# The 3D Rasterization Pipeline

COS 426

# 3D Rendering Scenarios

- Batch
  - One image generated with as much quality as possible for a particular set of rendering parameters
    - Take as much time as is needed (minutes)
    - Useful for photorealistism, movies, etc.

- Interactive
  - Images generated in fraction of a second (<1/10) with user input, animation, varying camera, etc.
    - Achieve highest quality possible in given time
    - Visualization, games, etc.

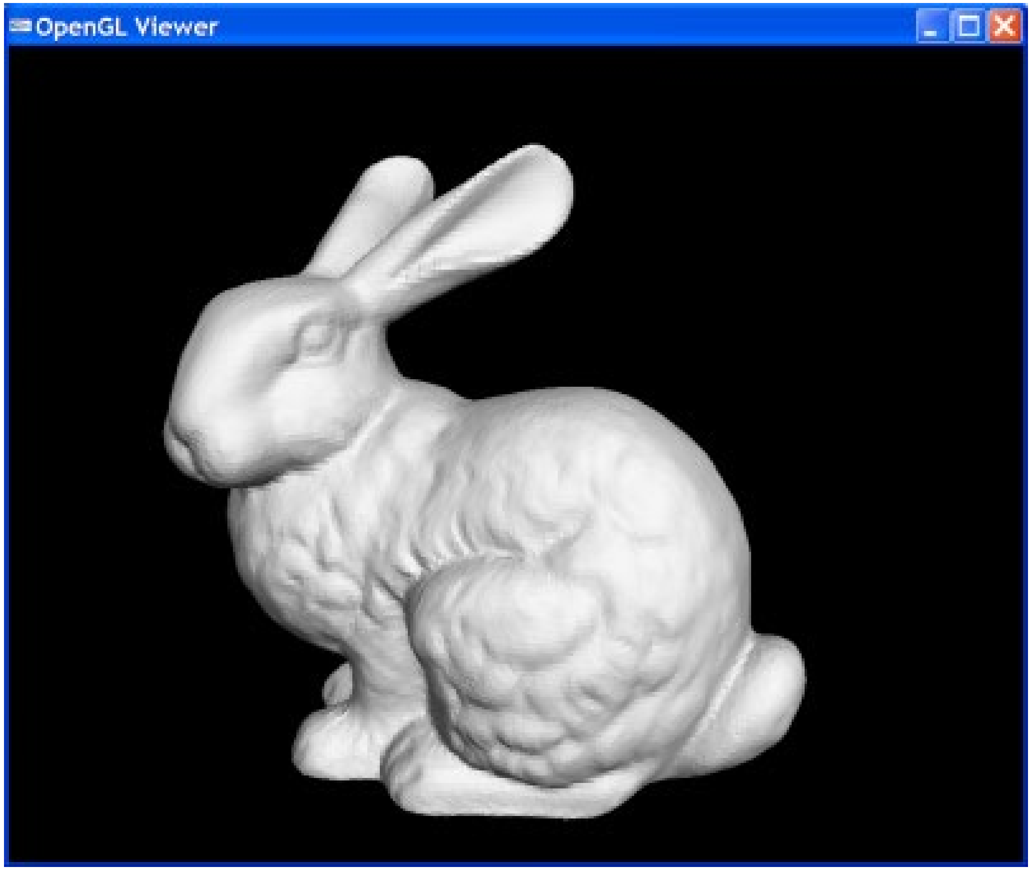
# 3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination


Bungie

# 3D Polygon Rendering

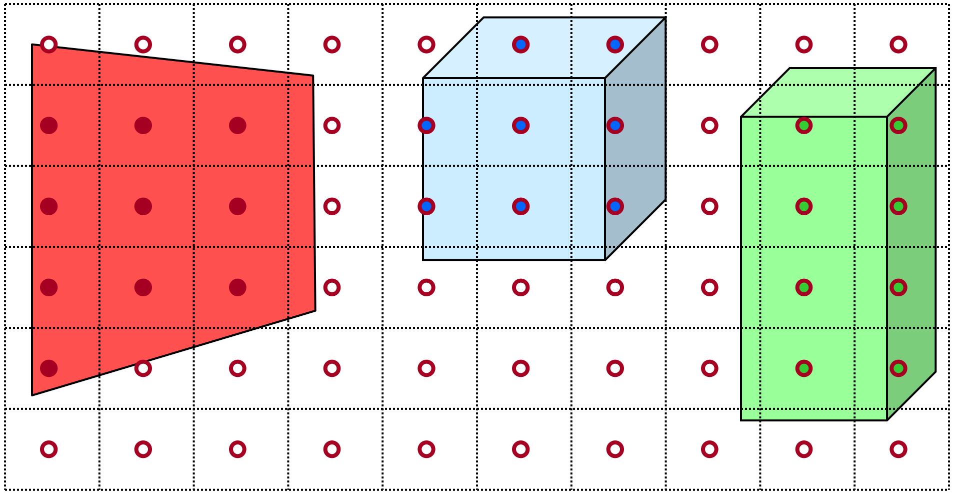
- Many applications use rendering of 3D polygons with direct illumination




meshview

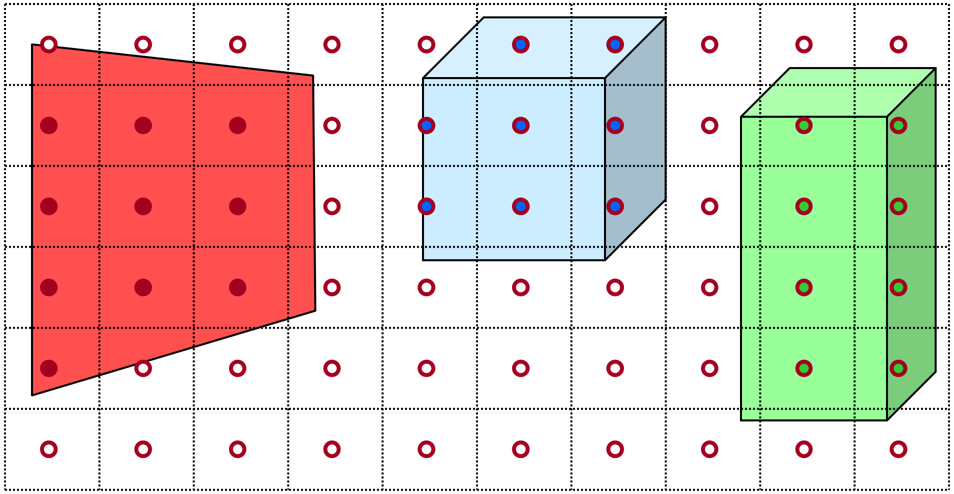
# Ray Casting Revisited

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on illumination



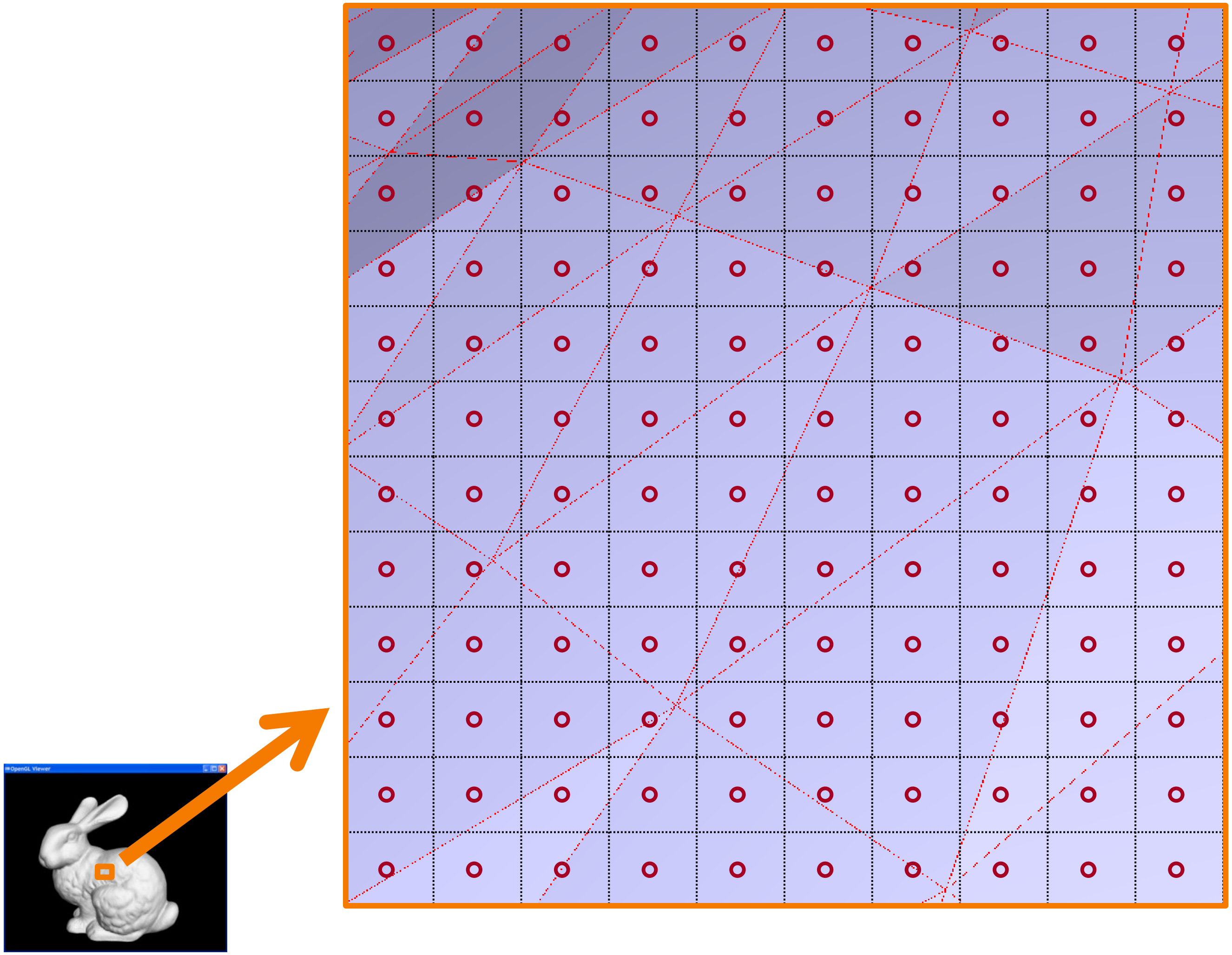
# 3D Polygon Rendering

- We can render polygons faster if we take advantage of spatial coherence



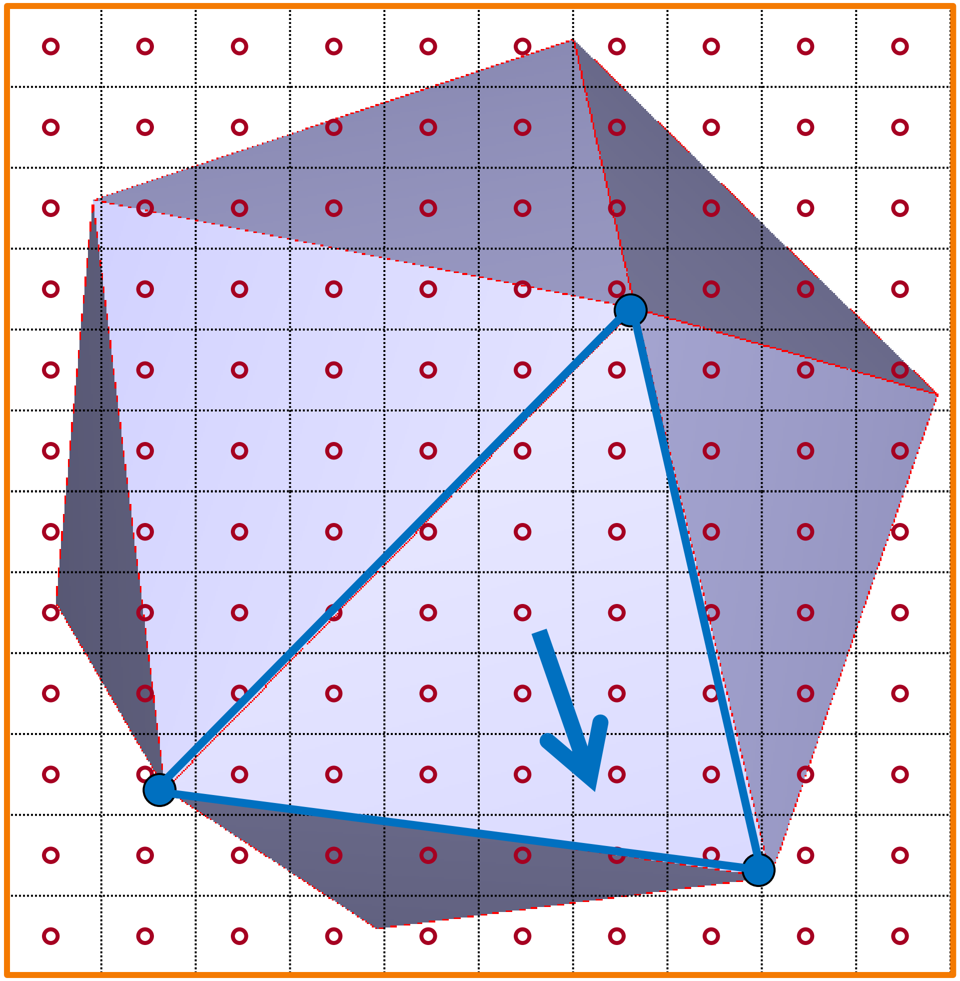
# 3D Polygon Rendering

- How?



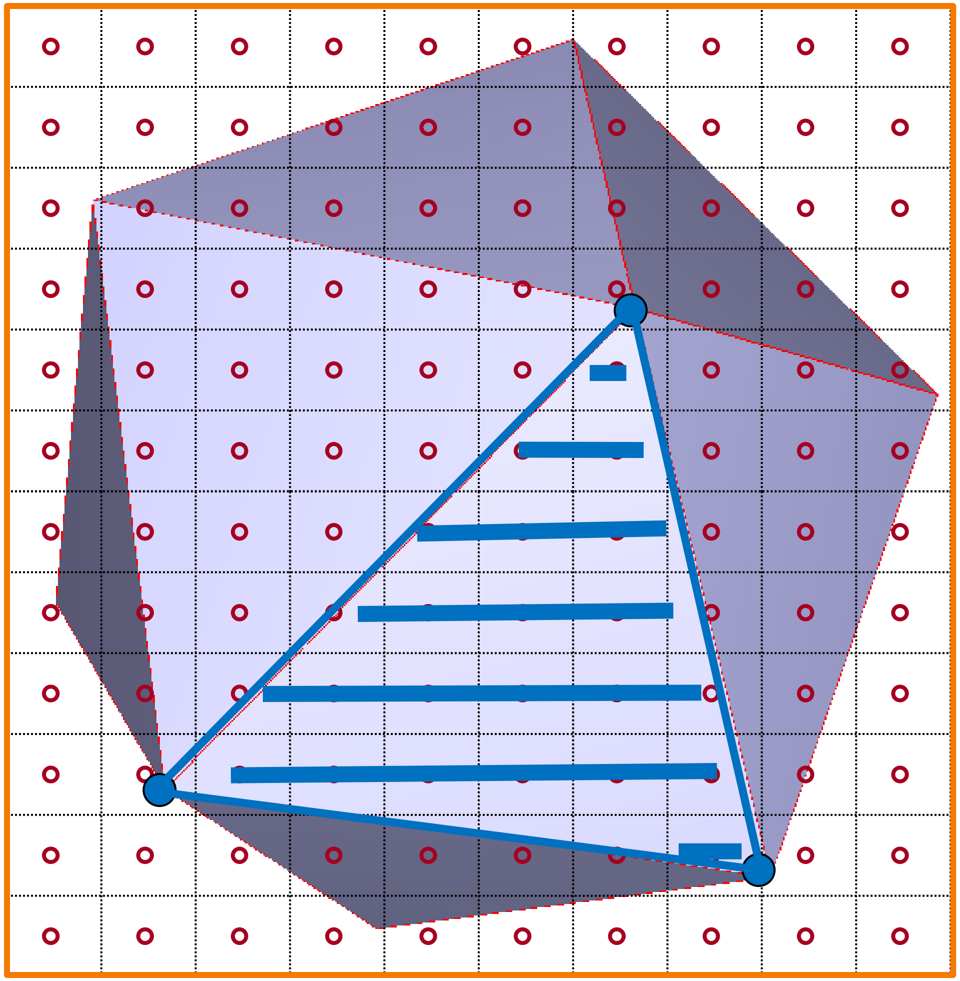
# 3D Polygon Rendering

- How?

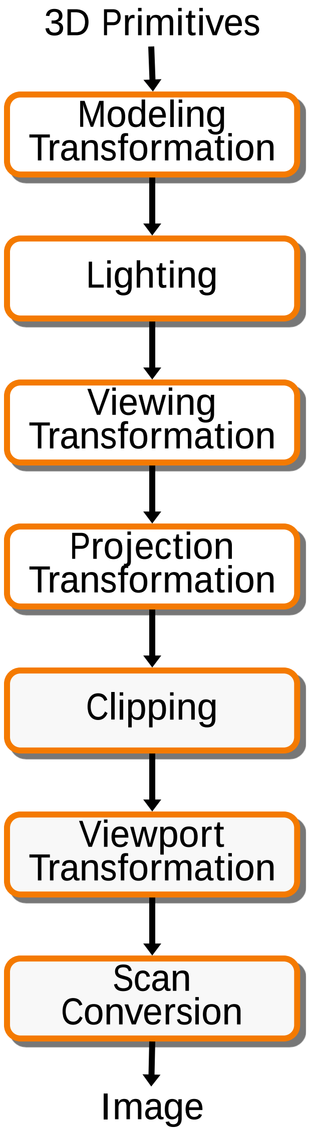


# 3D Polygon Rendering

- How?

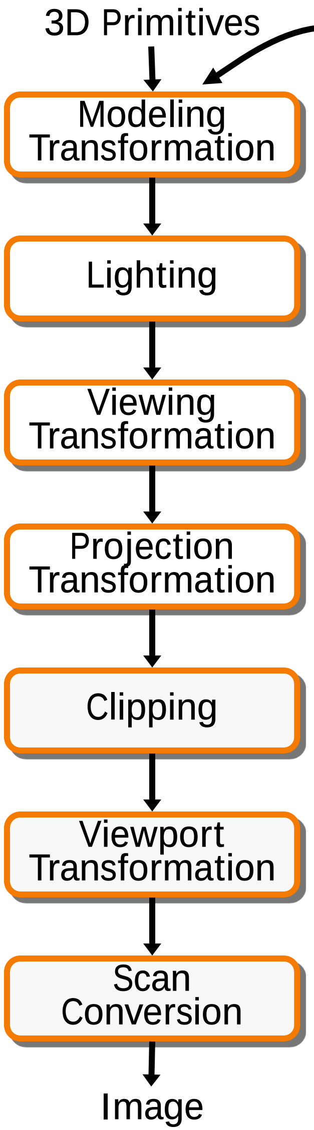


# 3D Rendering Pipeline (for direct illumination)




This is a pipelined sequence of operations to draw 3D primitives into a 2D image

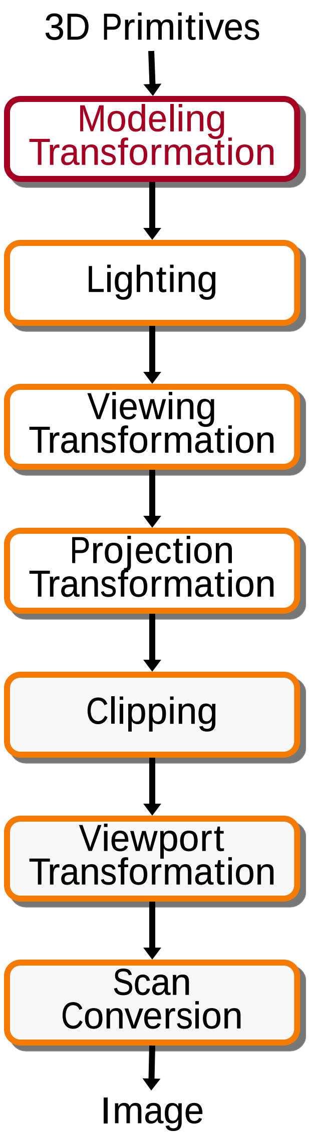
# 3D Rendering Pipeline (for direct illumination)




```
glBegin(GL_POLYGON);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(1.0, 0.0, 0.0);
glVertex3f(0.0, 1.0, 0.0);
glEnd();
```

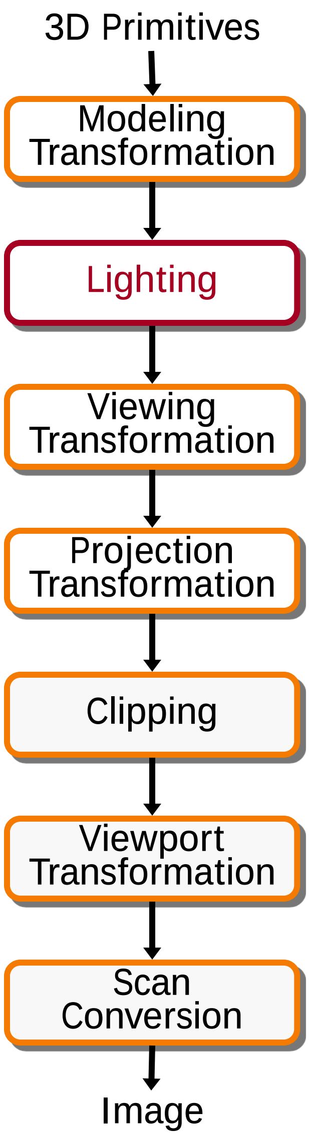
OpenGL executes steps of 3D rendering pipeline for each polygon

# 3D Rendering Pipeline (for direct illumination)




Transform into 3D world coordinate system

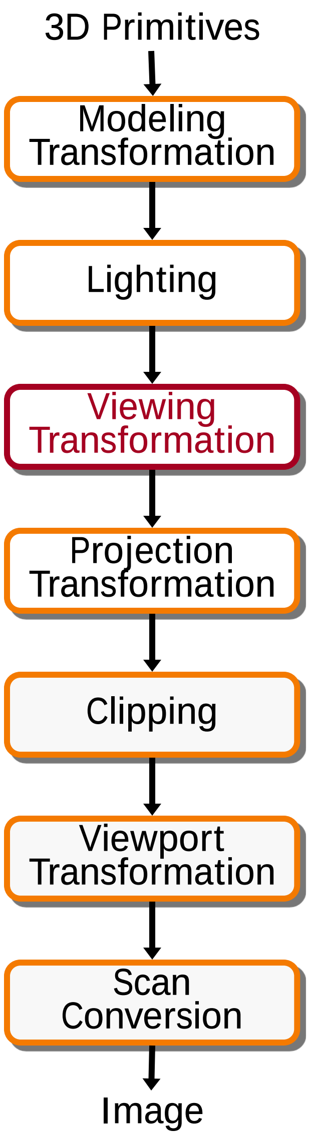
# 3D Rendering Pipeline (for direct illumination)




Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

# 3D Rendering Pipeline (for direct illumination)

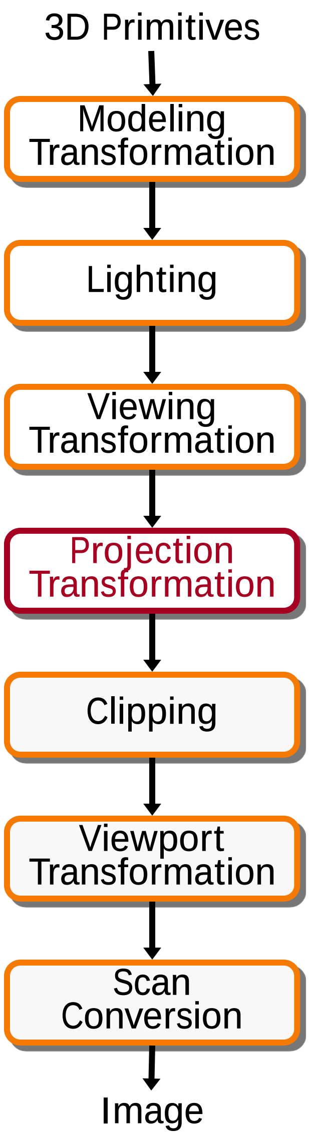



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

# 3D Rendering Pipeline (for direct illumination)



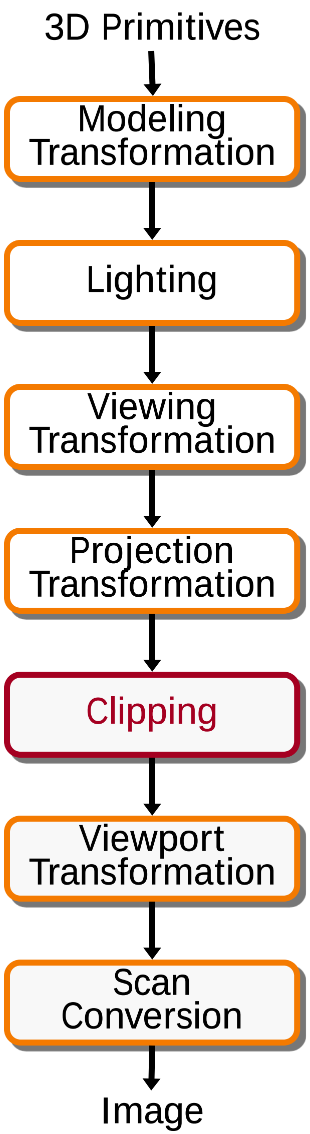

Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

# 3D Rendering Pipeline (for direct illumination)




Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera's view

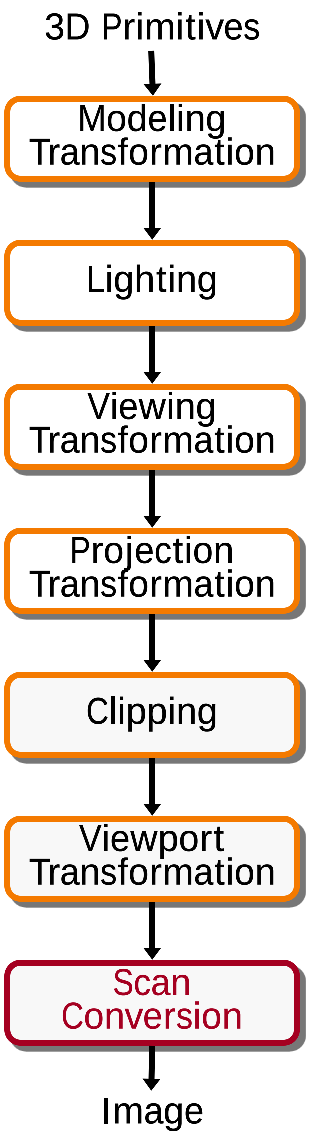
# 3D Rendering Pipeline (for direct illumination)

3D Primitives

- **Modeling Transformation** — Transform into 3D world coordinate system
- **Lighting** — Illuminate according to lighting and reflectance
- **Viewing Transformation** — Transform into 3D camera coordinate system
- **Projection Transformation** — Transform into 2D camera coordinate system
- **Clipping** — Clip primitives outside camera's view
- **Viewport Transformation** — Transform into image coordinate system
- **Scan Conversion**

Image

# 3D Rendering Pipeline (for direct illumination)




Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

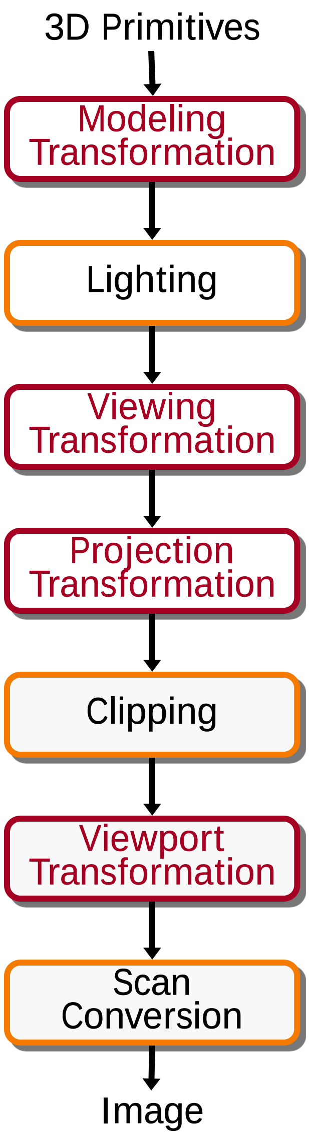
Transform into 2D camera coordinate system

Clip primitives outside camera's view

Transform into image coordinate system

Draw pixels (includes texturing, hidden surface, ...)

# 3D Rendering Pipeline (for direct illumination)




Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera's view

Transform into image coordinate system

Draw pixels (includes texturing, hidden surface, ...)

# Transformations

p(x,y,z)

↓ 3D Object Coordinates

**Modeling Transformation**

↓ 3D World Coordinates

**Viewing Transformation**

↓ 3D Camera Coordinates

**Projection Transformation**

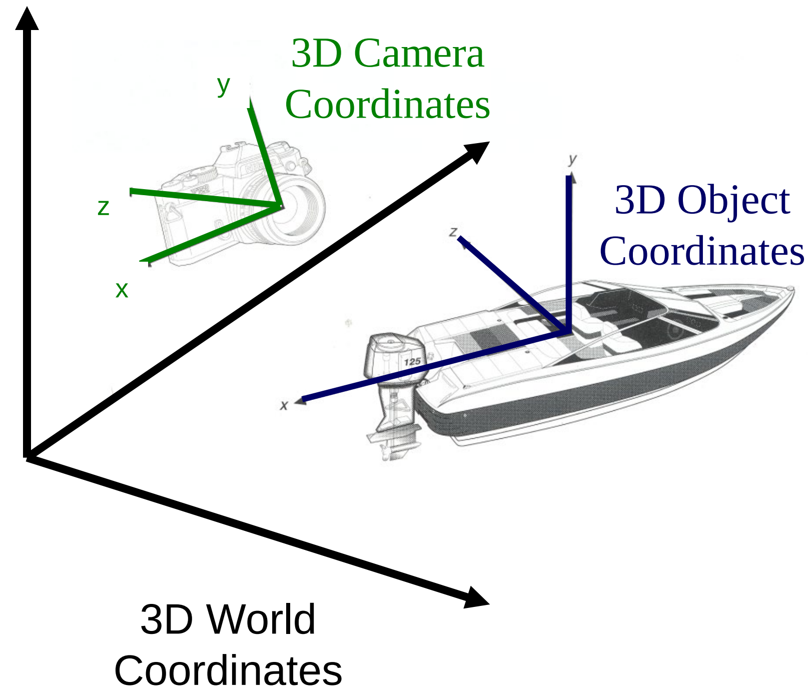
↓ 2D Screen Coordinates

**Viewport Transformation**

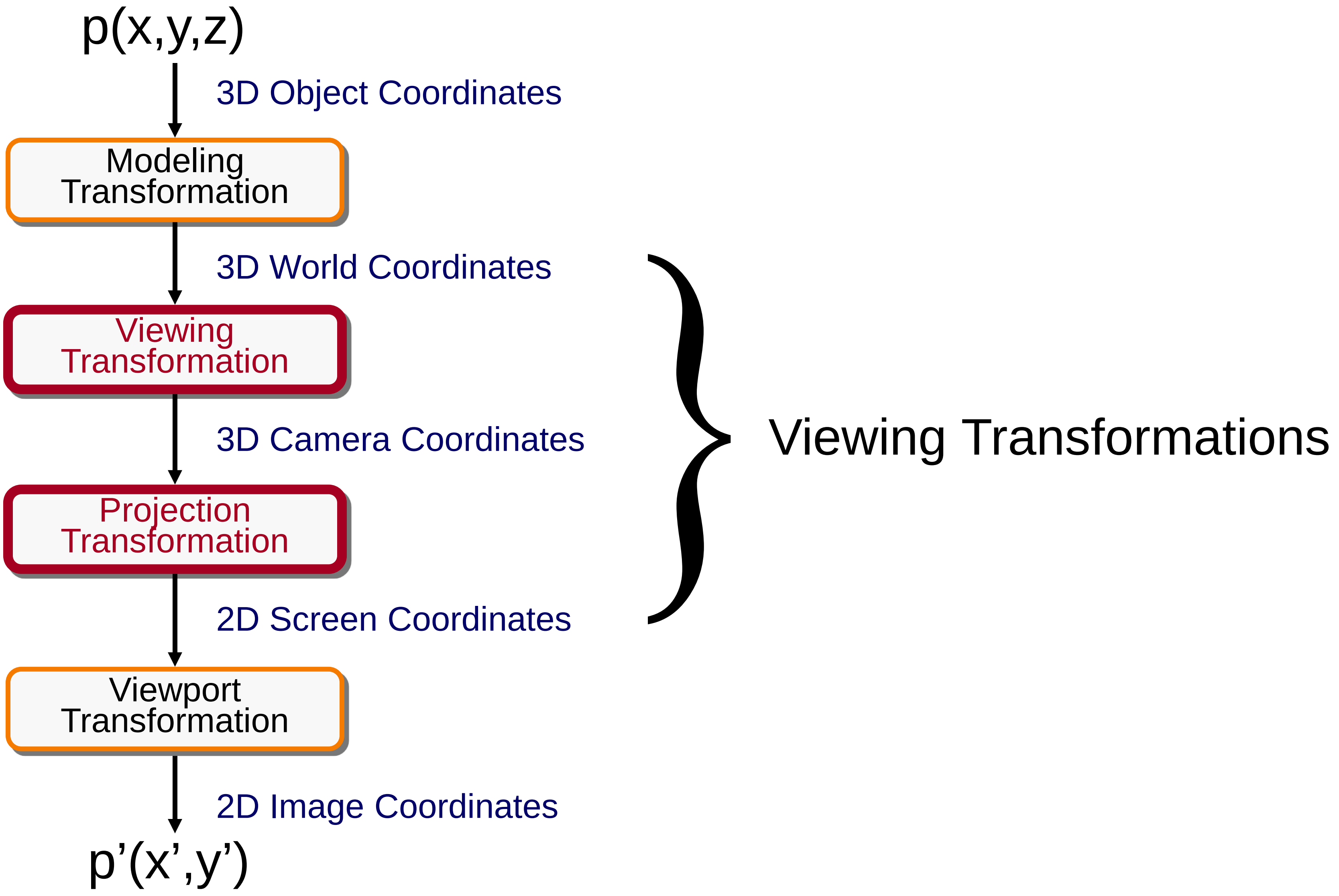
↓ 2D Image Coordinates

p'(x',y')

> Transformations map points from one coordinate system to another

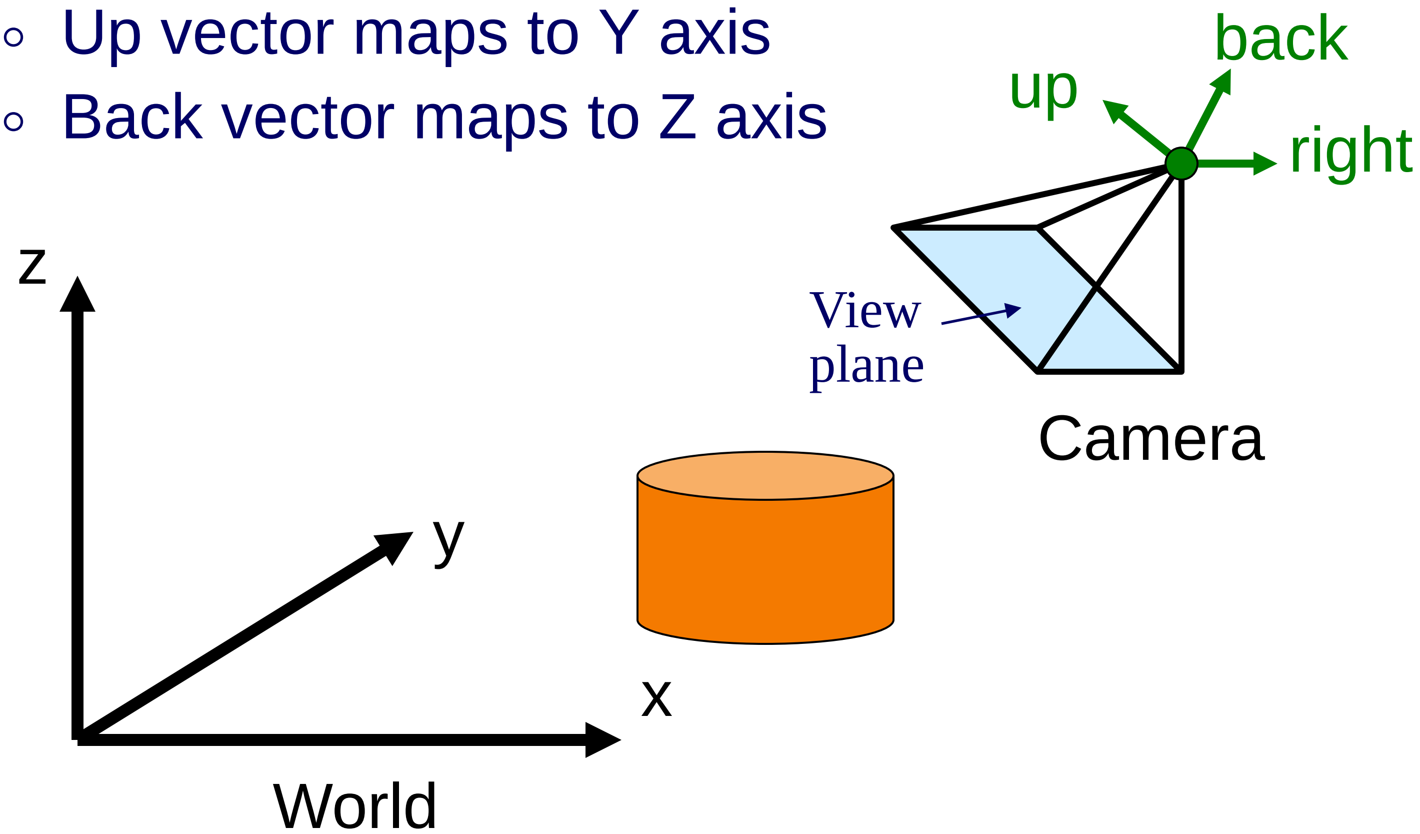


# Viewing Transformations



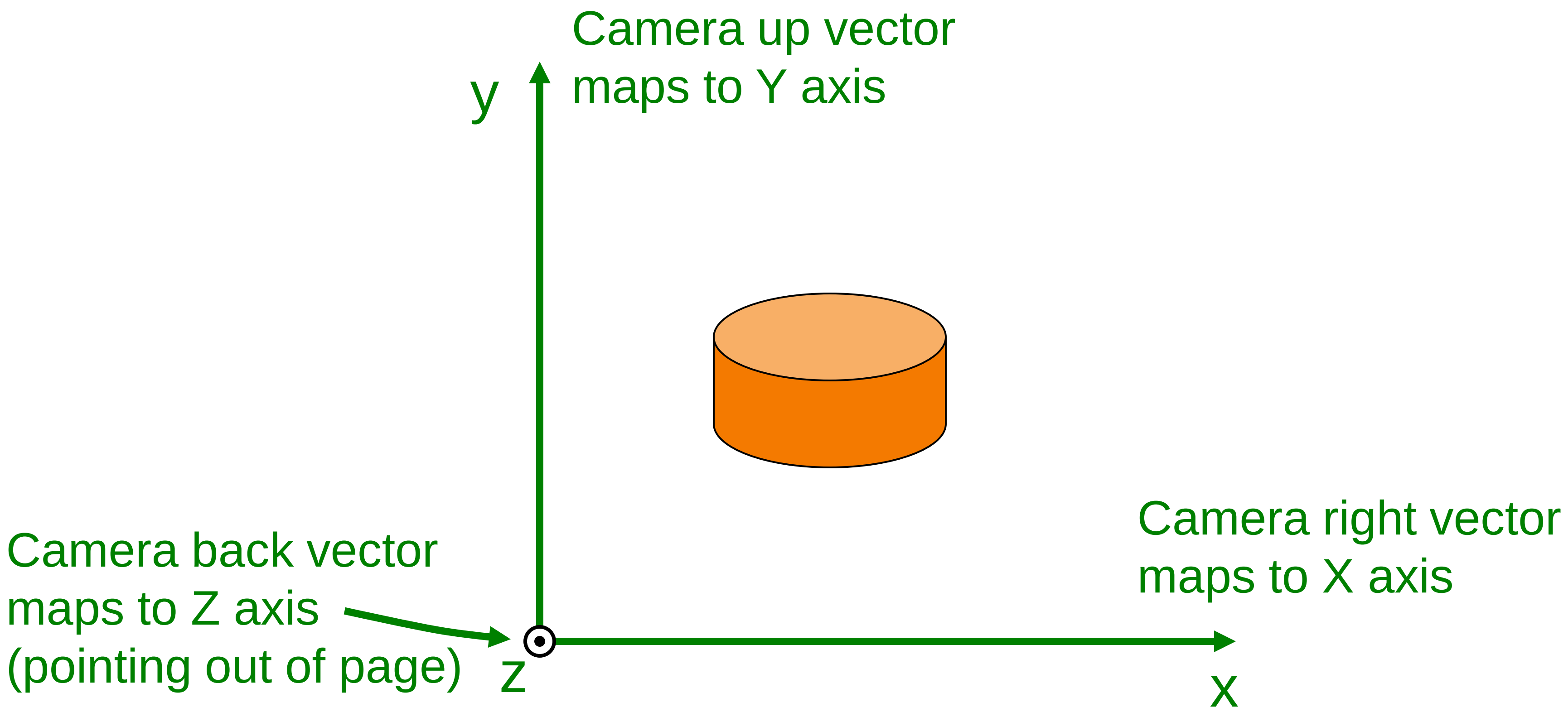
# Review: Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis



# Review: Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



# Finding the viewing transformation

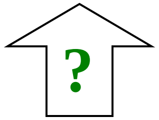
- We have the camera (in world coordinates)
- We want $T$ taking objects from world to camera

$$p^C = T\ p^W$$

- Trick: find $T^{-1}$ taking objects in camera to world

$$p^W = T^{-1} p^C$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$



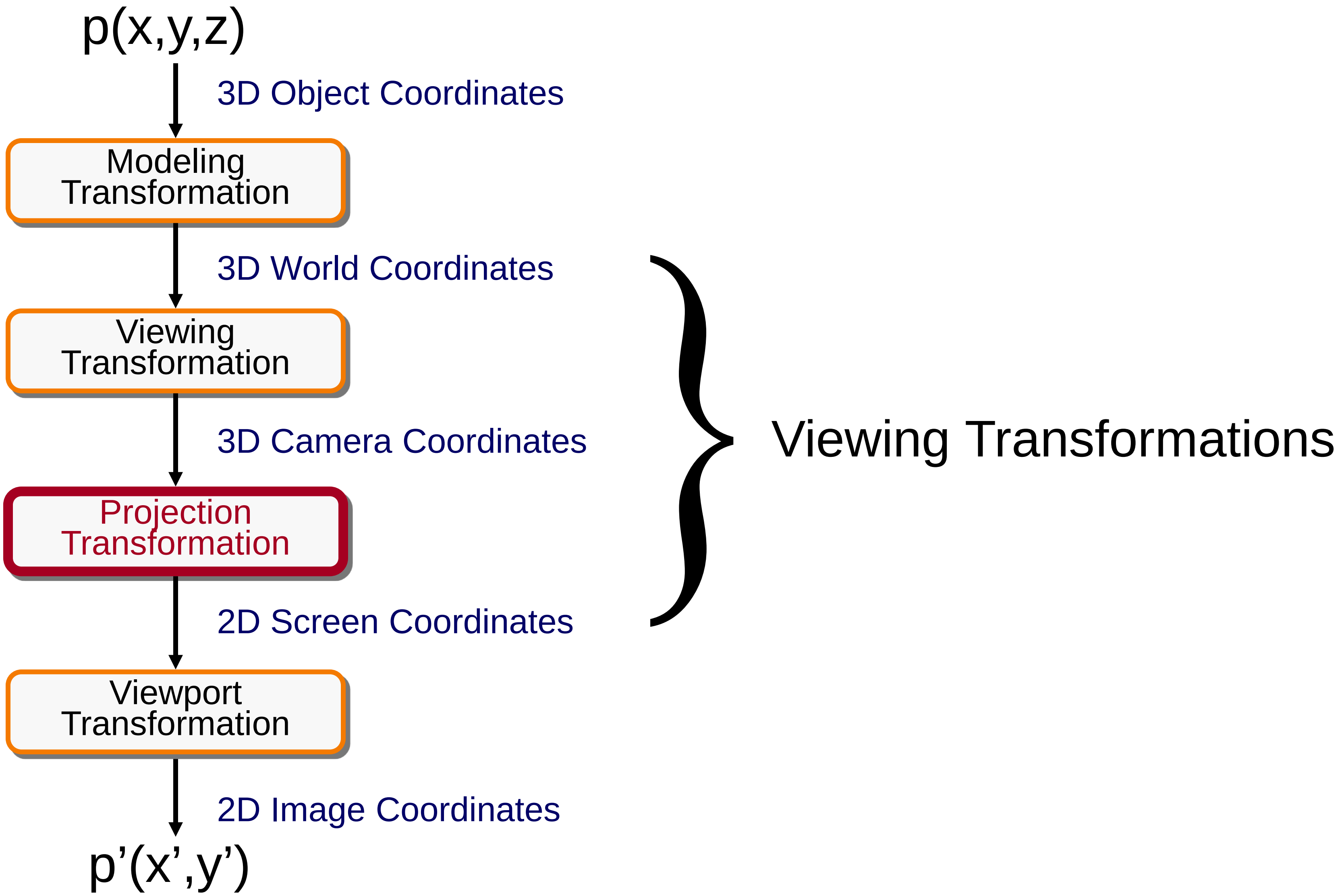
# Finding the Viewing Transformation

- Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

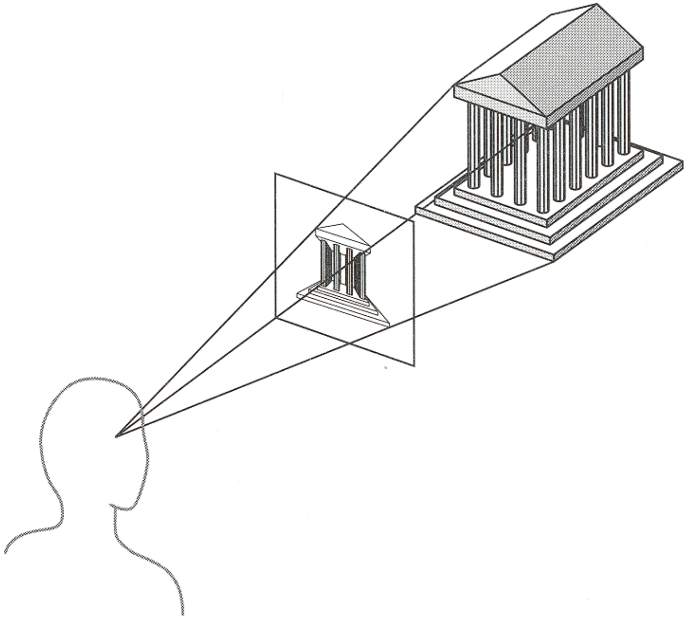
- This matrix is $T^{-1}$ so we invert it to get $T$ … easy!

