Polygonal Meshes

COS 426

3D Object Representations

Points
- Range image
- Point cloud

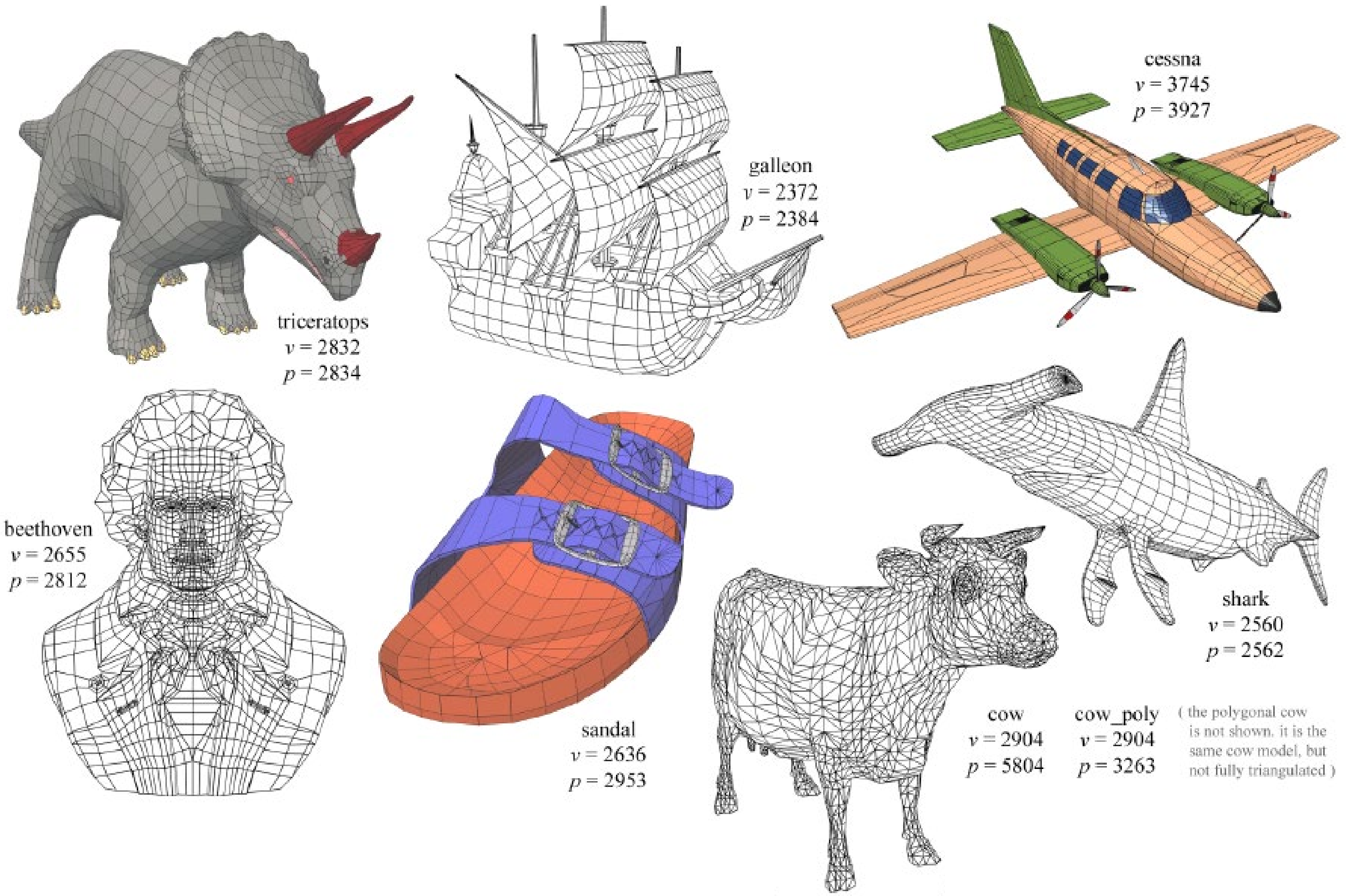
Surfaces
- Polygonal mesh
- Subdivision
- Parametric
- Implicit

