filtering



[Paris 2008]