# Viewing Transformations

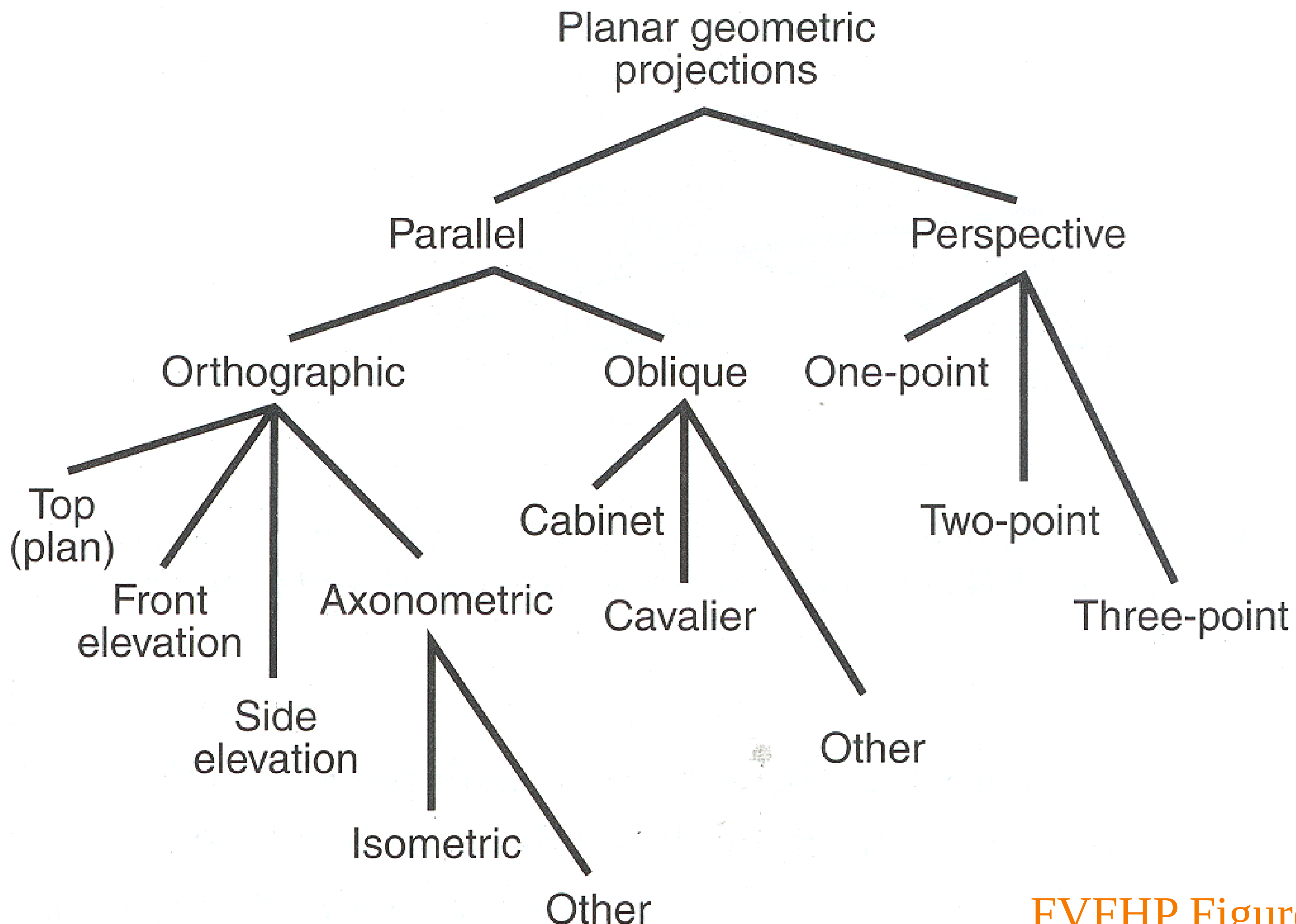


# Projection

- General definition:
  - Transform points in *n*-space to *m*-space (*m*<*n*)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

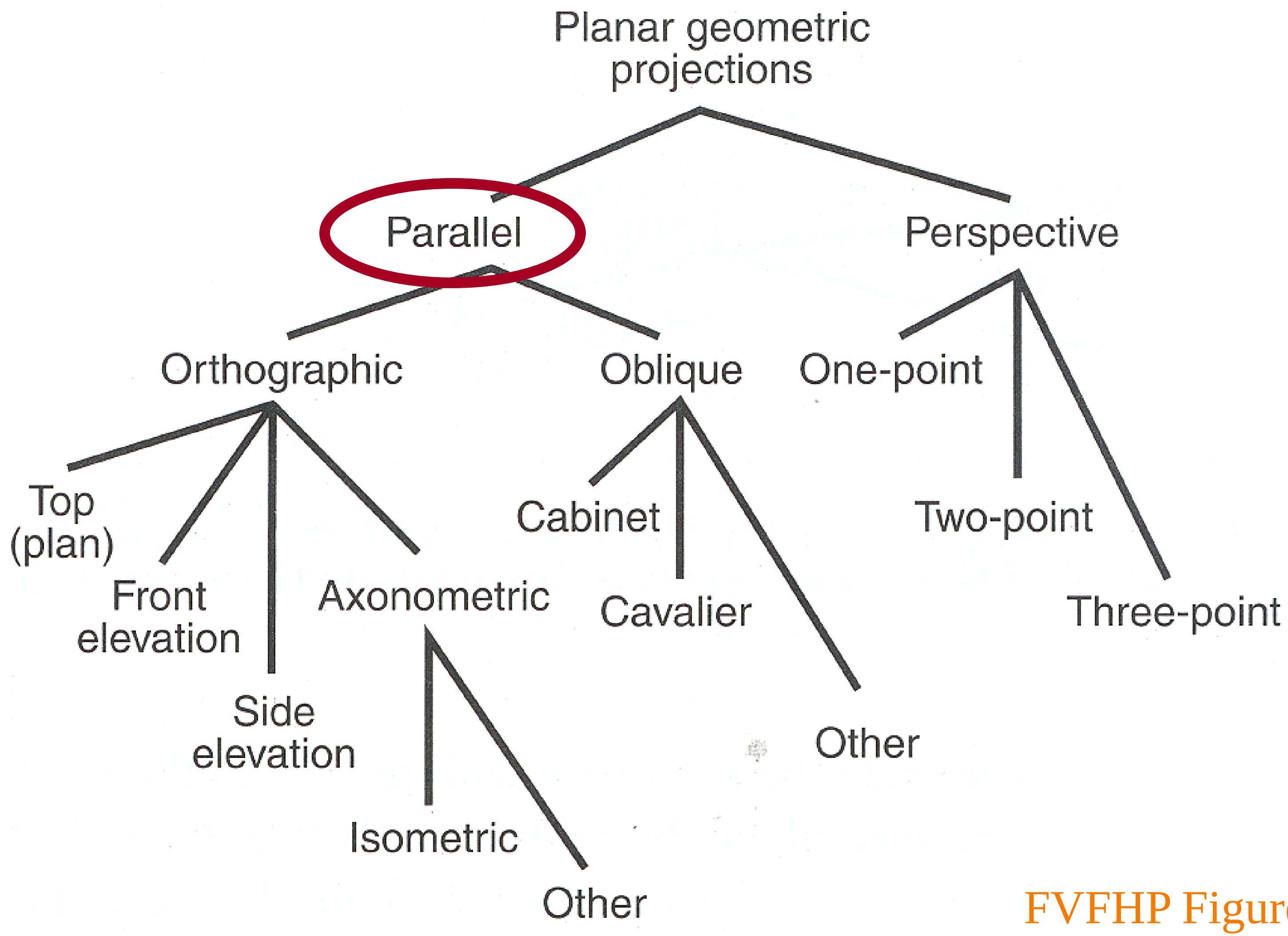


# Taxonomy of Projections




FVFHP Figure 6.10

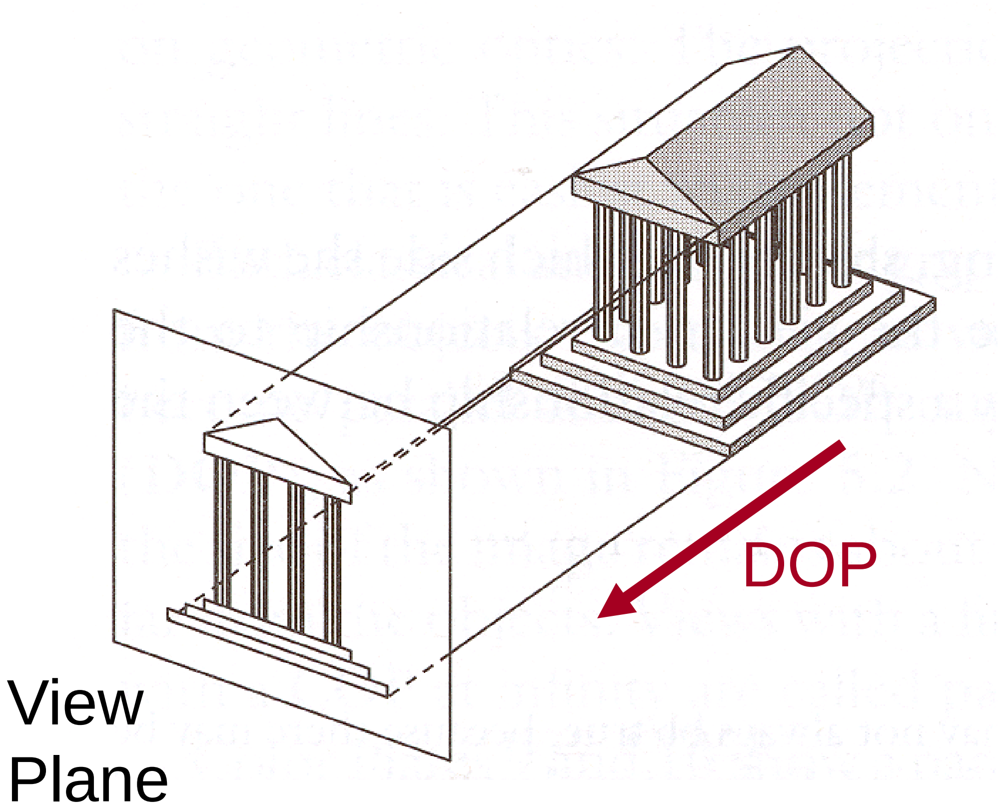
# Taxonomy of Projections




FVFHP Figure 6.10

# Parallel Projection

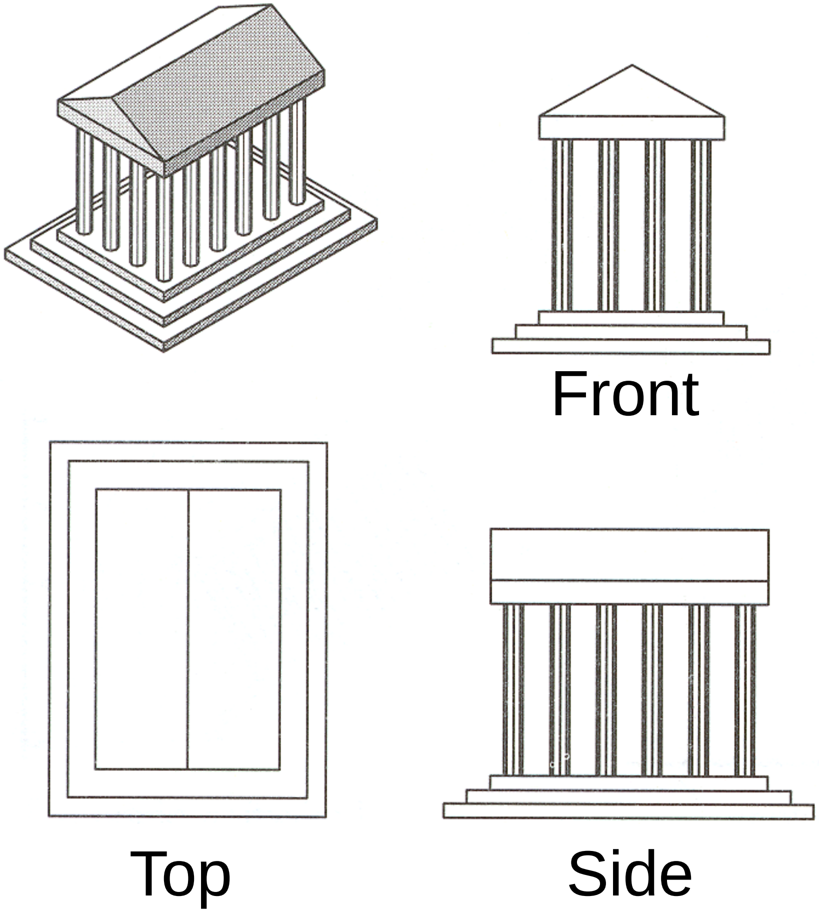
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points




Angel Figure 5.4

# Orthographic Projections

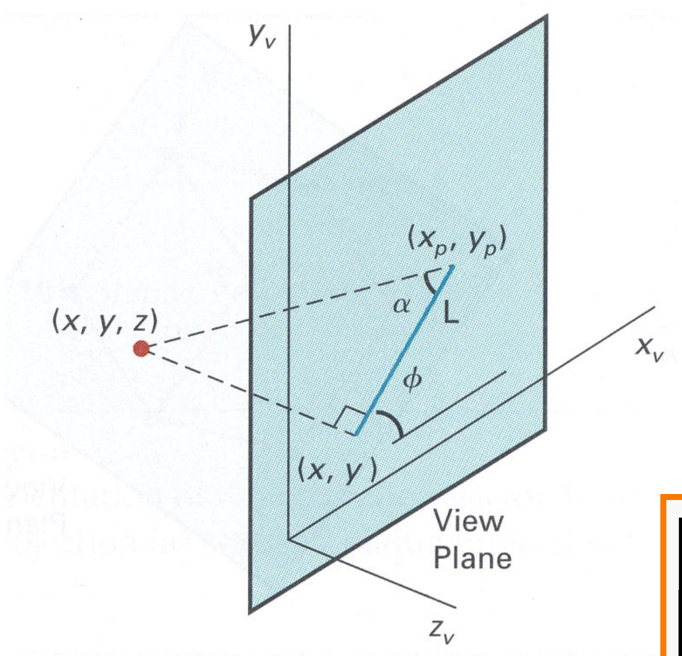
- DOP perpendicular to view plane




Angel Figure 5.5

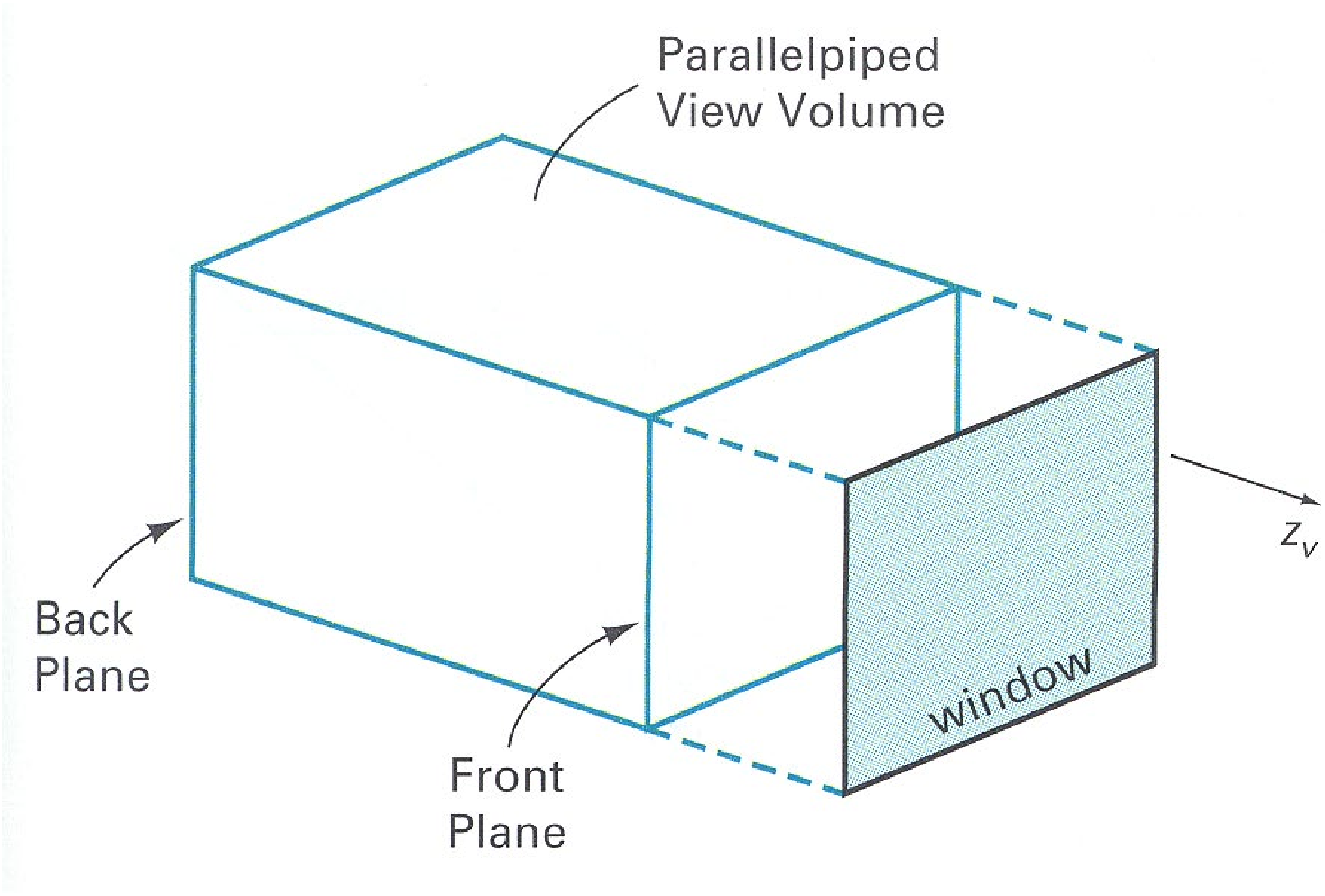
# Parallel Projection Matrix

- General parallel projection transformation:



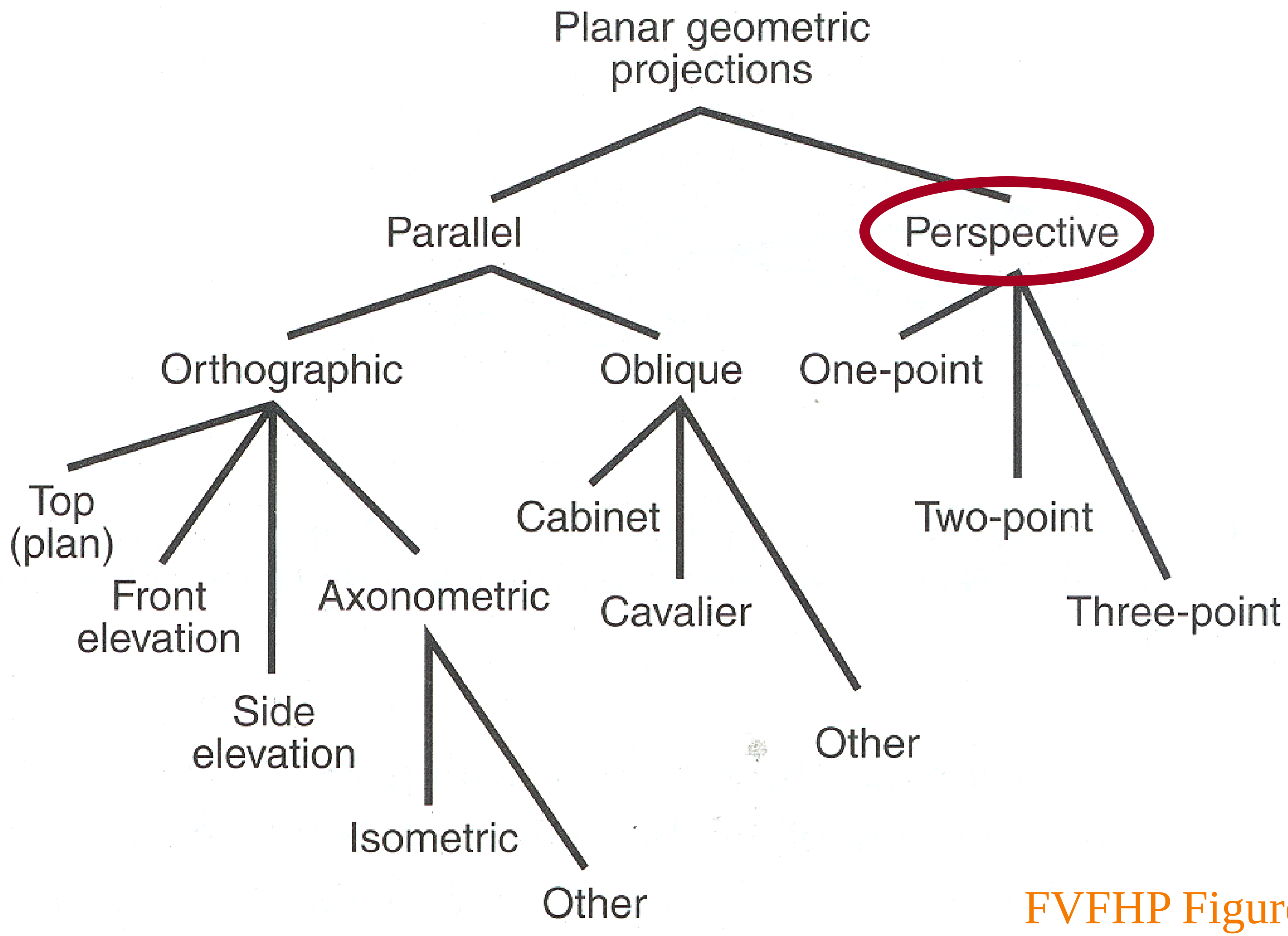

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L\cos\phi & 0 \\ 0 & 1 & L\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