Solids
- Voxels
- BSP tree
- CSG
- Sweep

High-level structures
- Scene graph
- Application specific

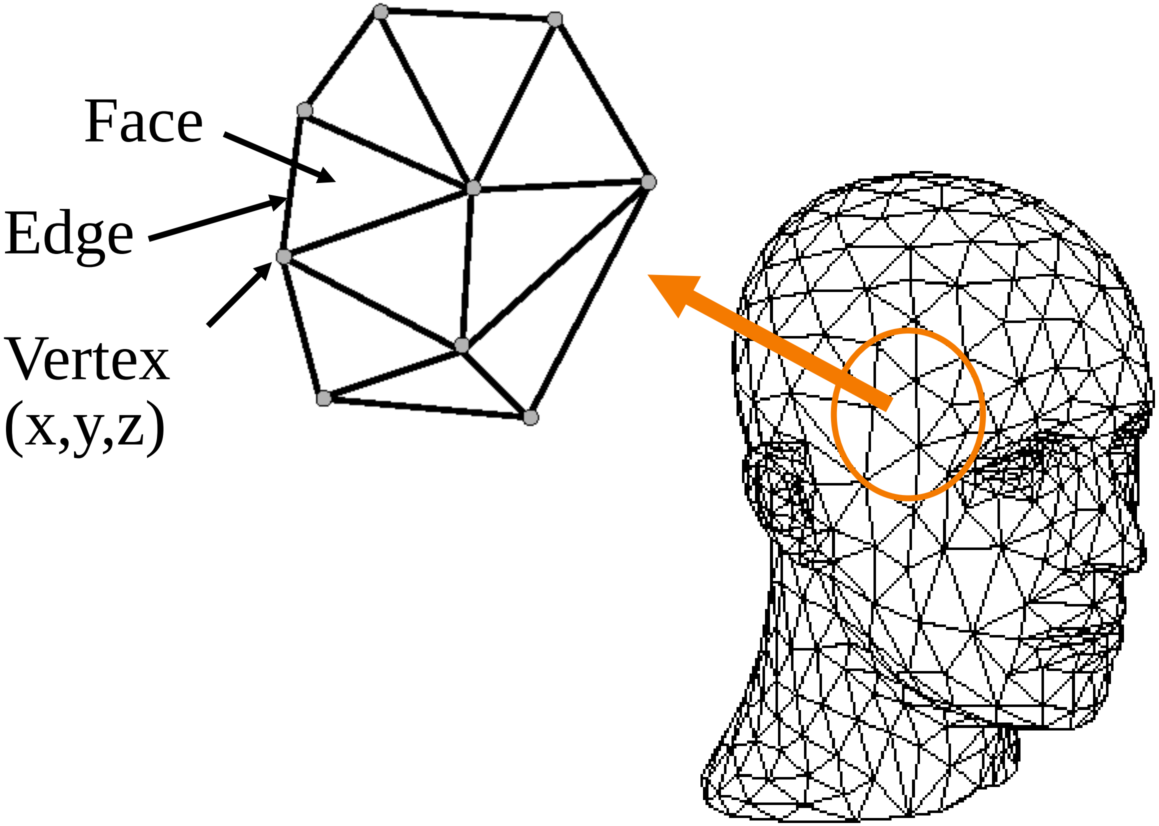
3D Polygonal Mesh

Set of polygons representing a 2D surface embedded in 3D



3D Polygonal Mesh

Geometry & topology



Zorin & Schroeder

Geometry background

Scene is usually approximated by 3D primitives

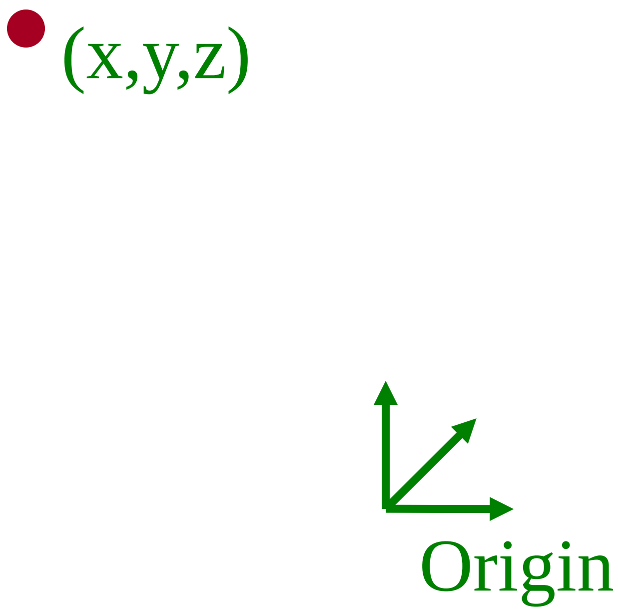
- Point
- Vector
- Line segment
- Ray
- Line
- Plane
- Polygon

3D Point

Specifies a location

- Represented by three coordinates
- Infinitely small

```
struct Point {
    Coordinate x;
    Coordinate y;
    Coordinate z;
};
```

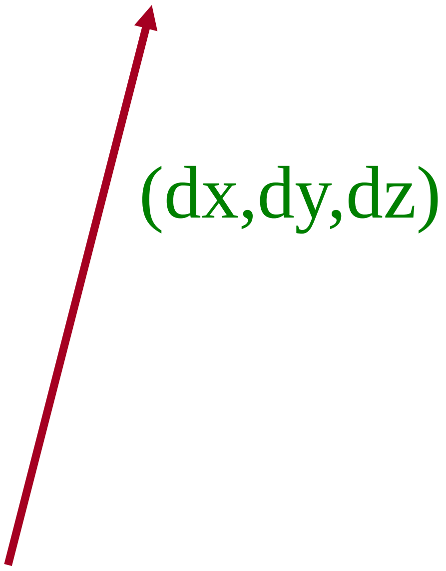


3D Vector

Specifies a direction and a magnitude

- Represented by three coordinates
- Magnitude ||V|| = sqrt(dx*dx + dy*dy + dz*dz)
- Has no location

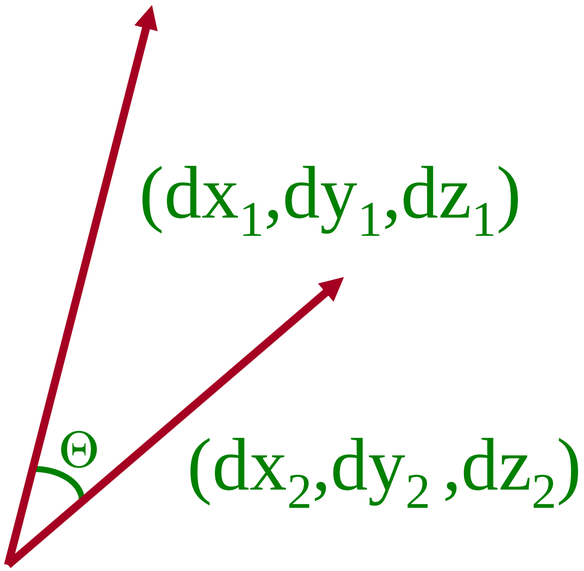
```
struct Vector {
    Coordinate dx;
    Coordinate dy;
    Coordinate dz;
};
```



3D Vector

Scalar / dot product of two 3D vectors

$$V_1 \cdot V_2 = dx_1 * dx_2 + dy_1 * dy_2 + dz_1 * dz_2 = \|V_1\| \, \|V_2\| \cos(\Theta)$$

