COS 426
Computer Graphics
Princeton University

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Topics

• Morphing

[Beier 1992]

• Bilateral Filtering

[Paris 2008]
Morphing

- **Beier and Neely, 1992:**
  - Align facial features
  - Blend colors
Morphing

• Beier and Neely, 1992:
  – Align facial features
  – Blend colors

Why align features?
Morphing

• Beier and Neely, 1992:
  – Align facial features
  – Blend colors

Why align features?
Morphing: Align Features

• Associate primitives: e.g. lines
Morphing: Align Features

• Associate primitives: e.g. lines

• Move primitives so that they align (at some intermediate location) -> warp accordingly
Morphing: Align Features

- A simple case: 1 image, 1 primitive:
Morphing: Align Features

- A simple case: 1 image, 1 primitive:

Original Image

Intermediate Location

Where this pixel should come from?

X

P'Q'

PQ
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

1. Find local coordinates: $u, v$
   \[ X = u \cdot PQ + v \cdot PQ \perp \]
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

1. Find local coordinates: $u,v$

   $$X = P + u \cdot PQ + v \cdot PQ'$$

2. Location in original image: $X' = P' + u \cdot P'Q' + v \cdot P'Q'_\perp$
Morphing: Align Features

- Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \]
Morphing: Align Features

- Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \]

- Line 1 did not move
Morphing: Align Features

• Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \]

- Find \( X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \)

\[ \approx 0.6 \quad =0 \quad \approx 0.4 \quad \neq 0 \]

Line 1 is longer and closer
Morphing

• Beier and Neely, 1992:
  – Align facial features
  – Blend colors
Topics

- Morphing

[Beier 1992]

- Bilateral Filtering

[Paris 2008]
Bilateral Filtering

Taken from SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/
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Input

Gaussian Blur

Bilateral Filtering

Edge-preserving
Bilateral Filtering

• How?
Bilateral Filtering

• How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]
Bilateral Filtering

• How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]

Filtered value at pixel x
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{\sigma_x \sigma_y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image
Bilateral Filtering

• How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Go over every pixel \( \xi \) in image

Later
Bilateral Filtering

• How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

$$h(x) = \frac{1}{\alpha^d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel $x$

Later

Go over every pixel $\xi$ in image

Value at a pixel

Is pixel similar to $x$?

Is pixel close to $x$?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x
Later
Go over every pixel \( \xi \) in image
Value at a pixel
Is pixel similar to \( x \) ?
Is pixel close to \( x \) ?

???
**Bilateral Filtering**

- **How?**

\[
h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(|\xi - x|) s(|\xi - x|) \, d\xi
\]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

???
Bilateral Filtering

• How?

\[ h(x) = k_a \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

\( 0 \approx 1 \)

???
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

\( \approx 1 \)

\( \approx 1 \)
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{C} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Later

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{\sigma} e^{-\frac{x^2}{2\sigma^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

\( \approx 1 \)

\( \approx 1 \)

\( \approx 0 \)

EDGE!
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Filtered value at pixel x

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

\( \approx 0 \)

\( \approx 0 \) FAR!

\( \approx 1 \)
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{2\pi \sigma^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x
Later
Go over every pixel \( \xi \) in image
Value at a pixel
Is pixel similar to \( x \) ?
Is pixel close to \( x \) ?
Bilateral Filtering

- How?

$$h(x) = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x
Later
Go over every pixel $\xi$ in image
Value at a pixel
Is pixel similar to $x$?
Is pixel close to $x$?
$\approx 1$
$\approx 0$
FAR!
$\approx 0$
EDGE!
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{\sqrt{2\pi\sigma_d^2}} \int \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Later

Filter value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

\[ c(\xi - x) = e^{-\frac{1}{2} \left( \frac{\| \xi - x \|}{\sigma_d} \right)} \]

\[ s(\xi - x) = e^{-\frac{1}{2} \left( \frac{\| f(\xi) - f(x) \|}{\sigma_r} \right)} \]
Bilateral Filtering

• How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \) ?

Is pixel close to \( x \) ?

Normalization:

\[ k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) d\xi \]
Bilateral Filtering

• In Practice?
  - Not going to infinity, but using a window
  - Using ‘for’ loops instead of integral
  - Similar for \( k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x)s(\xi - x)d\xi \)
Questions?

- Morphing

[Beier 1992]

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