# Parallel Projection View Volume




H&B Figure 12.30

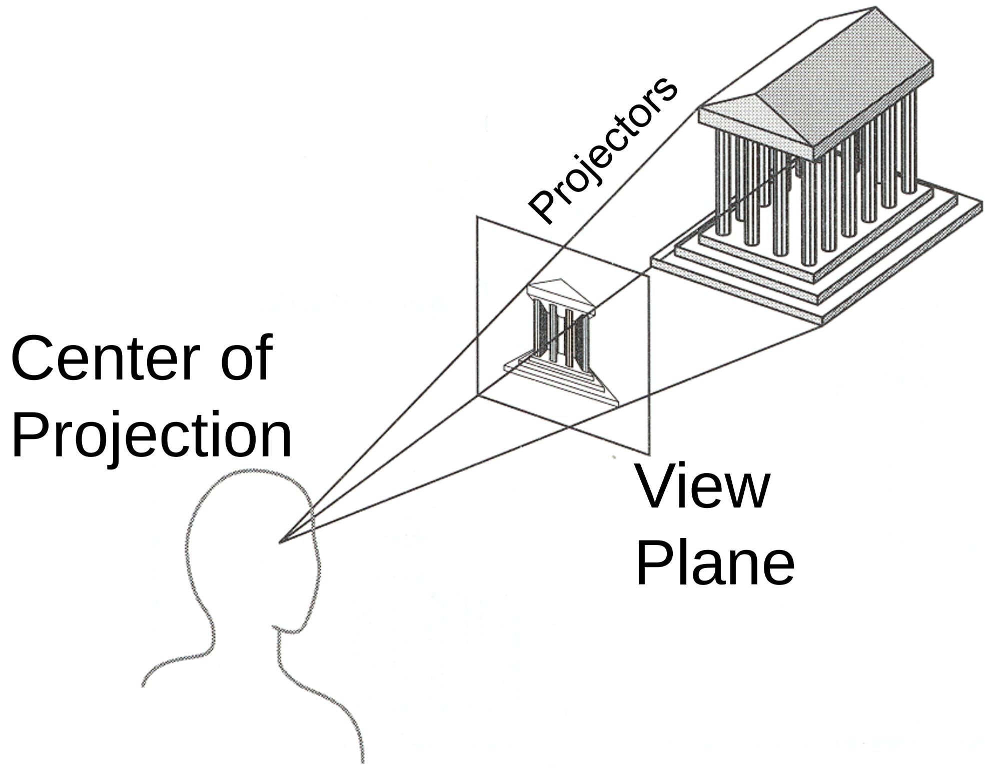
# Taxonomy of Projections




FVFHP Figure 6.10

# Perspective Projection

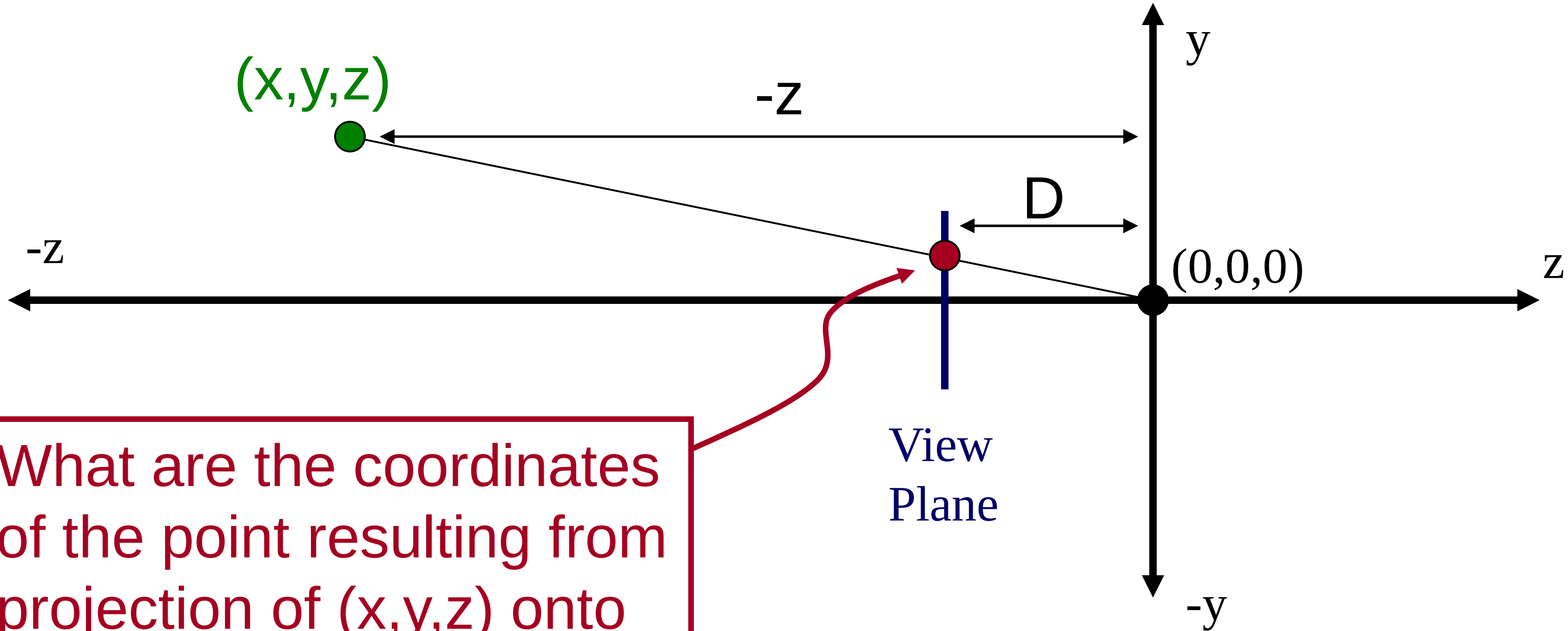
- Map points onto "view plane" along "projectors" emanating from "center of projection" (COP)




Angel Figure 5.9

# Perspective Projection

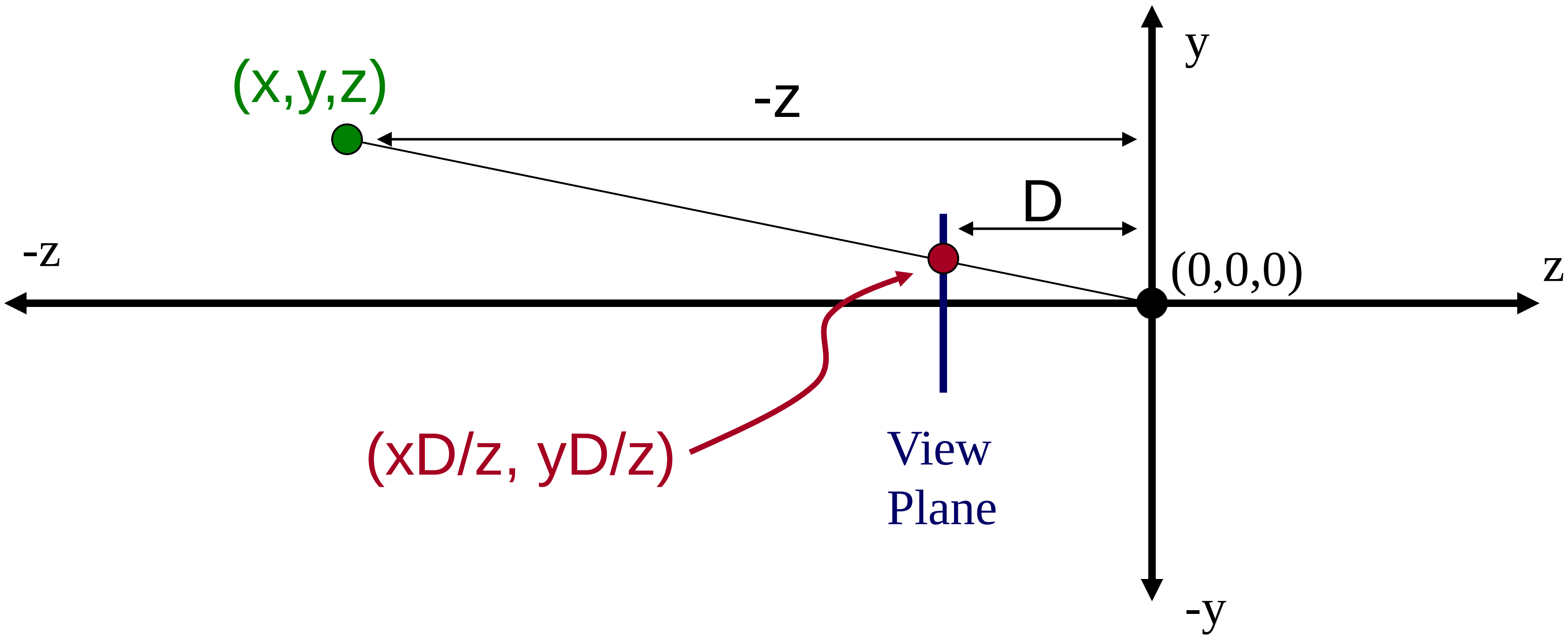
- Compute 2D coordinates from 3D coordinates with similar triangles




What are the coordinates of the point resulting from projection of (x,y,z) onto the view plane?

# Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



# Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

# Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c \qquad x_s = x' / w' \qquad x' = x_c$$
$$y_s = y_c D / z_c \qquad y_s = y' / w' \qquad y' = y_c$$
$$z_s = D \qquad z_s = z' / w' \qquad z' = z_c$$
$$w_s = 1 \qquad w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

# Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c \qquad x_s = x' / w' \qquad x' = x_c$$
$$y_s = y_c D / z_c \qquad y_s = y' / w' \qquad y' = y_c$$
$$z_s = D \qquad z_s = z' / w' \qquad z' = z_c$$
$$w_s = 1 \qquad w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

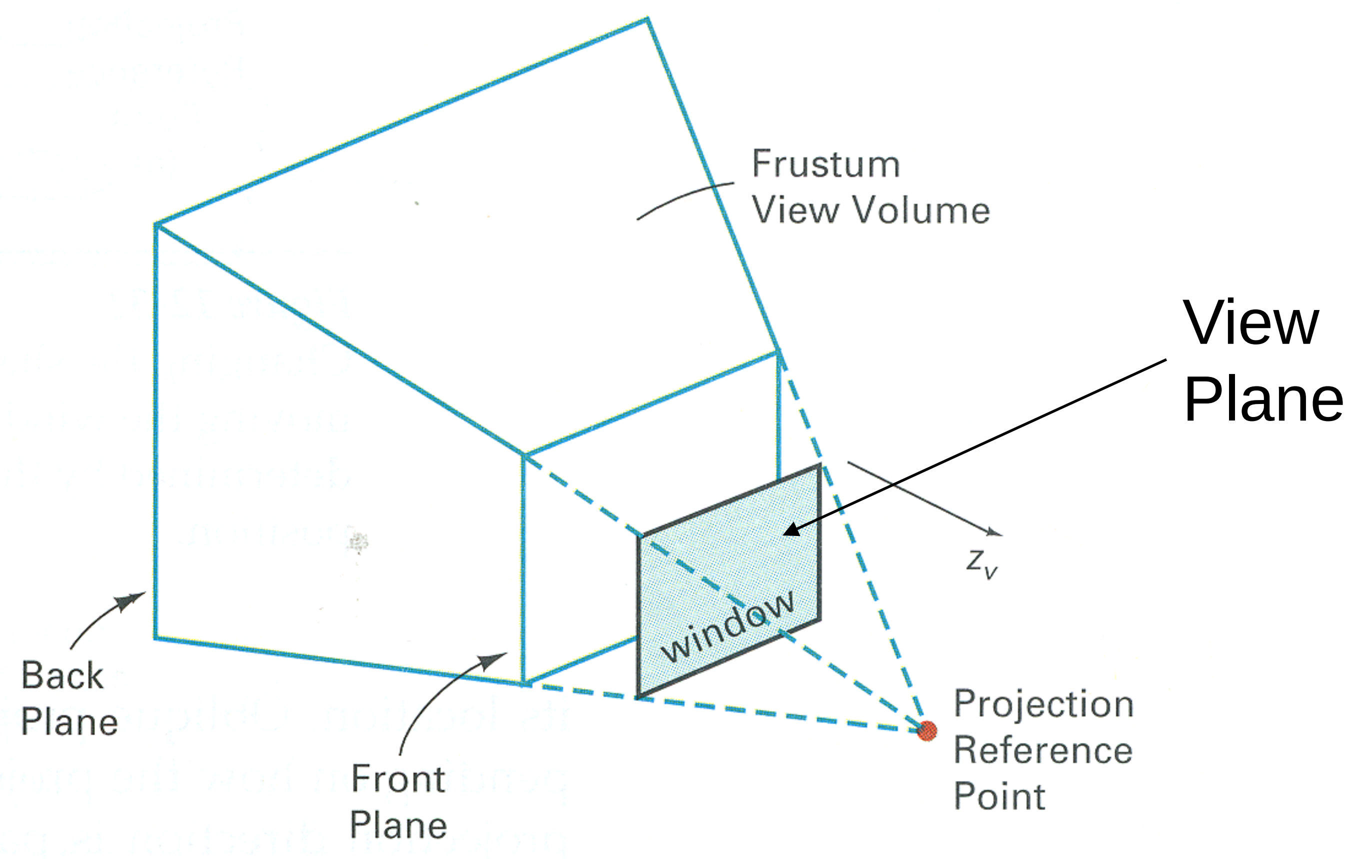
# Perspective Projection Matrix

- In practice, want to compute a value related to depth to include in *z*-buffer

$$
\begin{aligned}
x_s &= x_c D / z_c \\
y_s &= y_c D / z_c \\
z_s &= -D / z_c \\
w_s &= 1
\end{aligned}
\qquad
\begin{aligned}
x_s &= x' / w' \\
y_s &= y' / w' \\
z_s &= z' / w'
\end{aligned}
\qquad
\begin{aligned}
x' &= x_c \\
y' &= y_c \\
z' &= -1 \\
w' &= z_c / D
\end{aligned}
$$

$$
\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}
$$

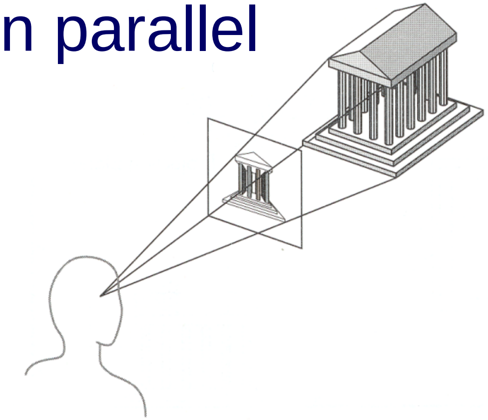
# Perspective Projection View Volume



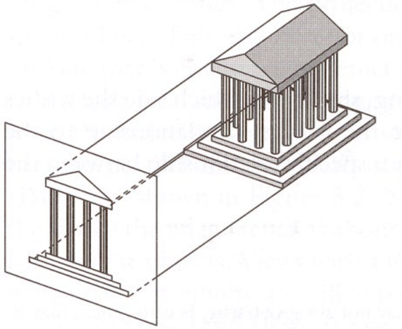

H&B Figure 12.30

# Perspective vs. Parallel

- Perspective projection
  - + Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - + Good for exact measurements
  - + Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



# Transformations

p(x,y,z)

↓ 3D Object Coordinates

**Modeling Transformation**

↓ 3D World Coordinates

**Viewing Transformation**

↓ 3D Camera Coordinates

**Projection Transformation**

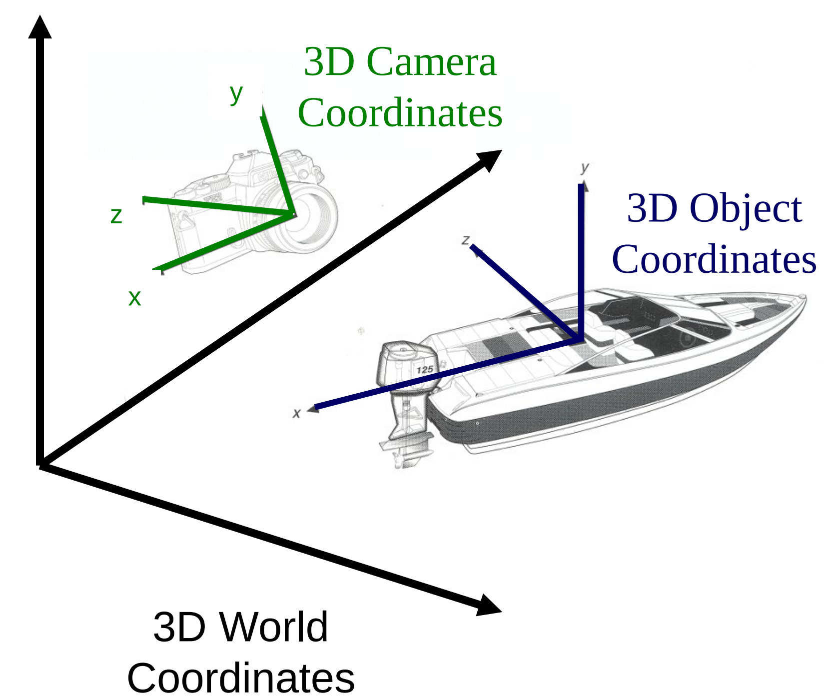
↓ 2D Screen Coordinates

**Viewport Transformation**

↓ 2D Image Coordinates

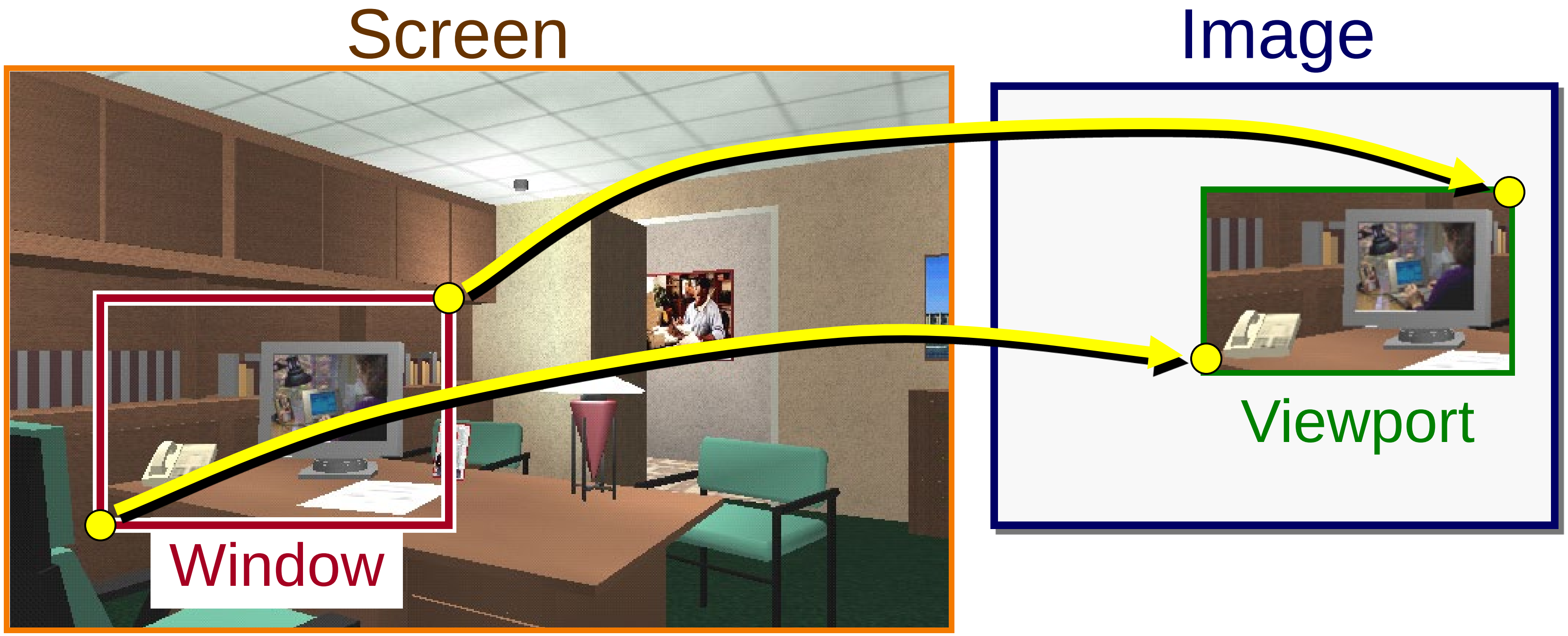
p'(x',y')