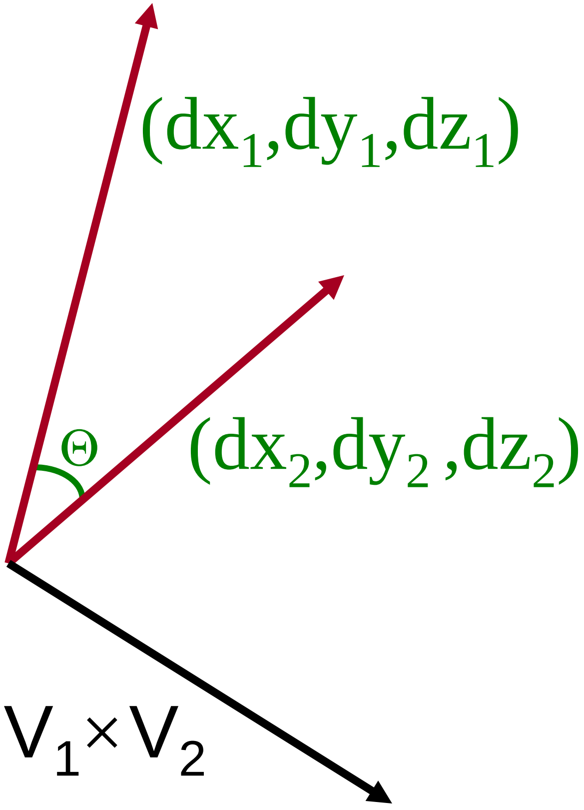


3D Vector

Cross product of two 3D vectors

$V_1 \times V_2 = (dy_1dx_2 - dz_1dy_2, dz_1dx_2 - dx_1dz_2, dx_1dy_2 - dy_1dx_2)$

- vector perpendicular to both V_1 and V_2
- magnitude is $||V_1|| \, ||V_2|| \sin(\Theta)$

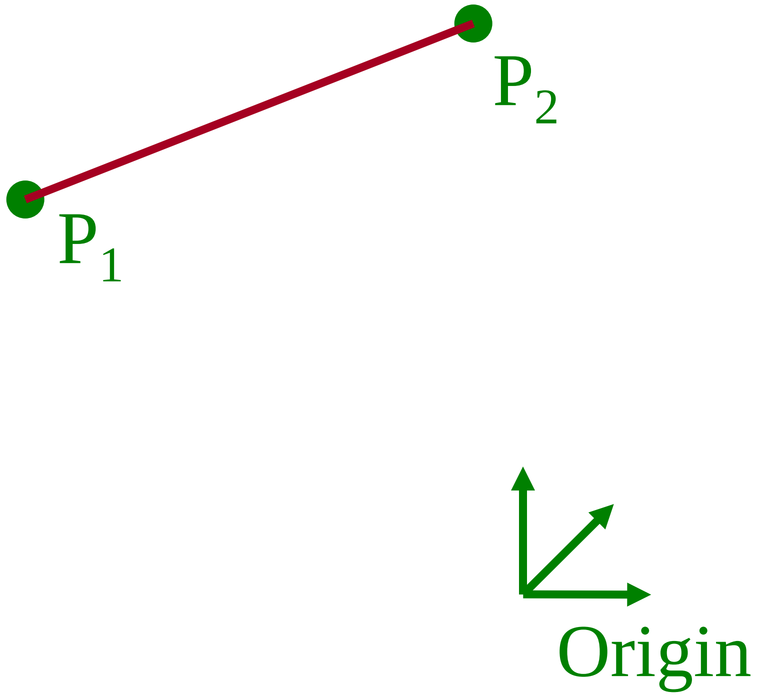


3D Line Segment

Linear path between two points

- Parametric representation:
  - » $p = P_1 + t\,(P_2 - P_1), \quad (0 \leq t \leq 1)$

```
struct Segment {
    Point P1;
    Point P2;
};
```

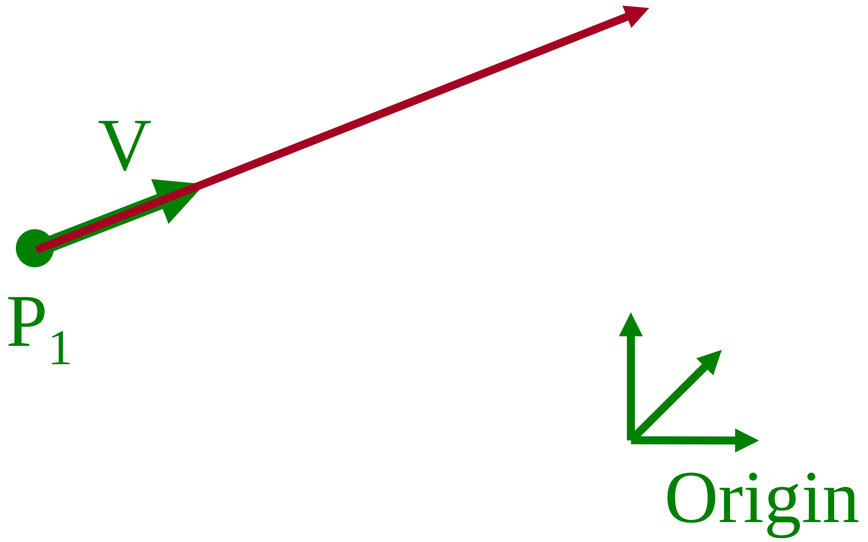


3D Ray

Line segment with one endpoint at infinity

- Parametric representation:
  - » $p = P_1 + t\,V, \quad (0 \le t < \infty)$

```
struct Ray {
    Point P1;
    Vector V;
};
```

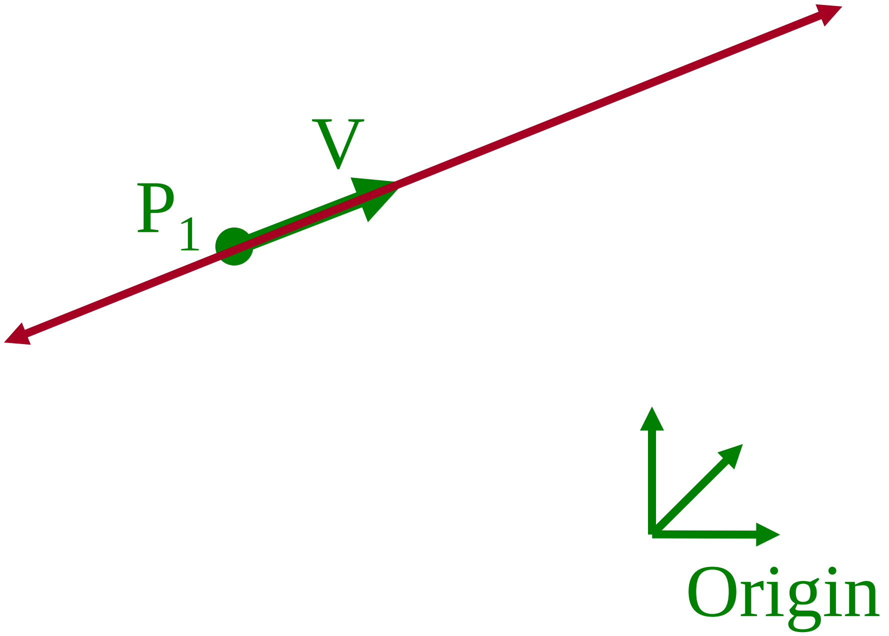


3D Line

Line segment with both endpoints at infinity

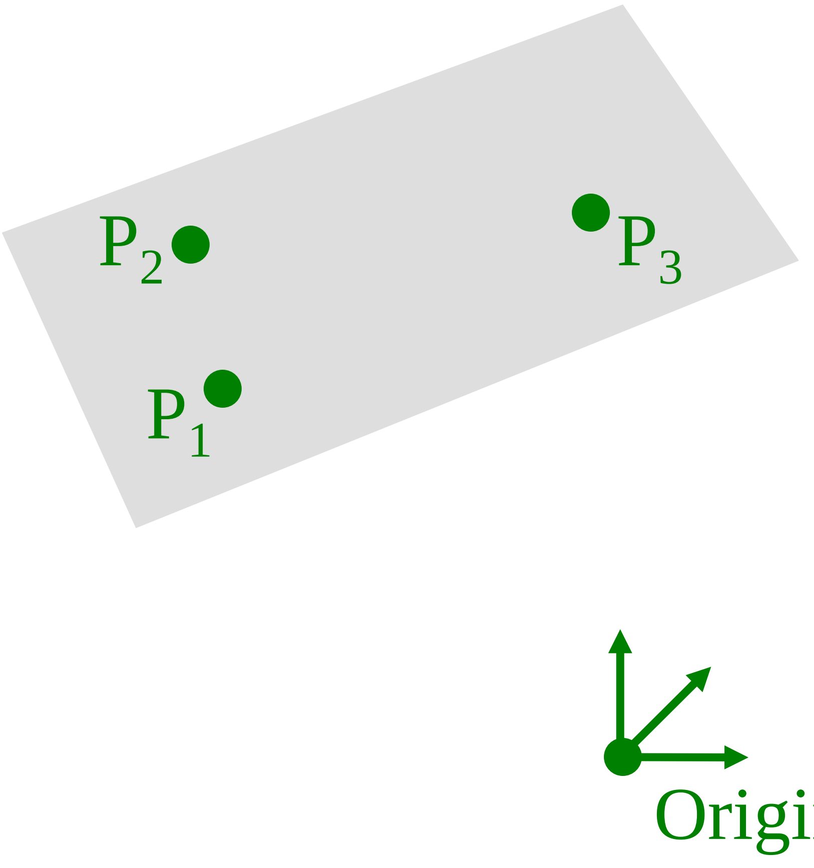
- Parametric representation:
  - » $p = P_1 + t\,V, \quad (-\infty < t < \infty)$

```
struct Line {
    Point P1;
    Vector V;
};
```



3D Plane

Defined by three points



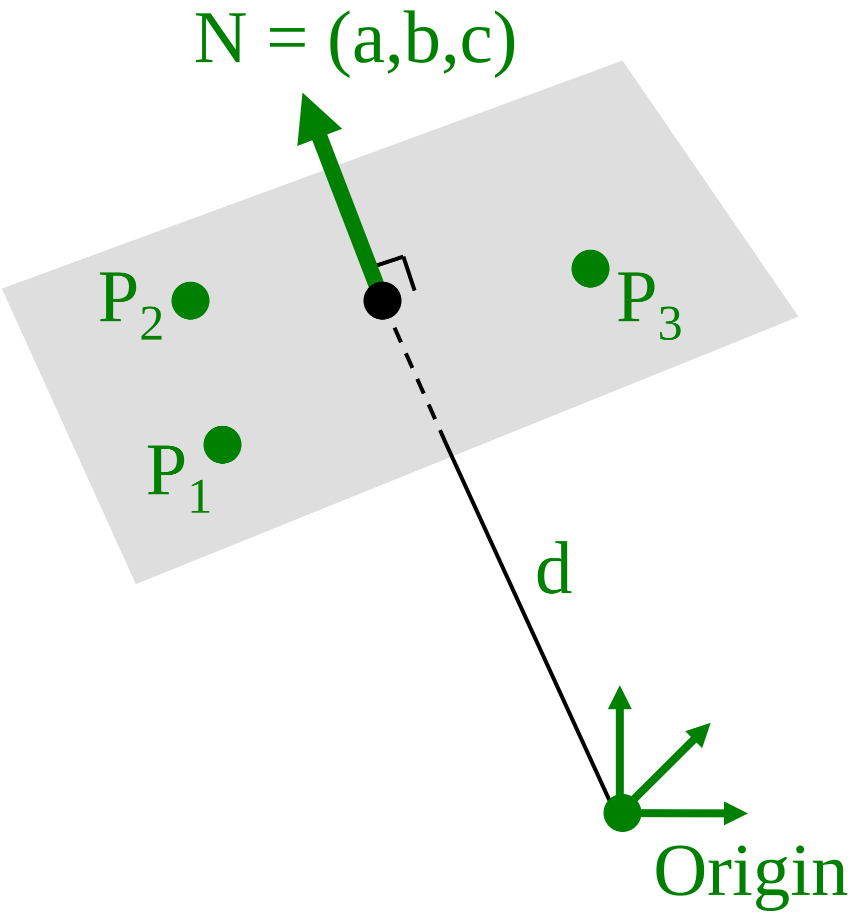
3D Plane

Defined by three points

- Implicit representation:
  - » $ax + by + cz + d = 0$

    OR
  - » $P \cdot N + d = 0$

```
struct Plane {
    Vector N;
    float d;
};
```