> Transformations map points from one coordinate system to another



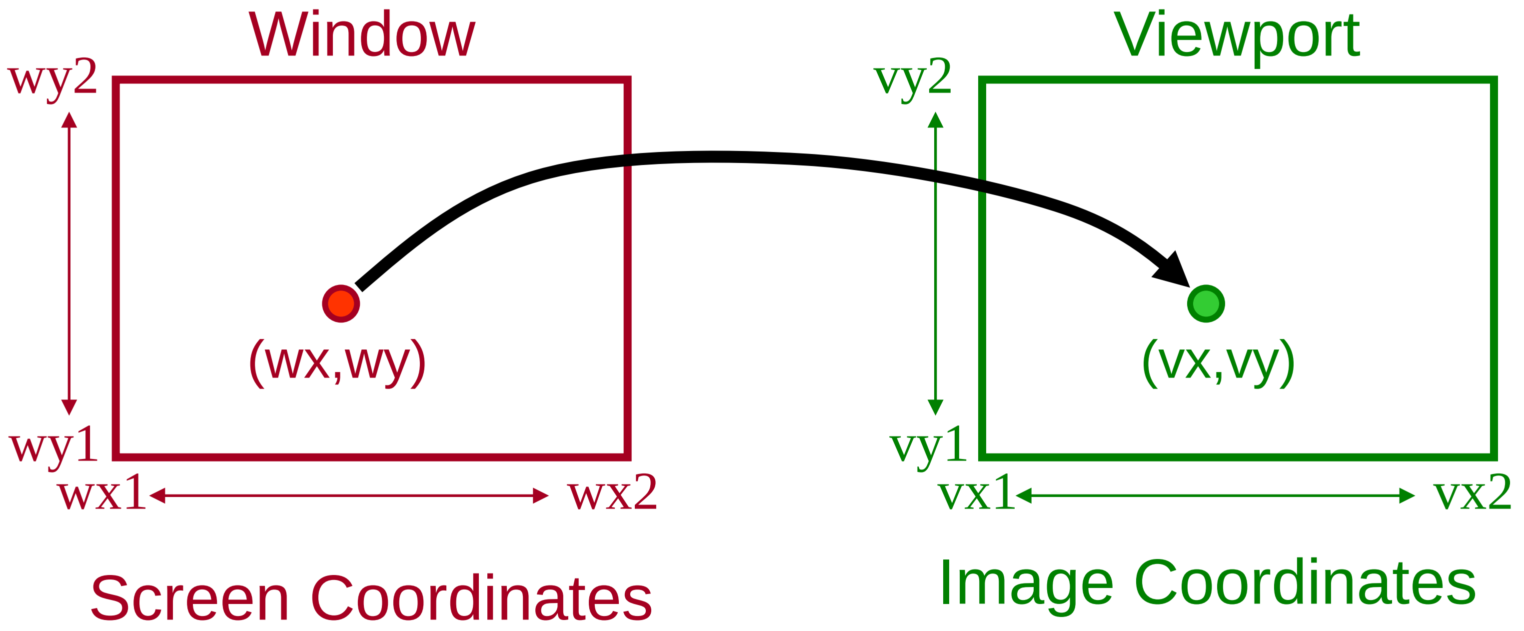
# Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



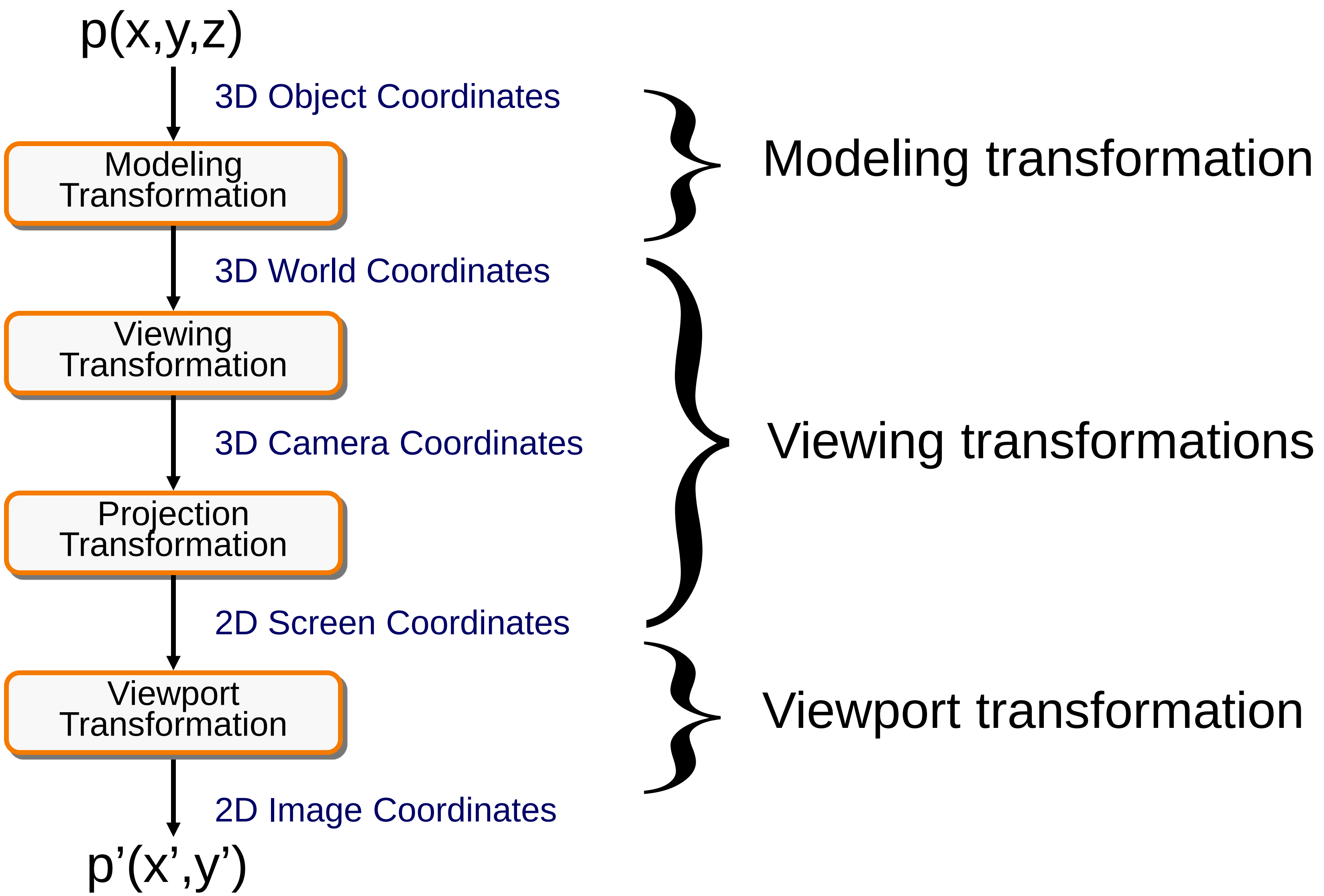
# Viewport Transformation

- Window-to-viewport mapping

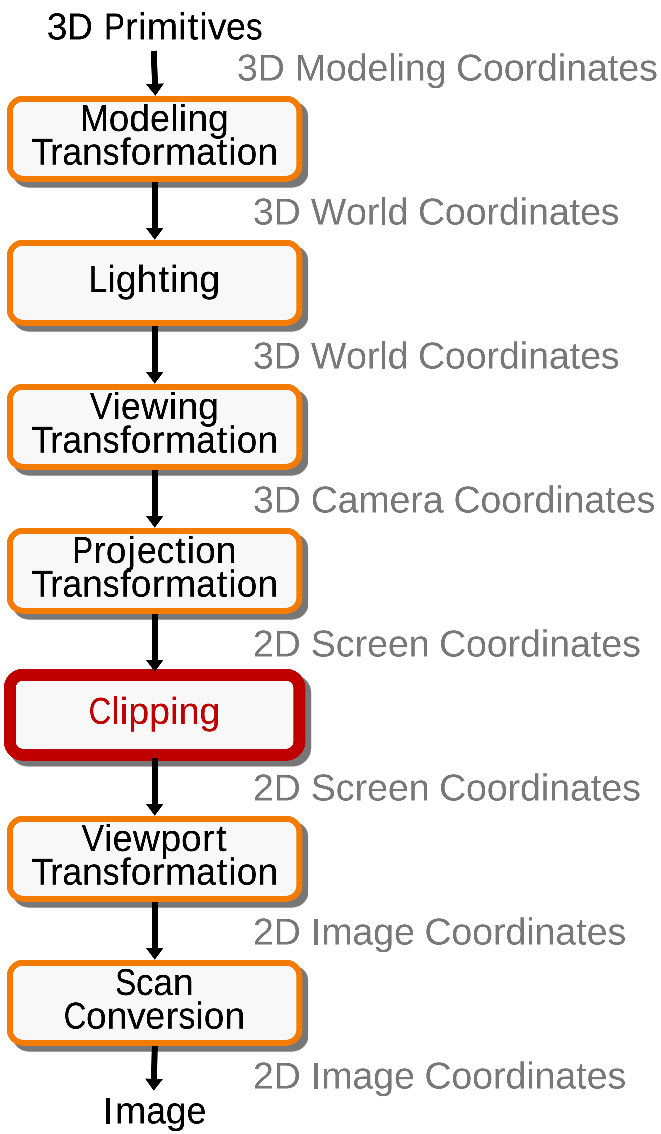


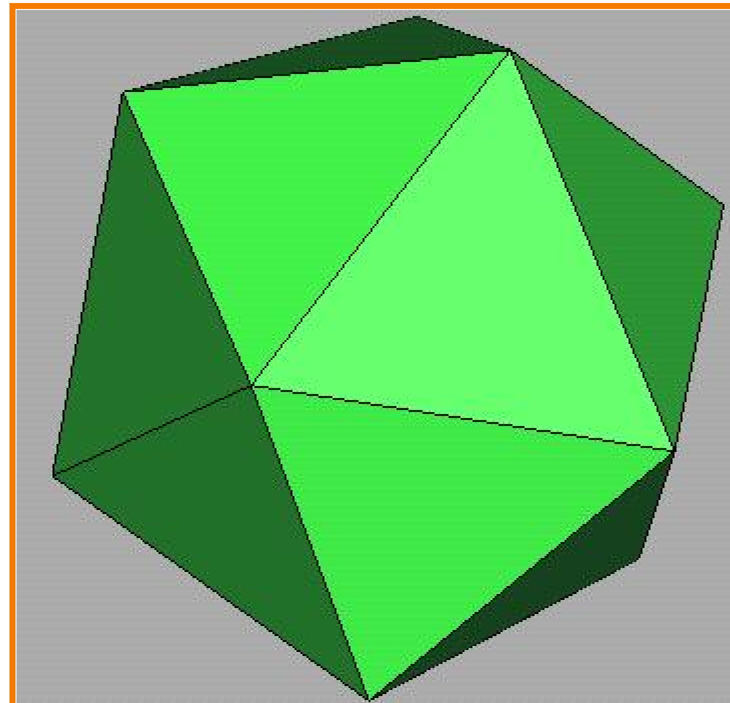

```
vx = vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1);
vy = vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1);
```

# Summary of Transformations



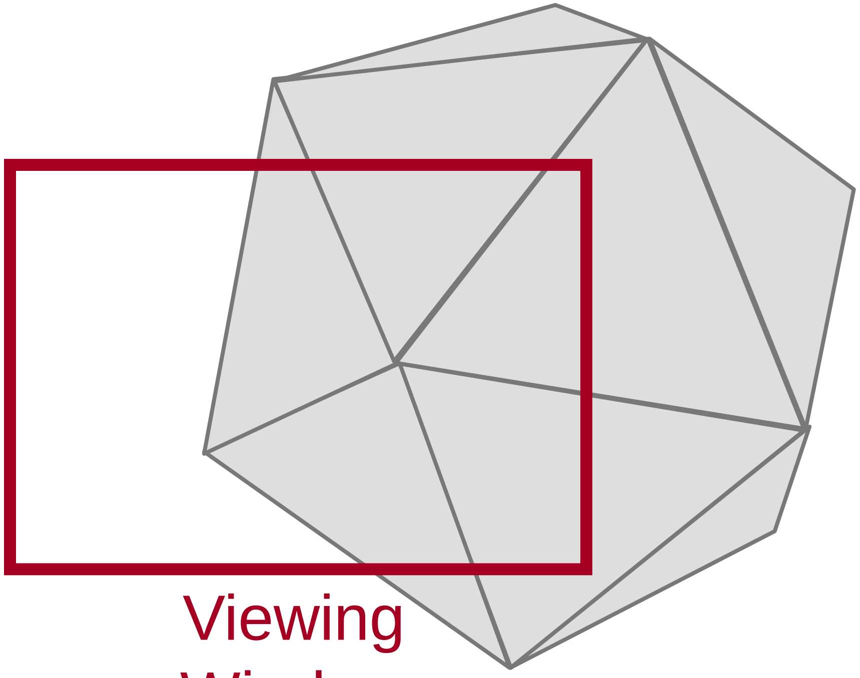
# 3D Rendering Pipeline (for direct illumination)



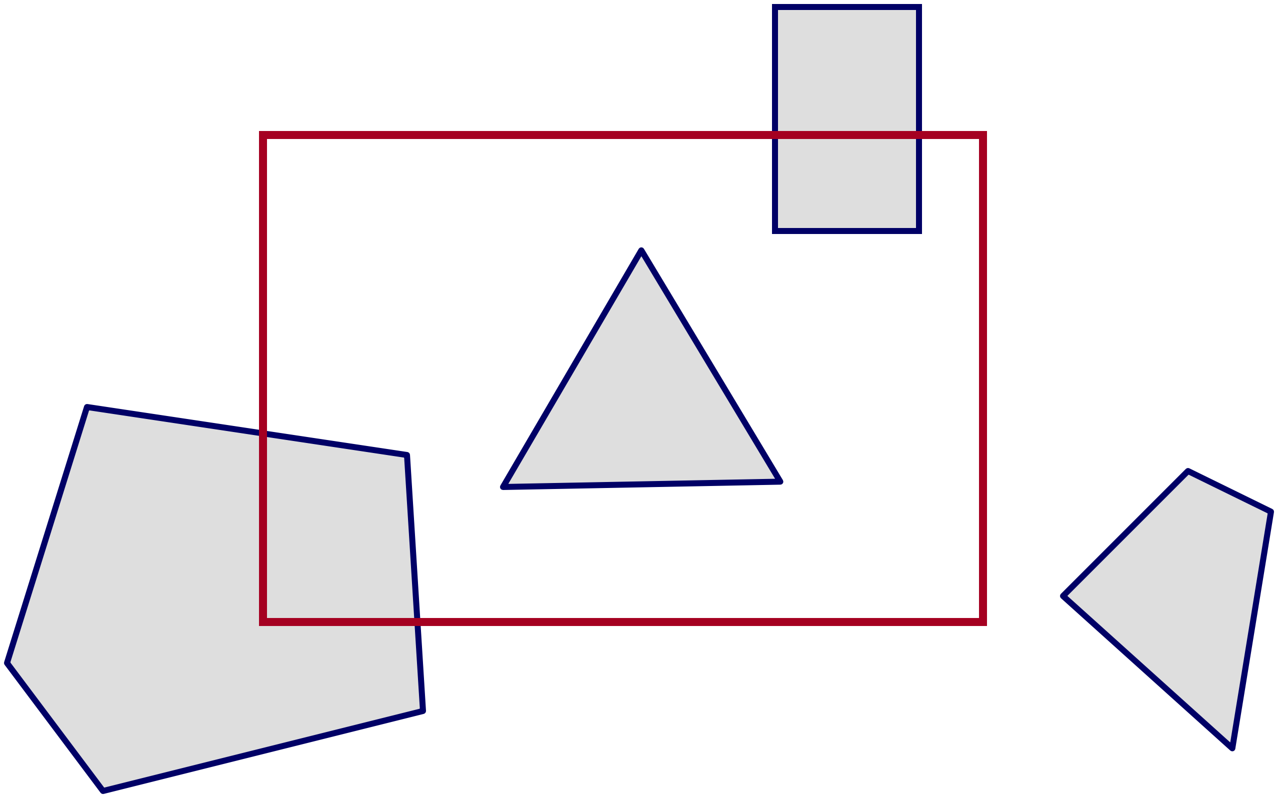
# Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window



# Polygon Clipping

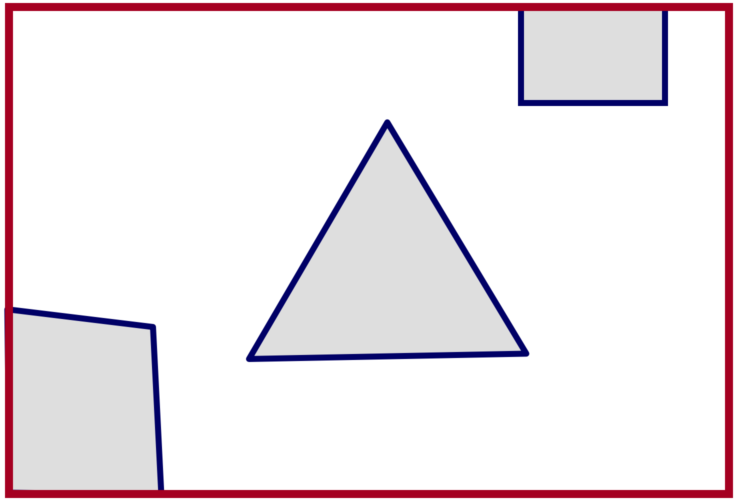
- Find the part of a polygon inside the clip window?



Before Clipping

# Polygon Clipping

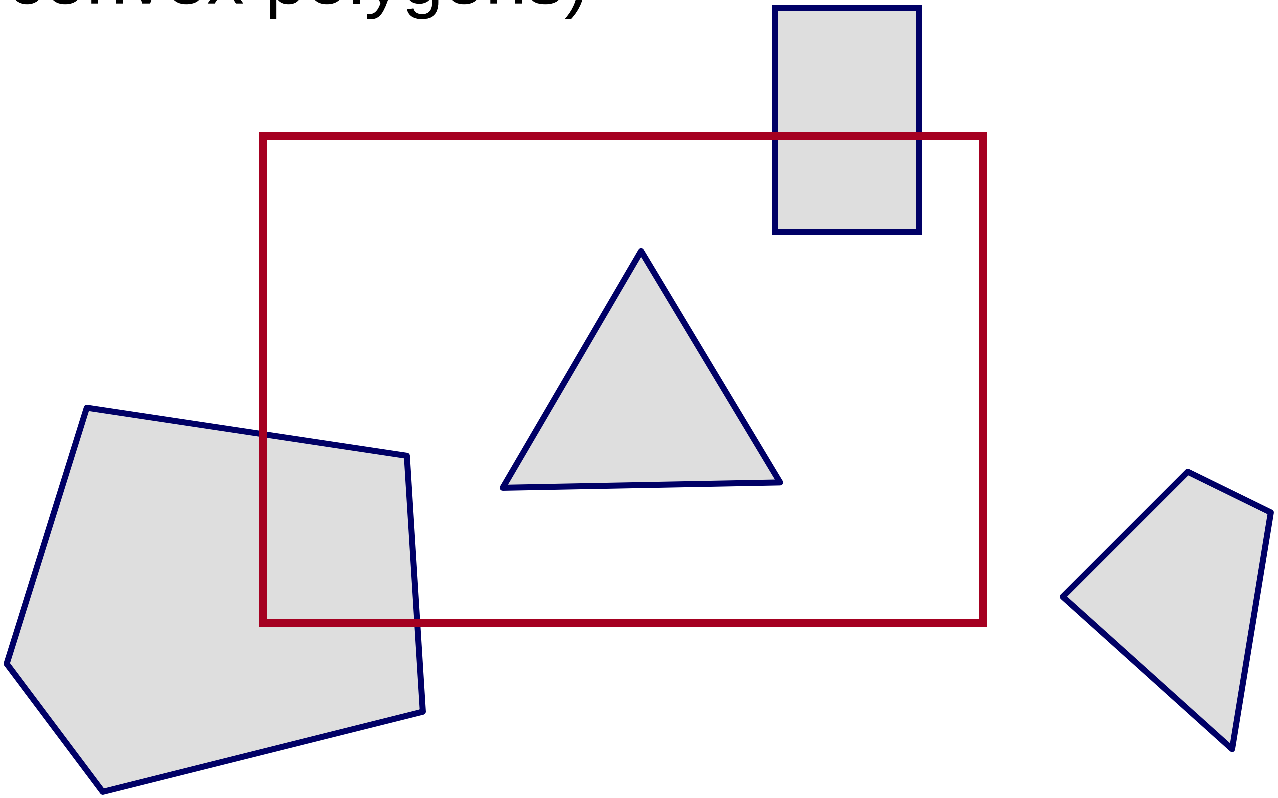
- Find the part of a polygon inside the clip window?