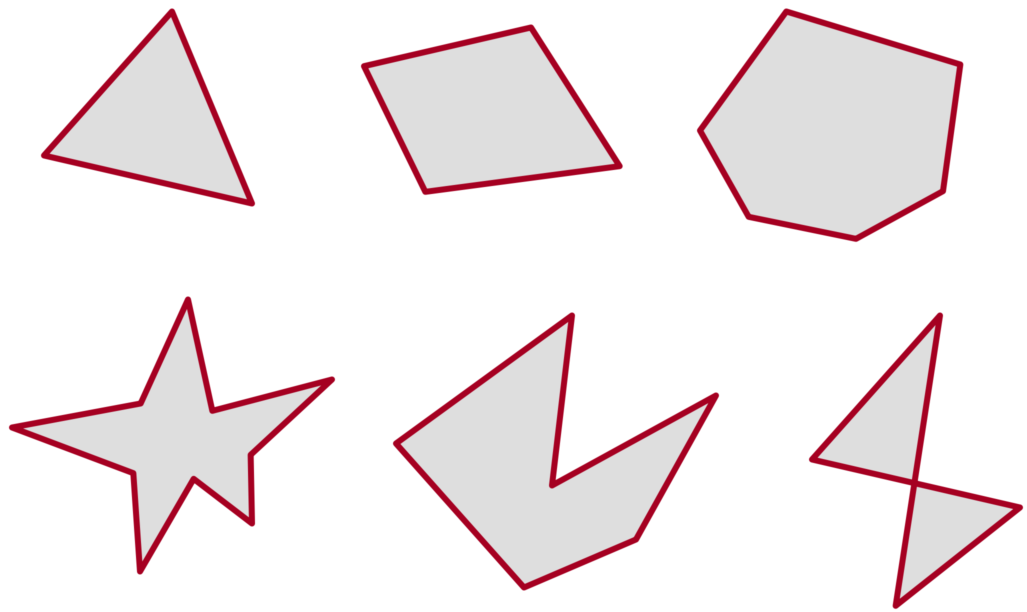


- N is the plane normal
  - » Unit-length
  - » Perpendicular to plane

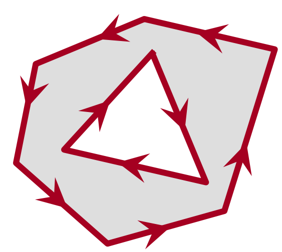
3D Polygon

Region "inside" a sequence of coplanar points

```
struct Polygon {
      vector<Point> points;
};
```



- Points in counter-clockwise order (defines normal)



- Winding rule determines inside/outside

3D Polygonal Mesh

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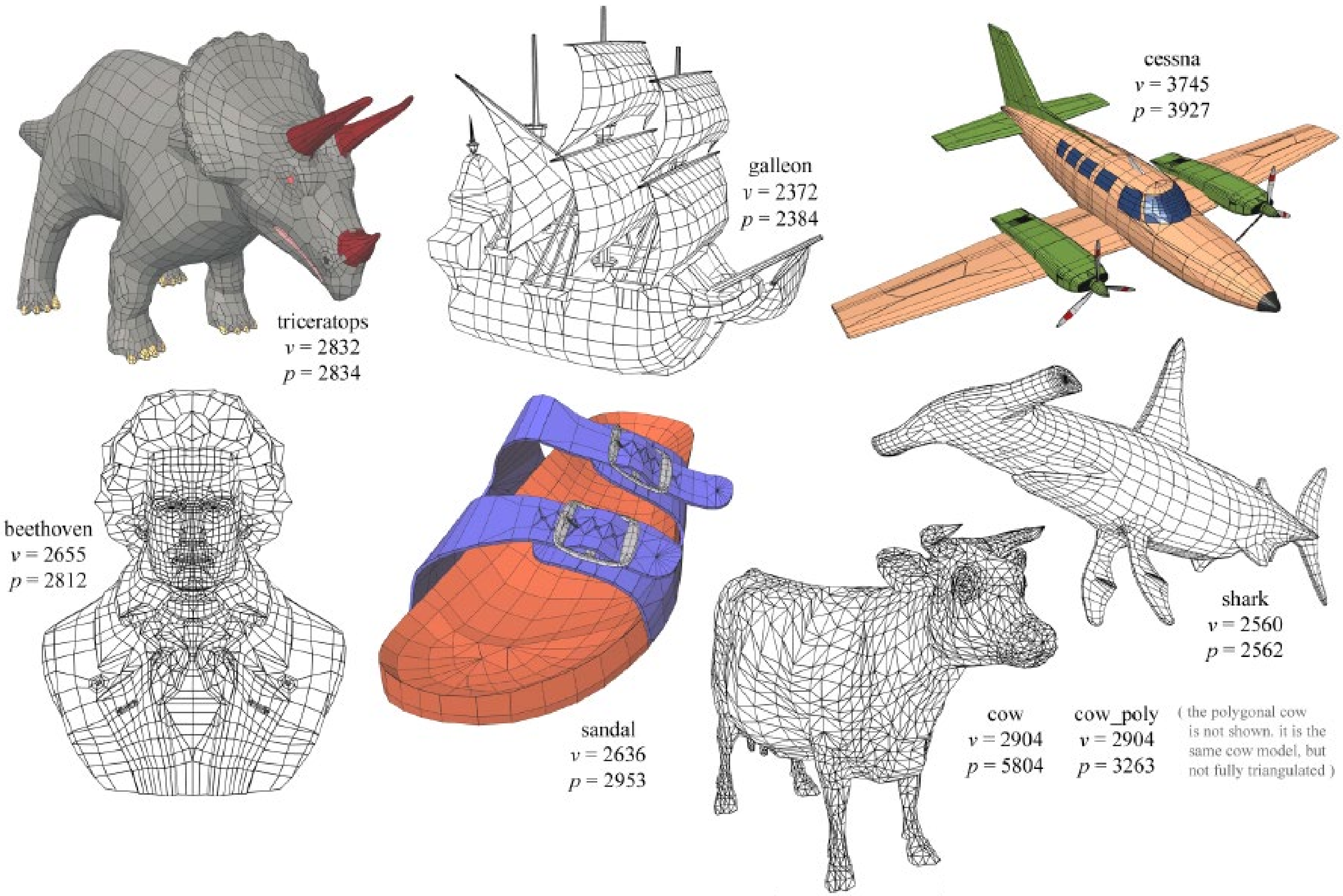
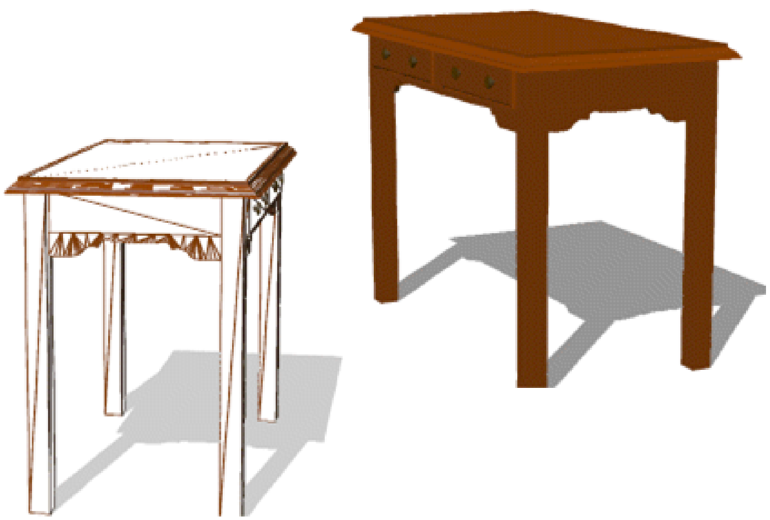
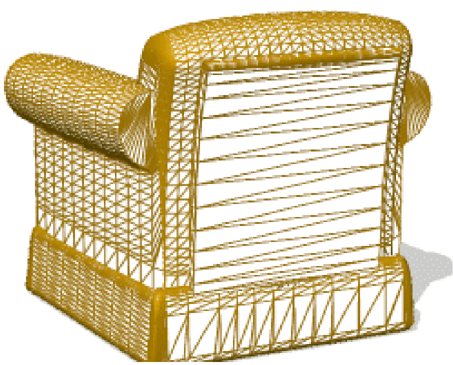


3D Polygonal Meshes

Why are they of interest?

- Simple, common representation
- Rendering with hardware support
- Output of many acquisition tools
- Input to many simulation/analysis tools





Viewpoint

3D Polygonal Meshes

Properties

- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.

Viewpoint

Outline

Acquisition ←

Processing

Representation

Polygonal Mesh Acquisition

Interactive modeling
- Polygon editors
- Interchange formats

Scanners
- Laser range scanners
- Geological survey
- CAT, MRI, etc. (isosurfaces)

Simulations
- Physical processes

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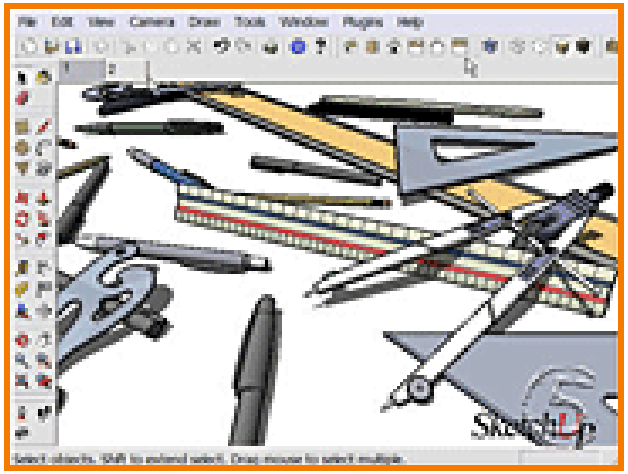
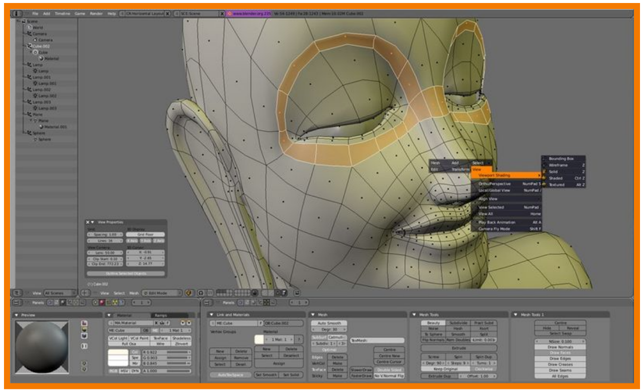


Sketchup



Blender

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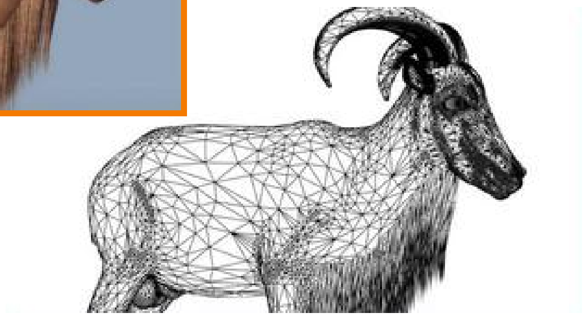