After Clipping

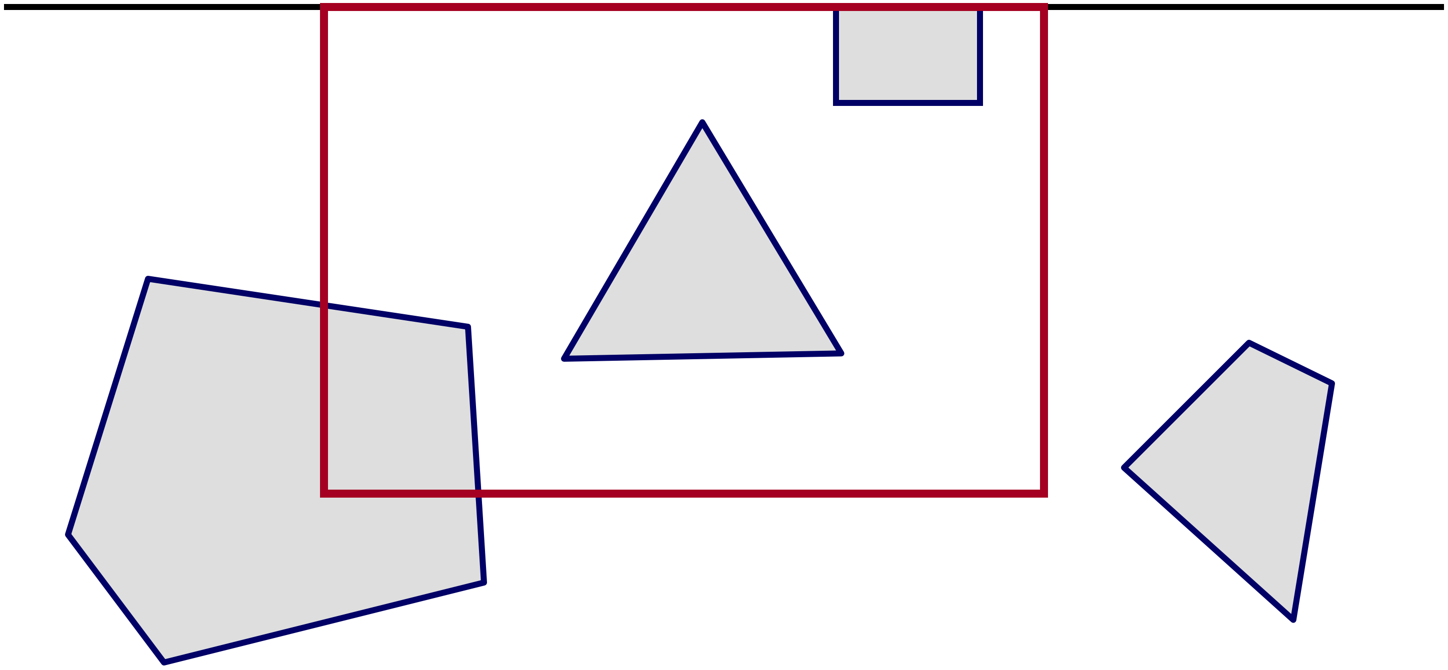
# Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time (for convex polygons)



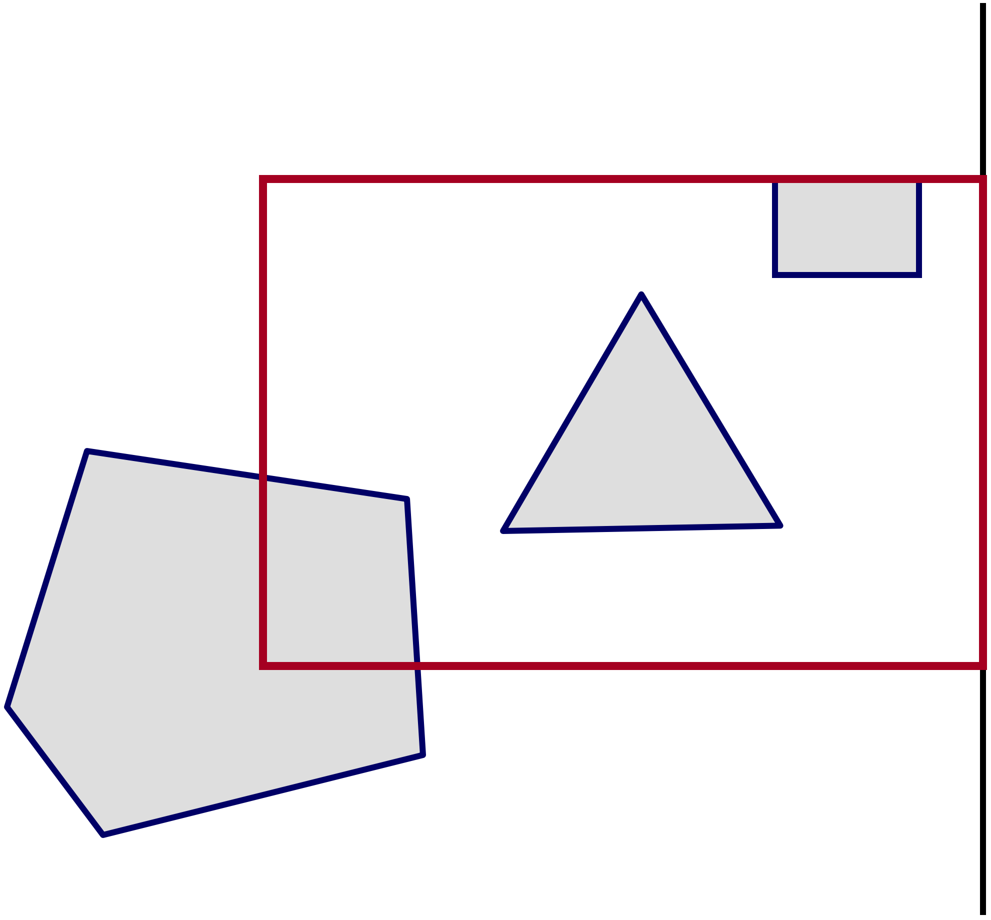
# Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



# Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



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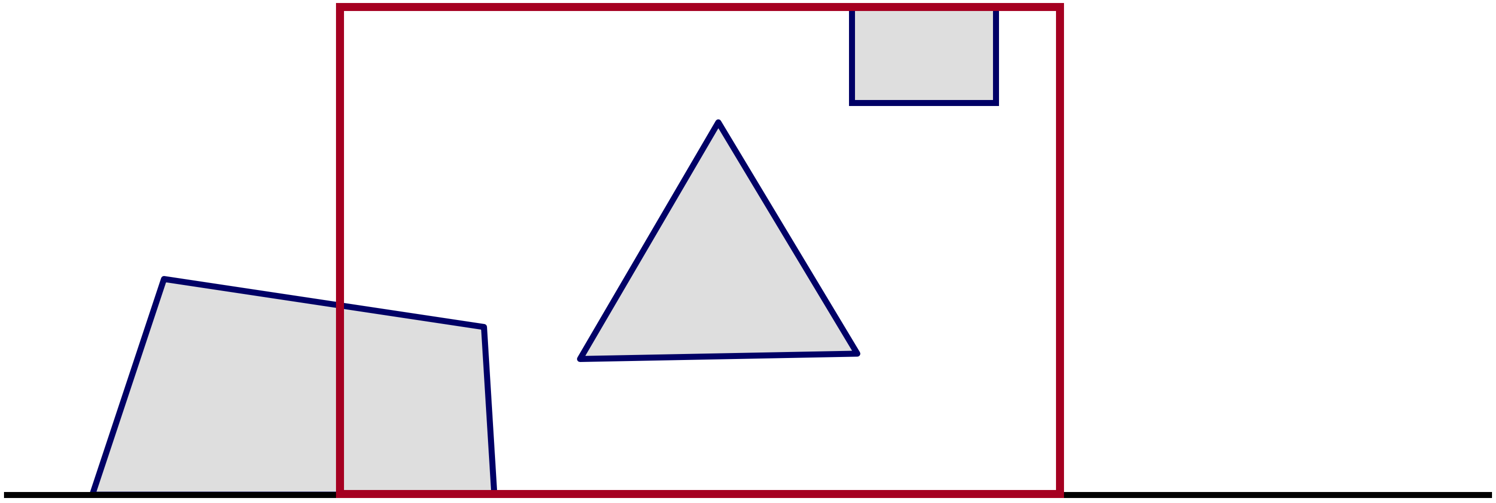

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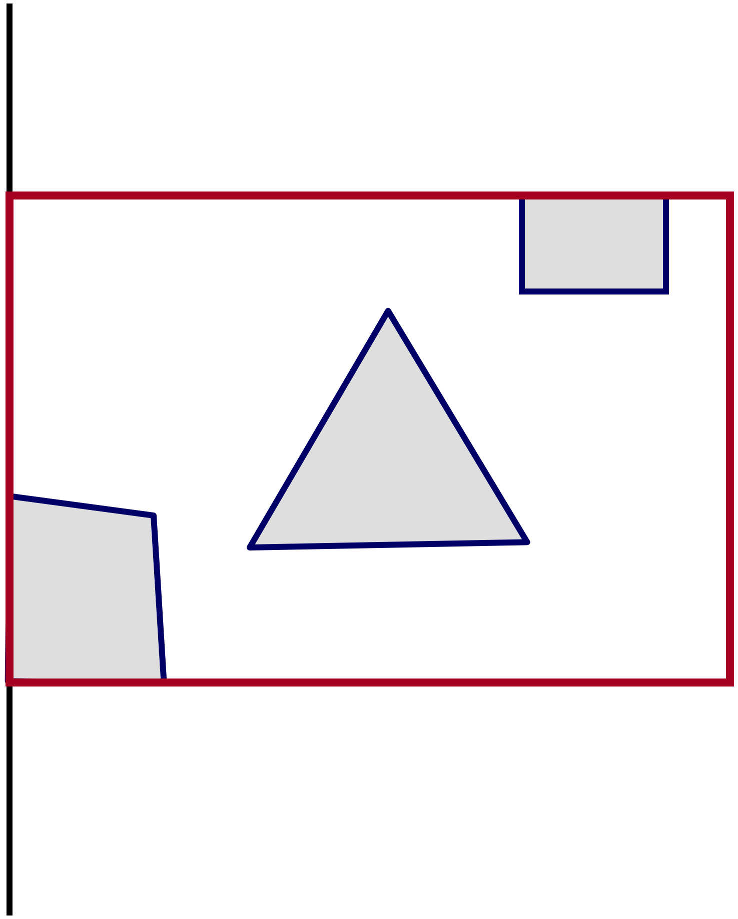
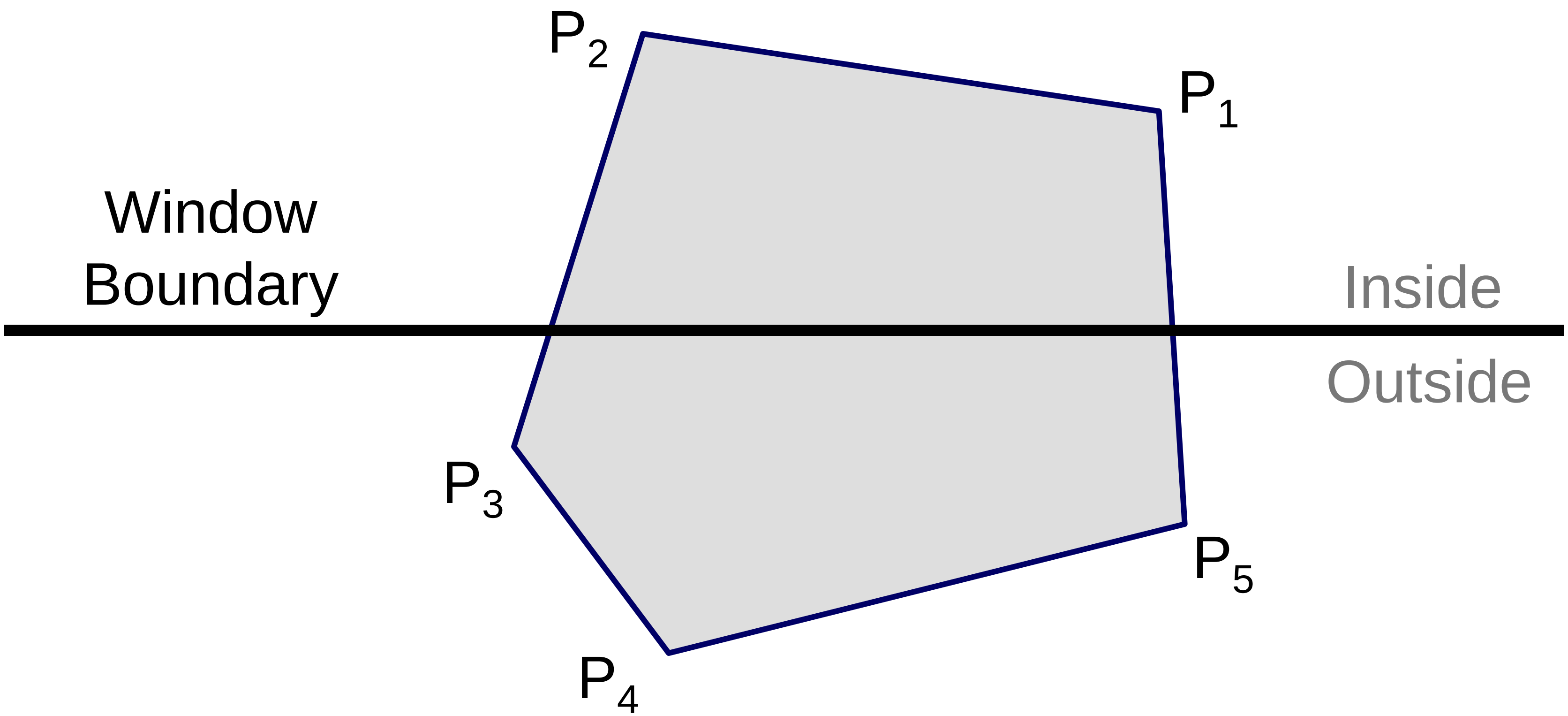


# Clipping to a Boundary

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary



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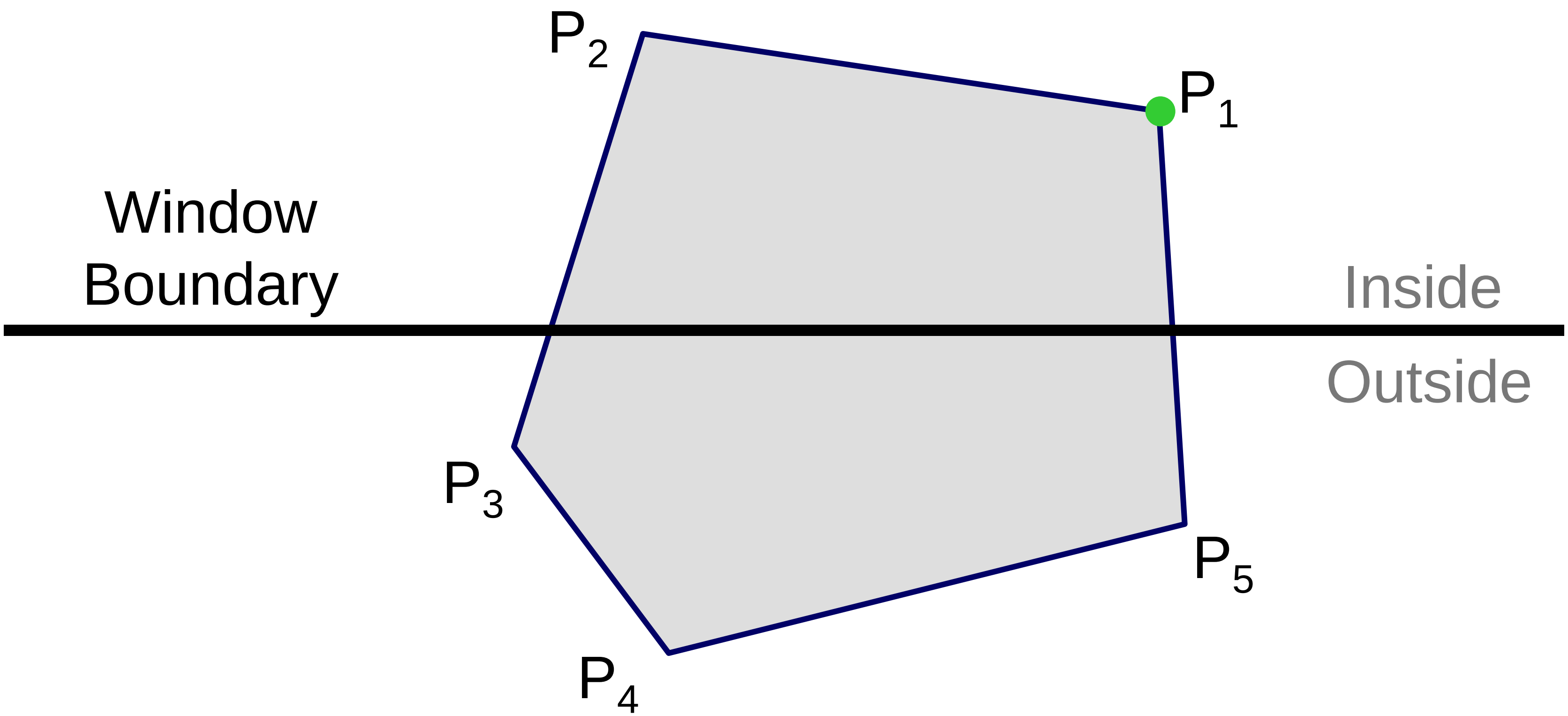

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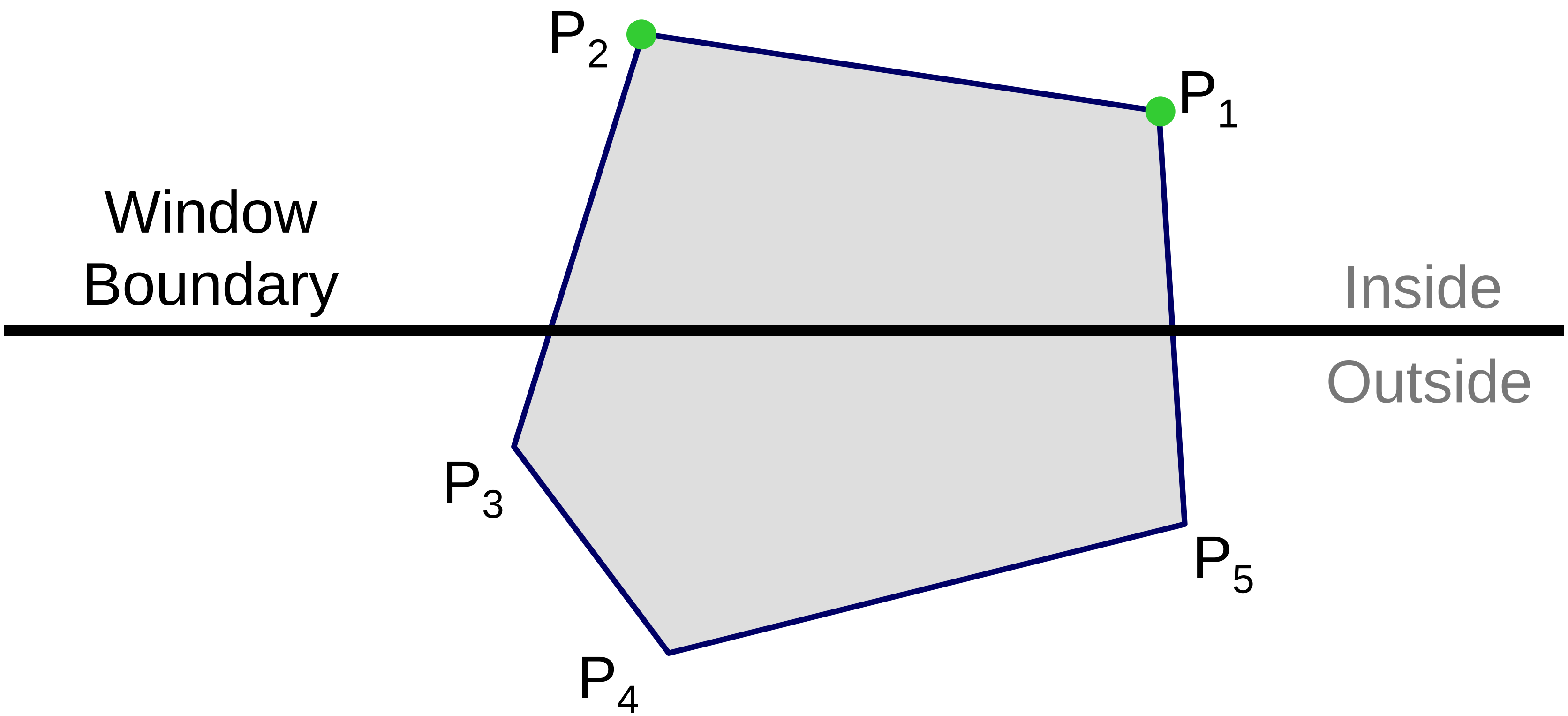

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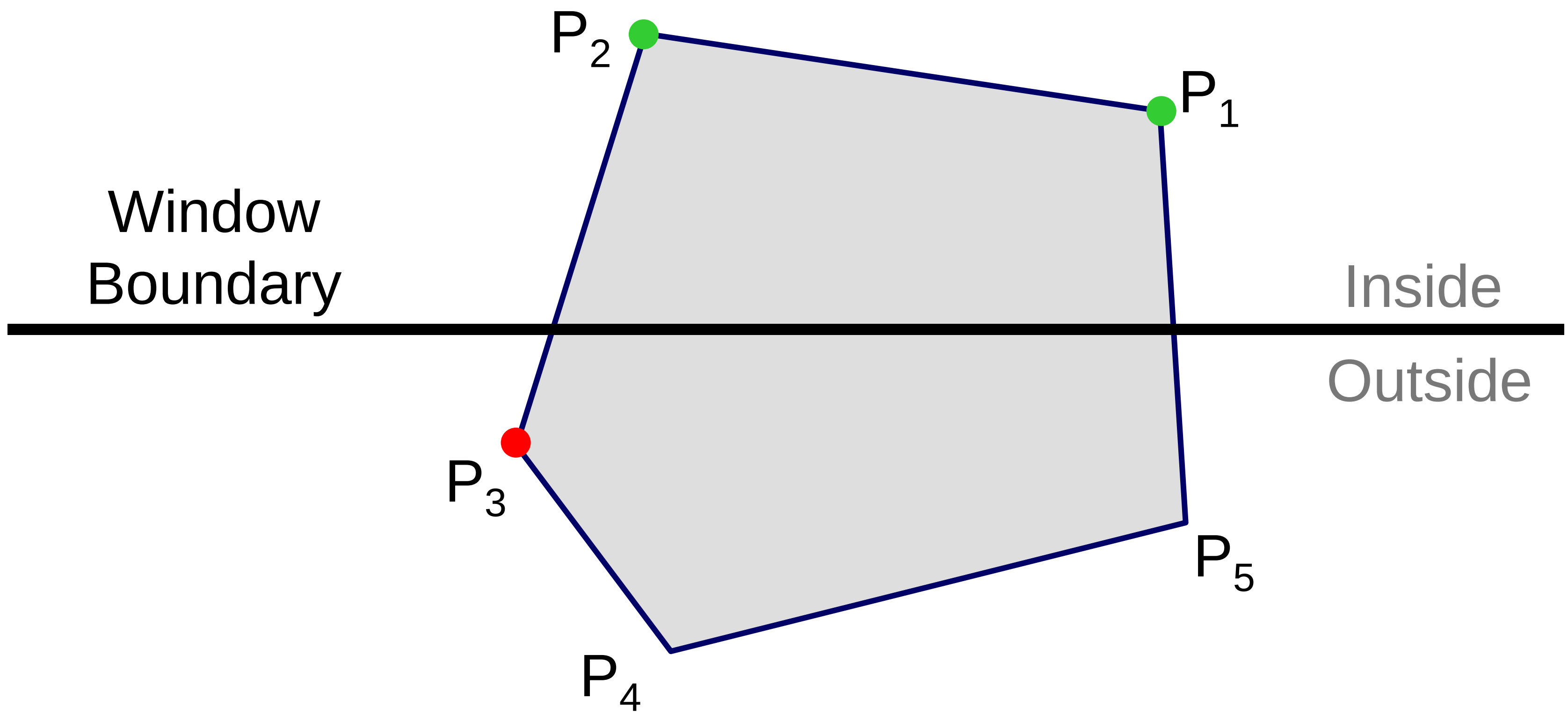

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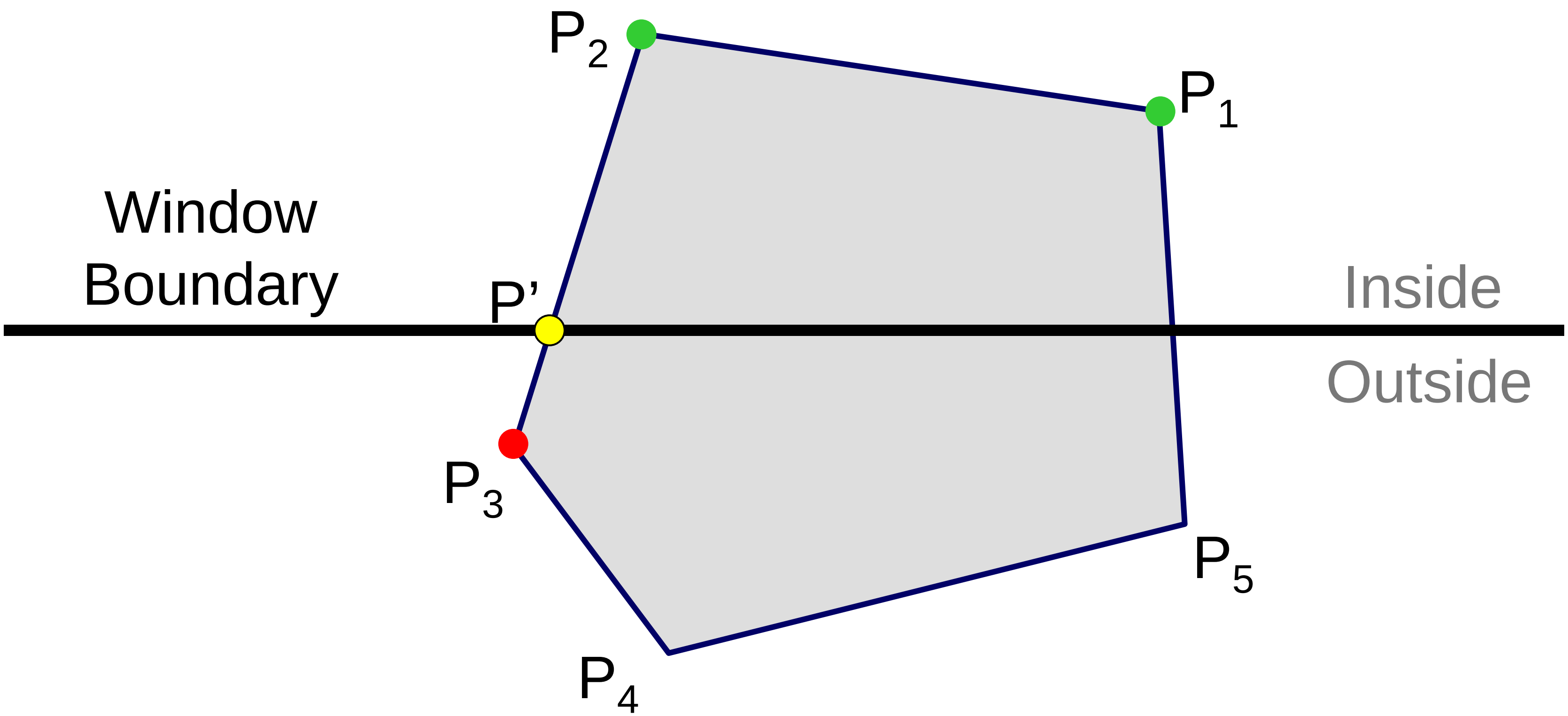

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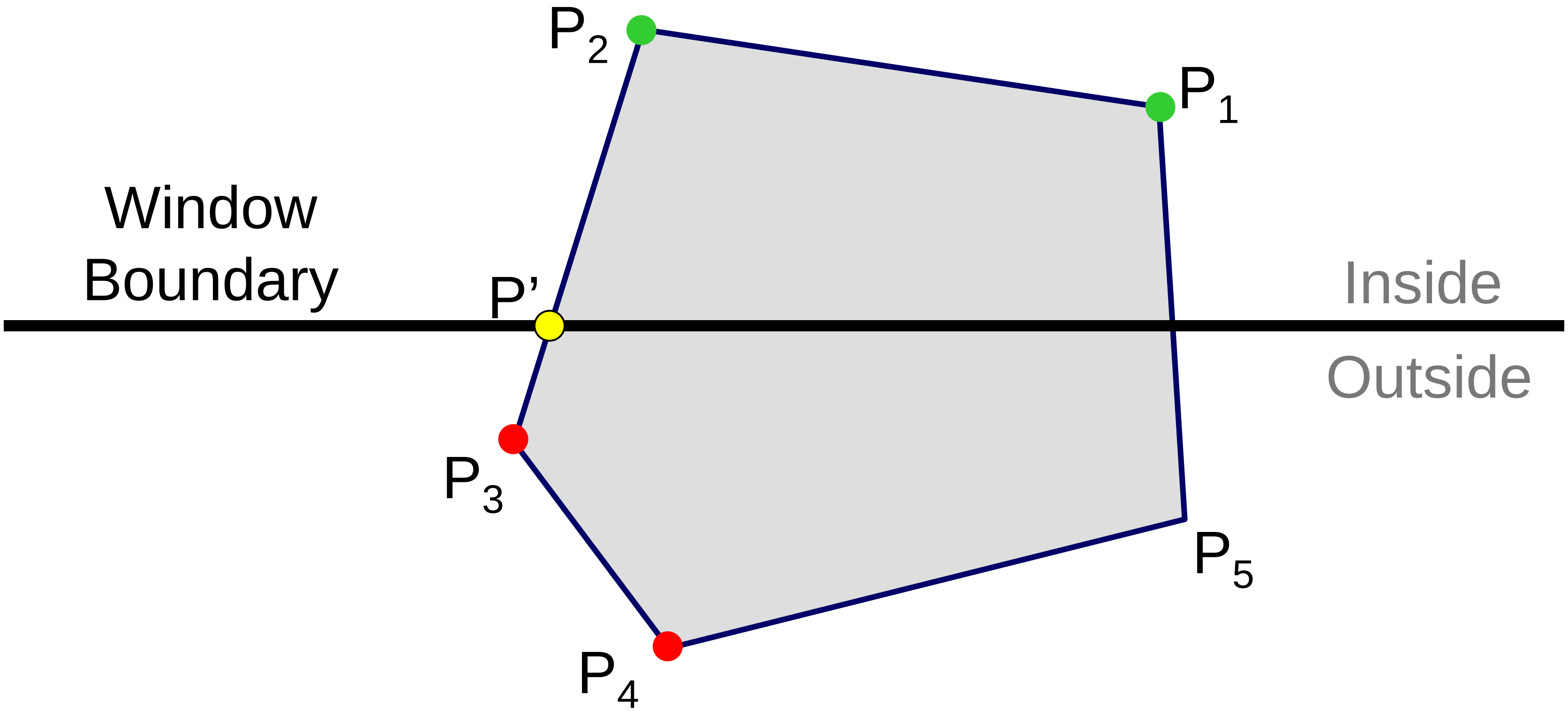

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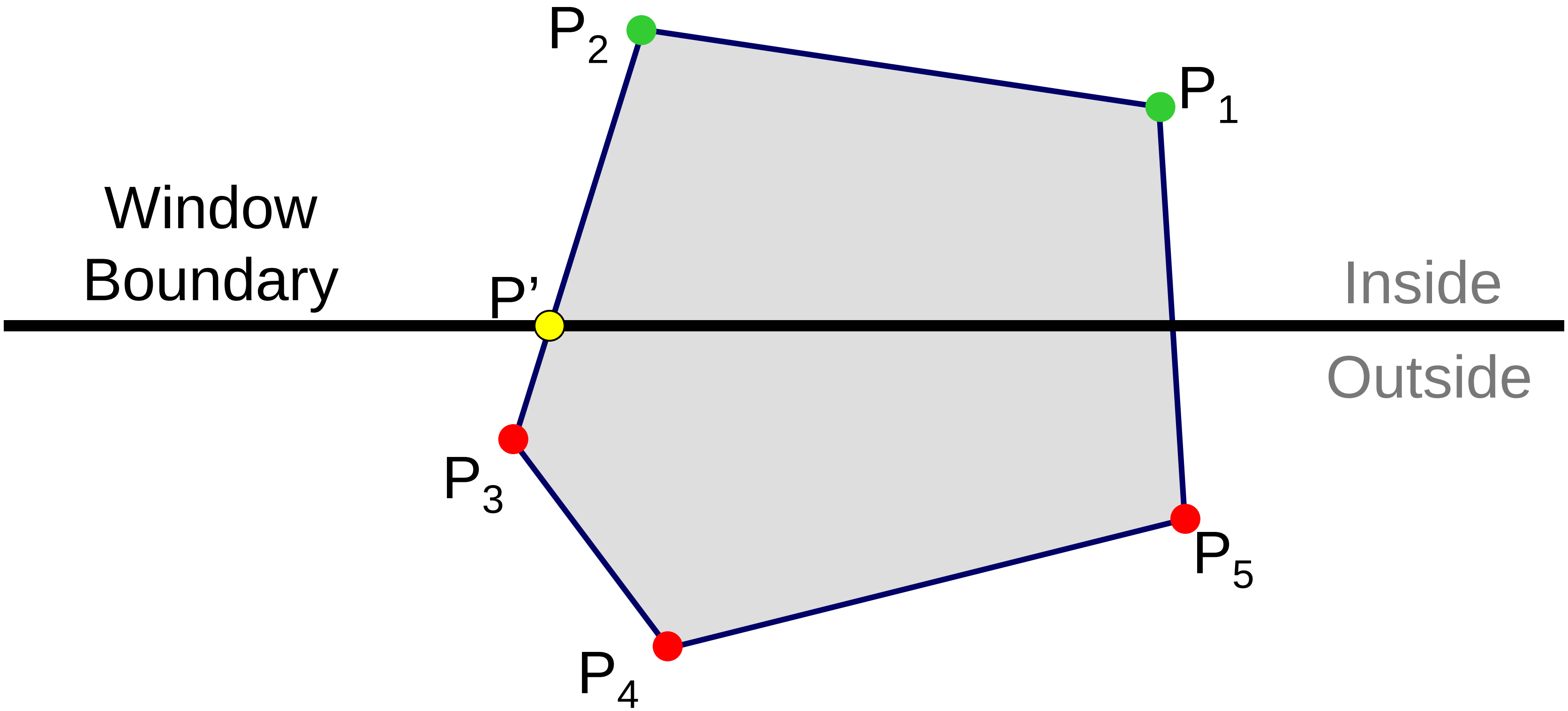

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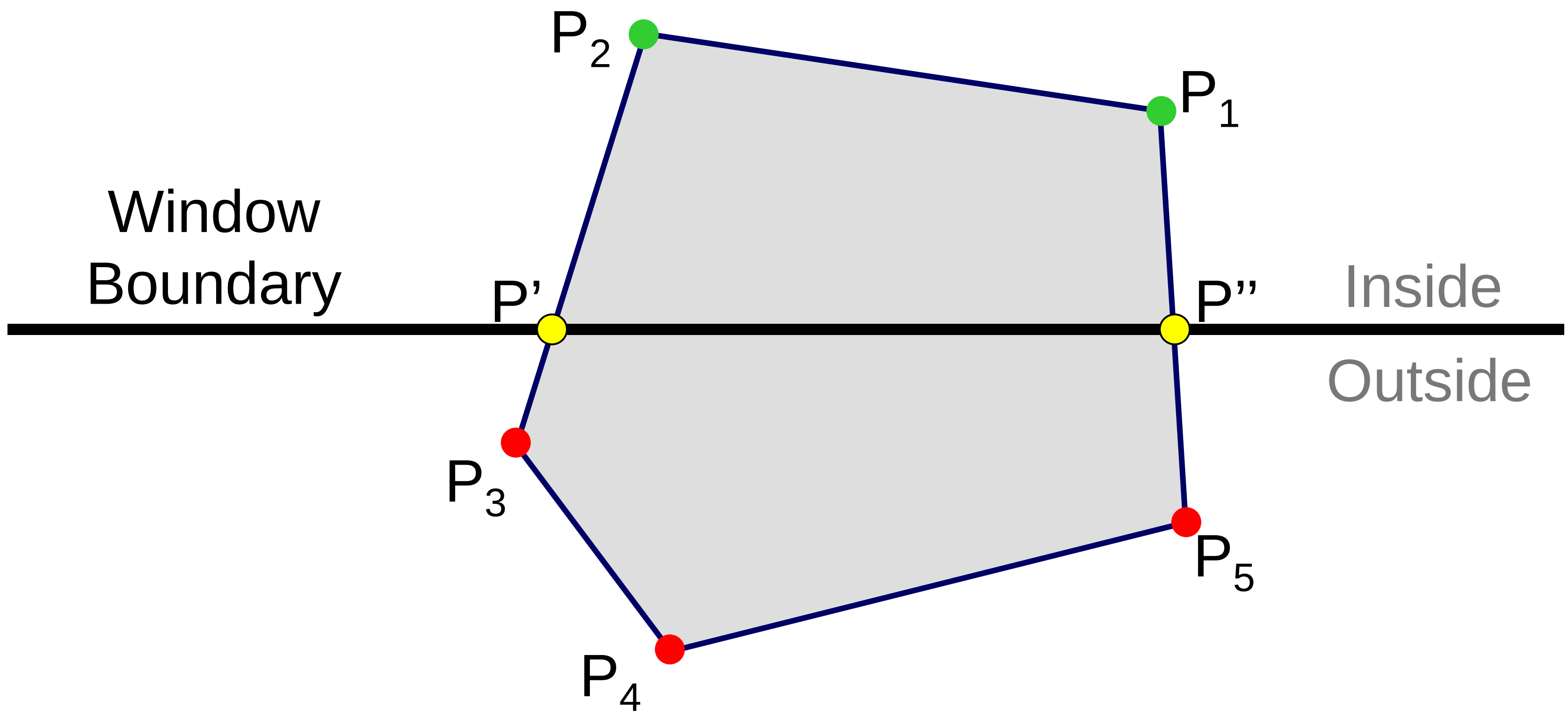

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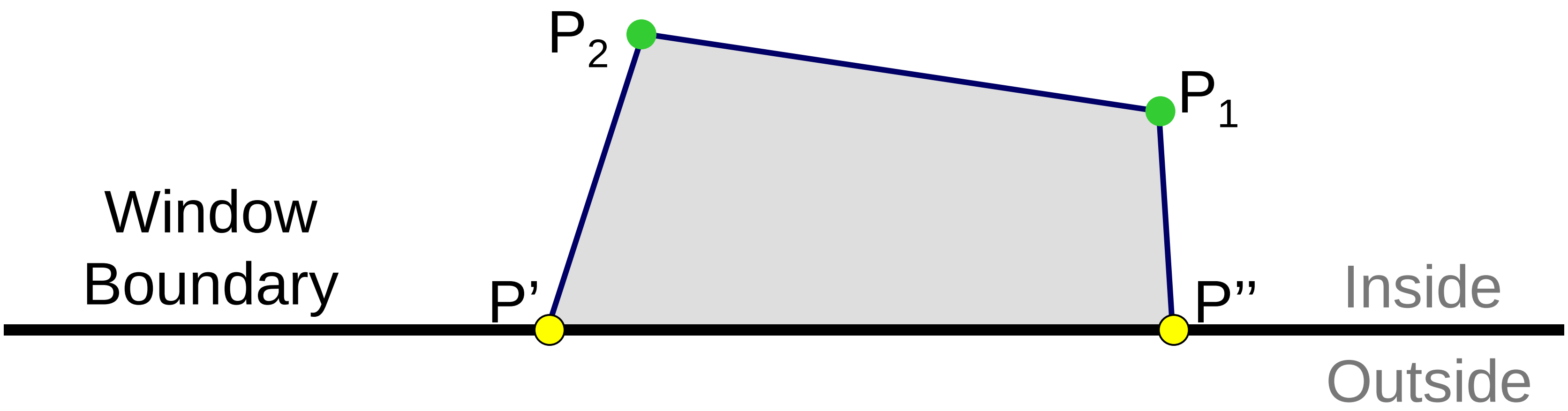
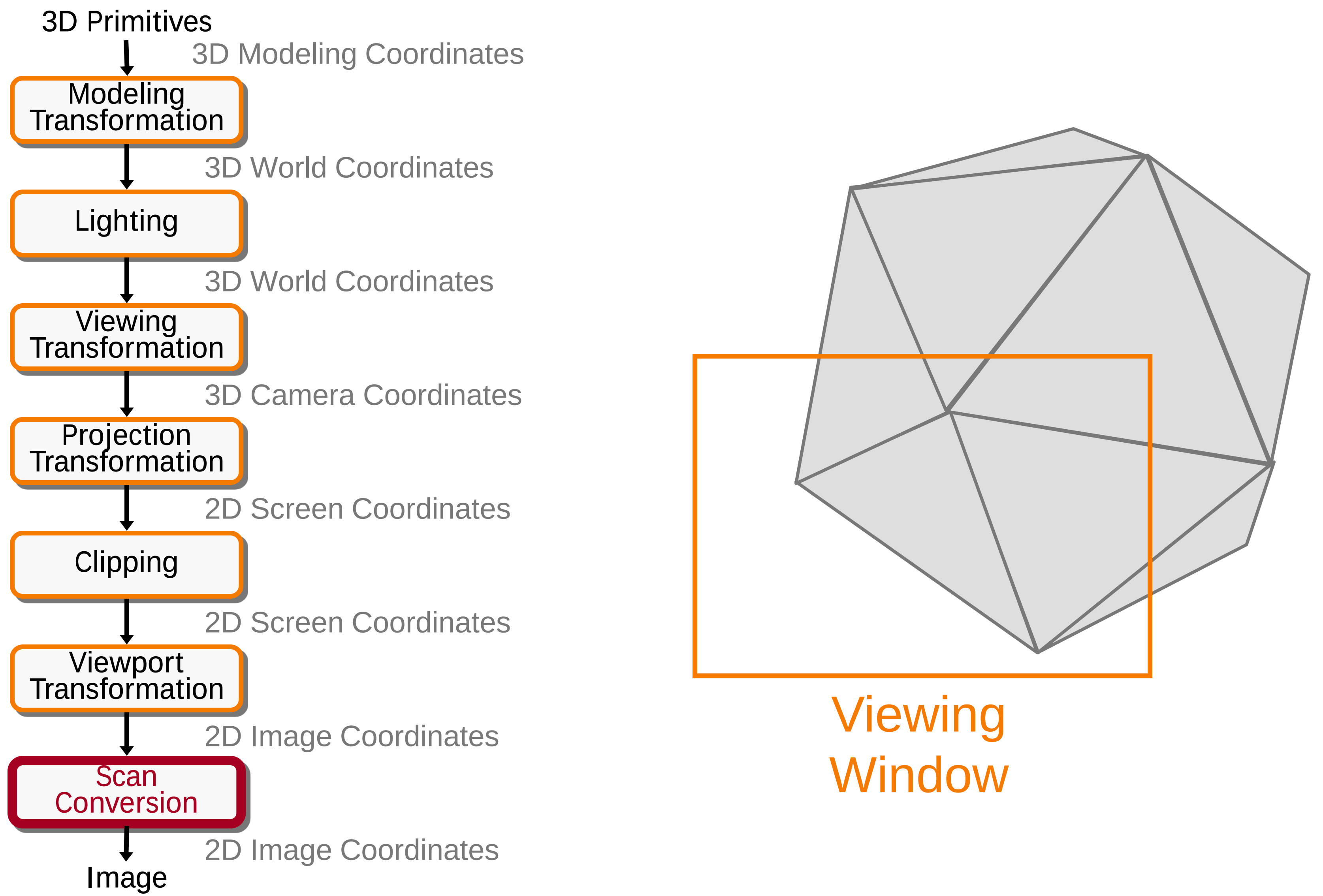