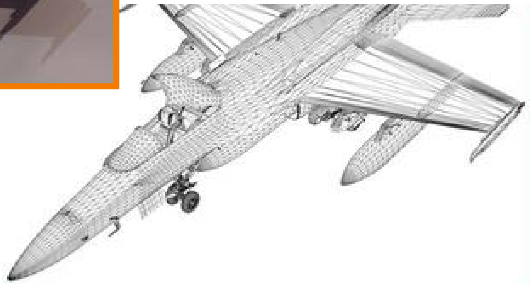

Jose Maria De Espona

Polygonal Mesh Acquisition

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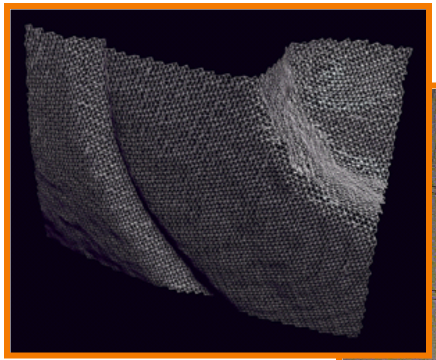

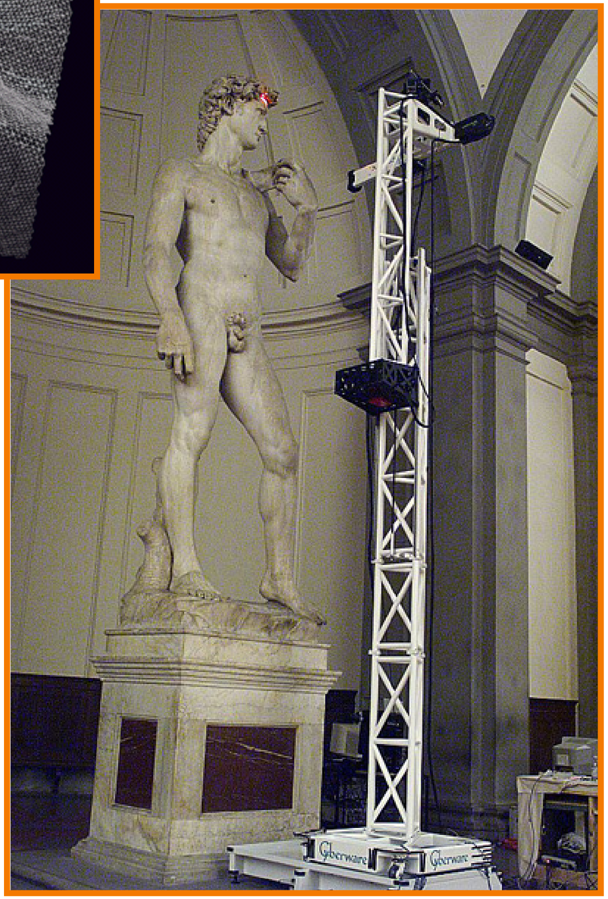

Digital Michelangelo Project
Stanford

Polygonal Mesh Acquisition

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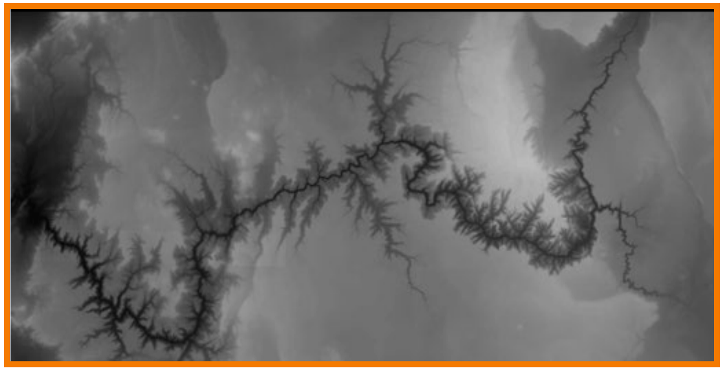