# 3D Rendering Pipeline (for direct illumination)



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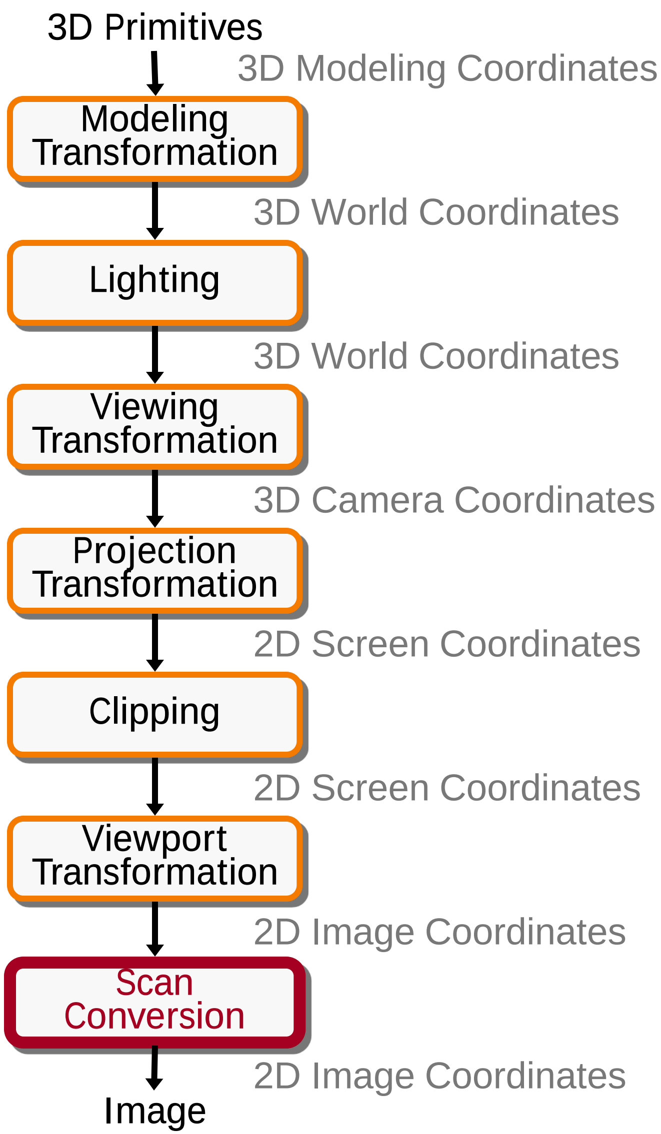
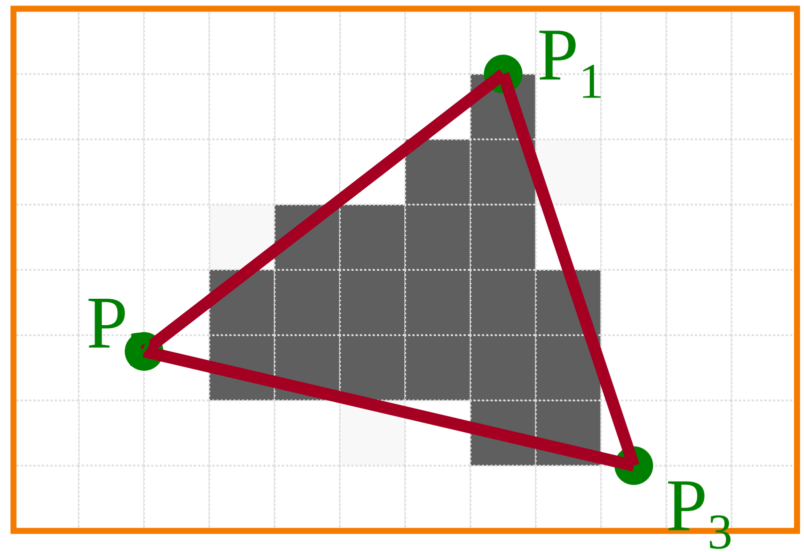
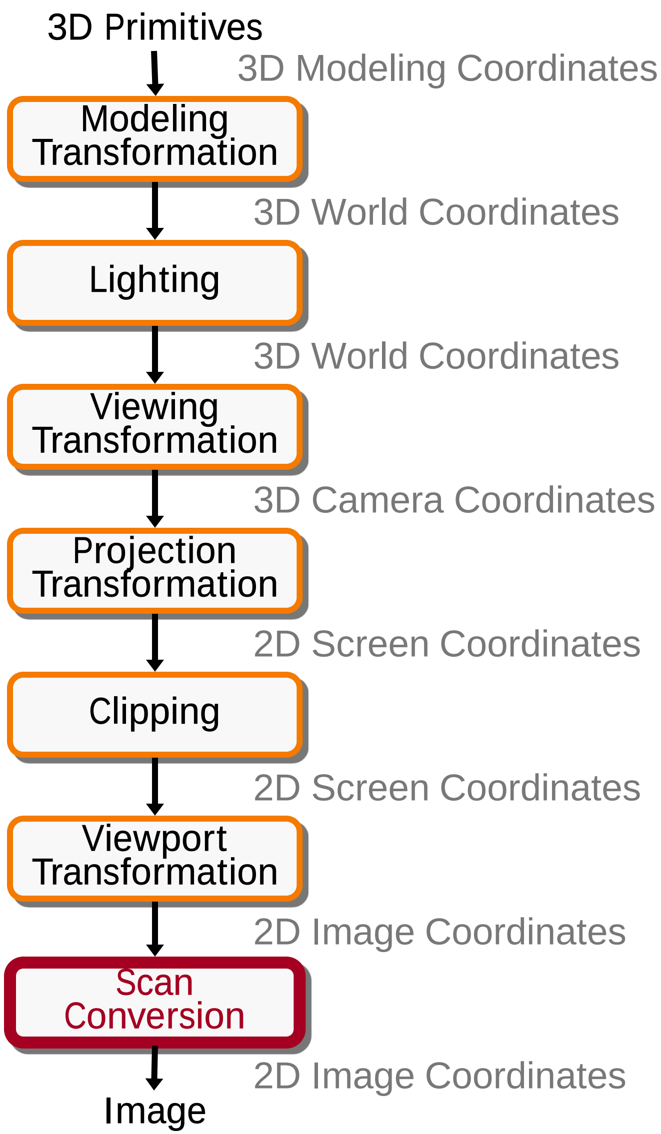
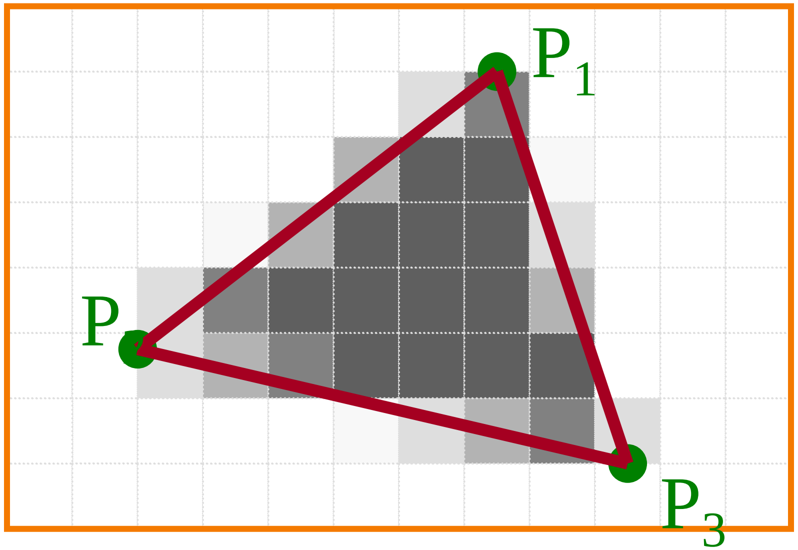



Standard (aliased)
Scan Conversion

# 3D Rendering Pipeline (for direct illumination)



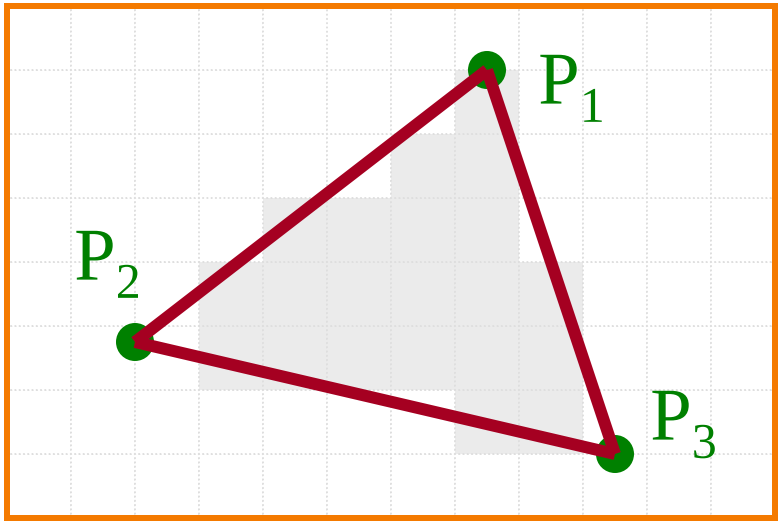

Antialiased
Scan Conversion

# Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

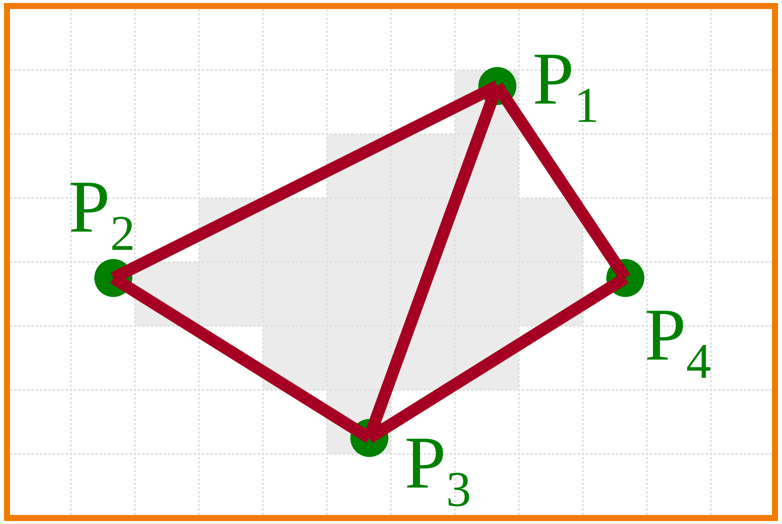
```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle



# Triangle Scan Conversion

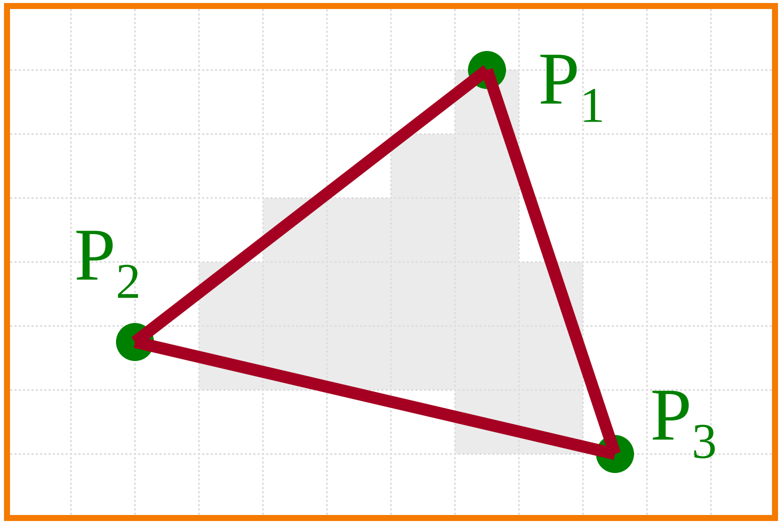
- Properties of a good algorithm
  - Symmetric
  - Straight edges
  - No cracks between adjacent primitives
  - (Antialiased edges)
  - FAST!



# Simple Algorithm

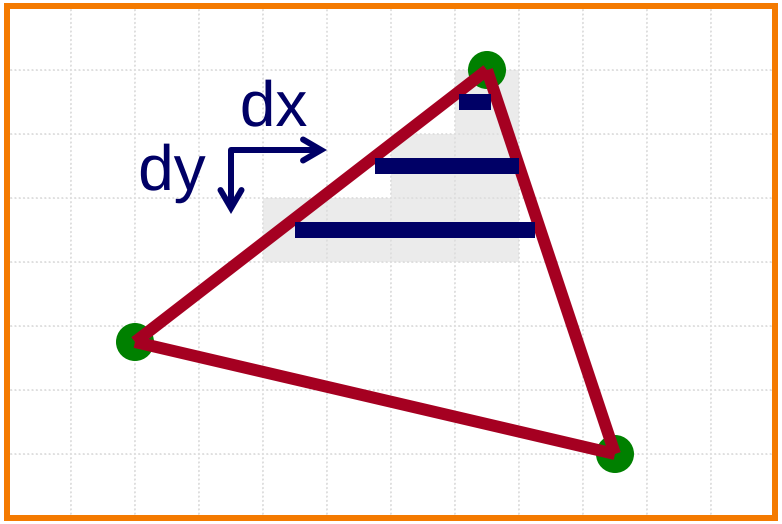
- Color all pixels inside triangle

```
void ScanTriangle(Triangle T, Color rgba){
   for each pixel P in bbox(T){
       if (Inside(T, P))
           SetPixel(P.x, P.y, rgba);
   }
}
```



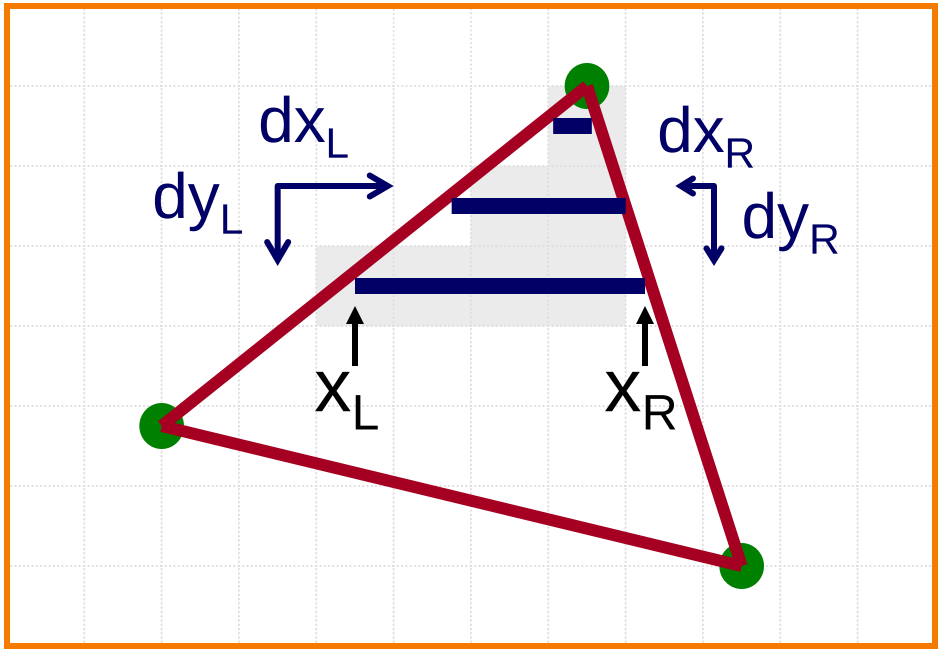
# Triangle Sweep-Line Algorithm

- Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order
- Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally



# Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba){
   for each edge pair {
      initialize xL, xR;
      compute dxL/dyL and dxR/dyR;
      for each scanline at y
         for (int x = xL; x <= xR; x++)
            SetPixel(x, y, rgba);
      xL += dxL/dyL;
      xR += dxR/dyR;
   }
}
```



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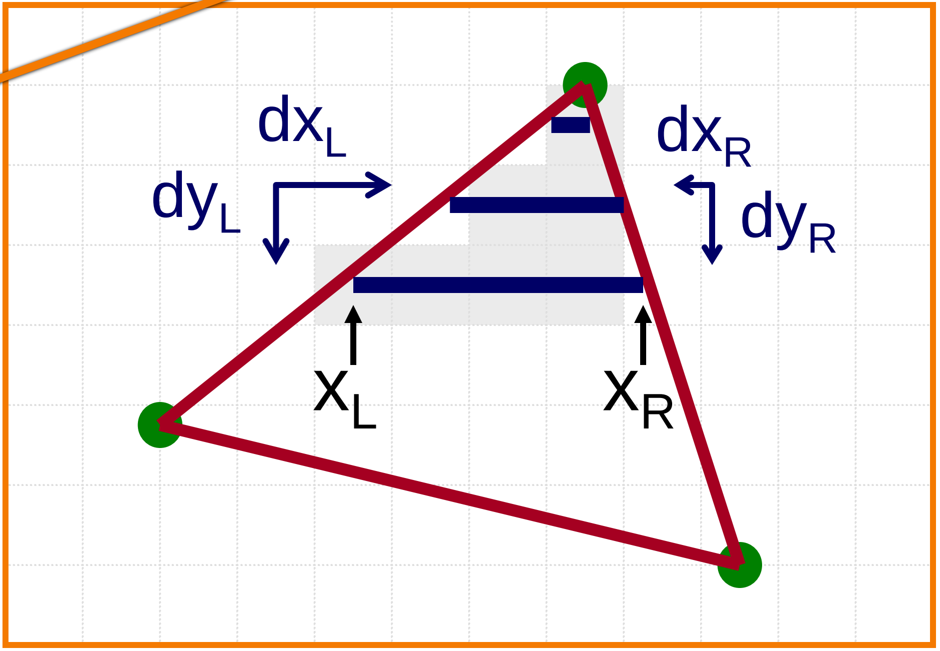
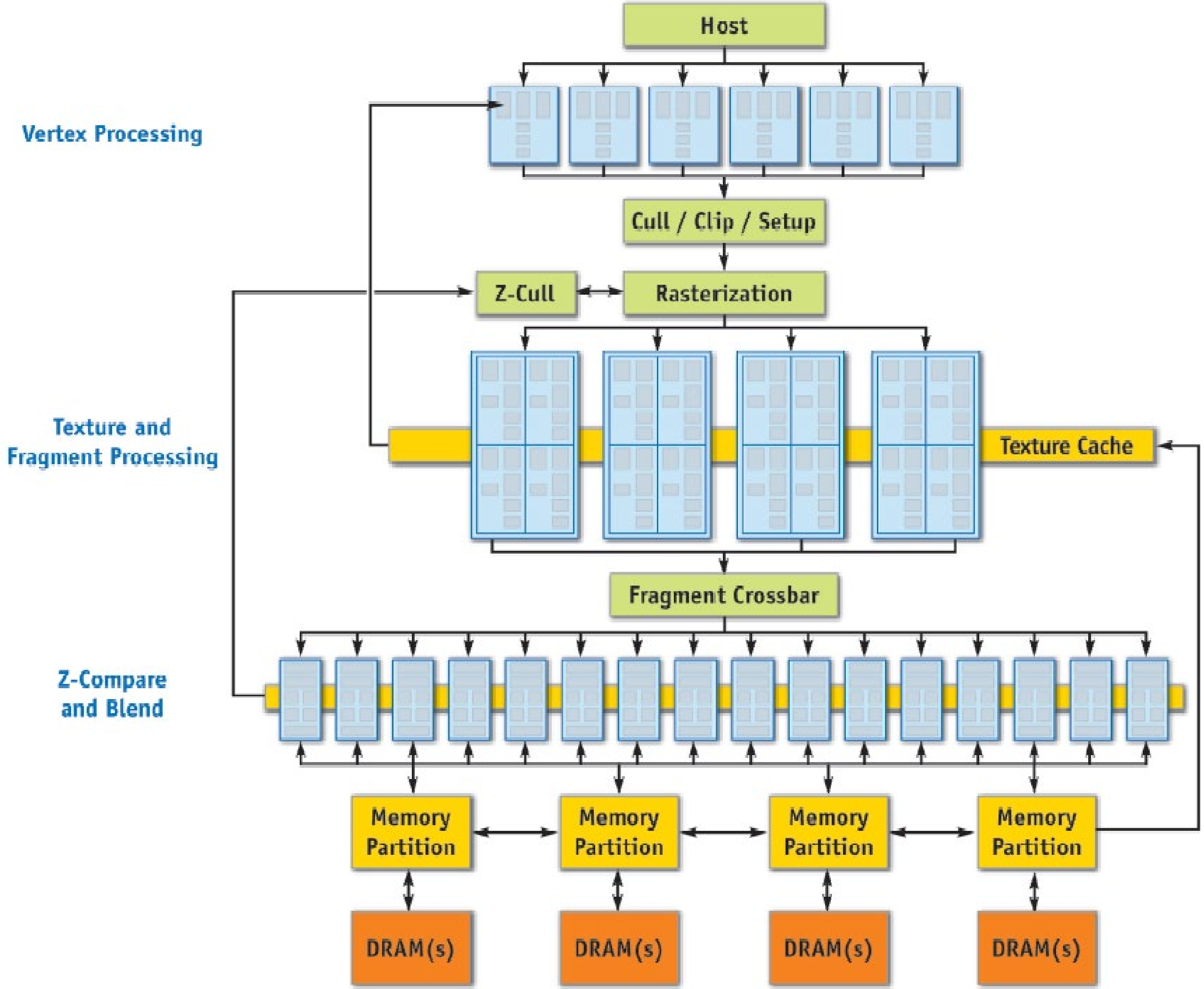
            SetPixel(x, y, rgba);
      xL += dxL/dyL;
      xR += dxR/dyR;
   }
}
```

Minimize computation in inner loops



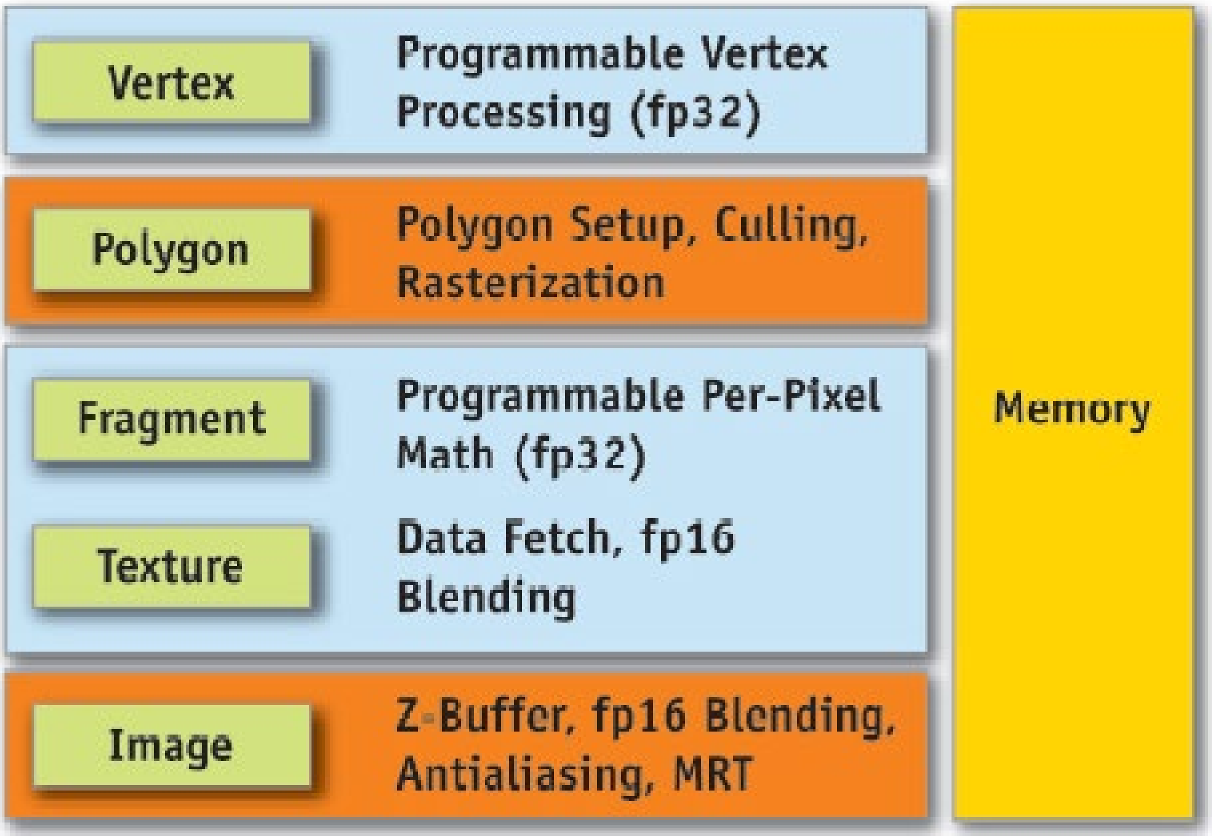
# GPU Architecture




GeForce 6 Series Architecture

GPU Gems 2, NVIDIA

# GPU Architecture




GeForce 6 Series Architecture

GPU Gems 2, NVIDIA