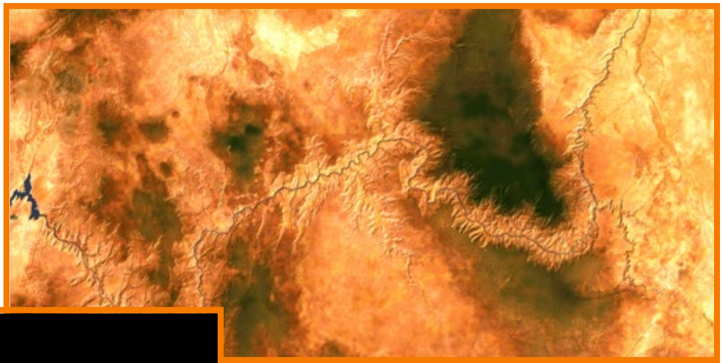

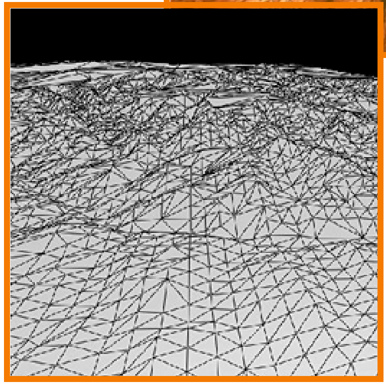

Large Geometric Model Repository Georgia Tech

Polygonal Mesh Acquisition

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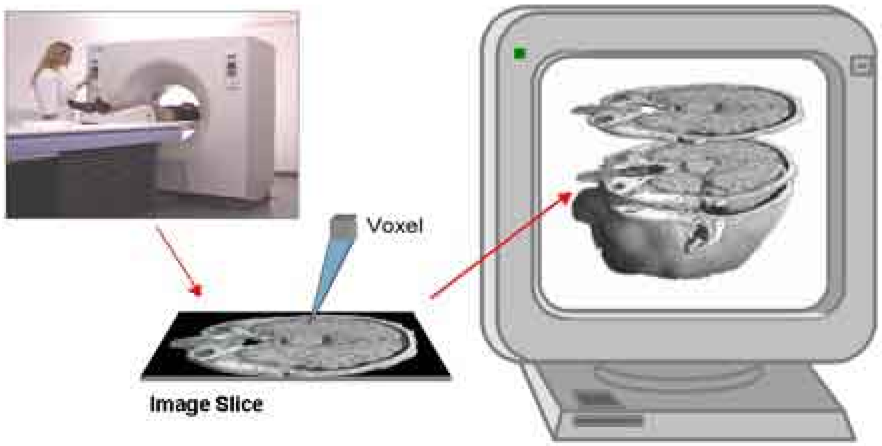

www.volumegraphics.com

SUNY Stony Brook

Polygonal Mesh Acquisition

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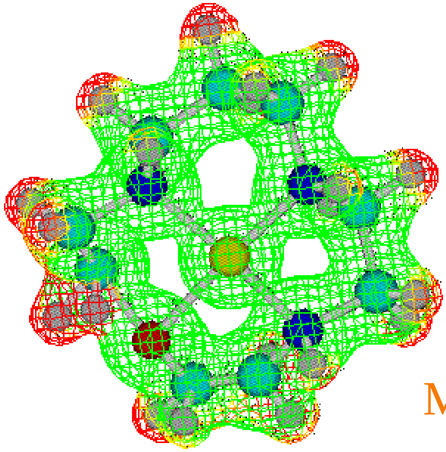
SGI



MIT

Outline

Acquisition

Processing ←

Representation

Polygonal Mesh Processing

Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- Bevel

Polygonal Mesh Processing

Analysis
- ➢ Normals
- ◦ Curvature

Warps
- ◦ Rotate
- ◦ Deform

Filters
- ◦ Smooth
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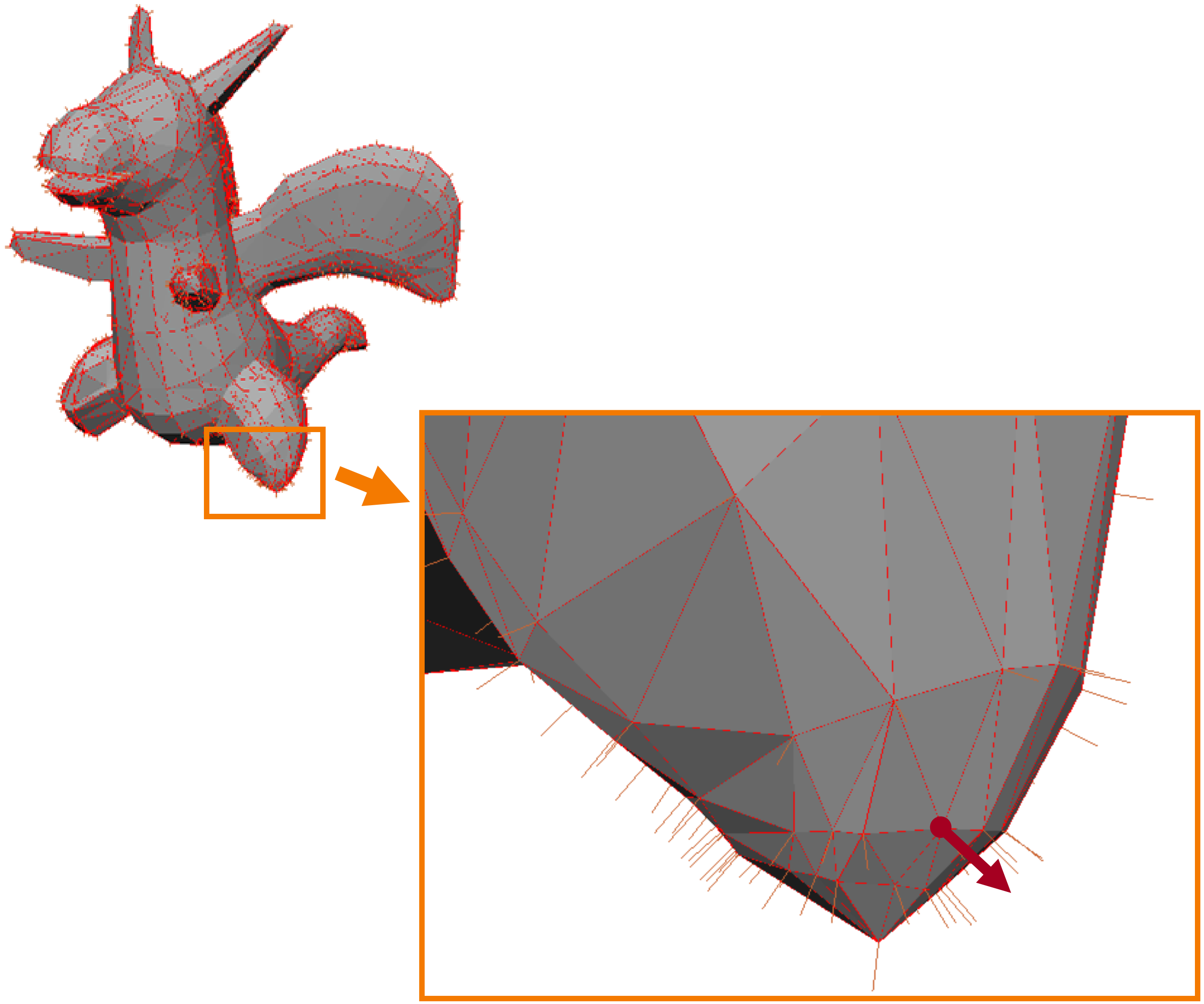
- ◦ Bevel



Polygonal Mesh Processing

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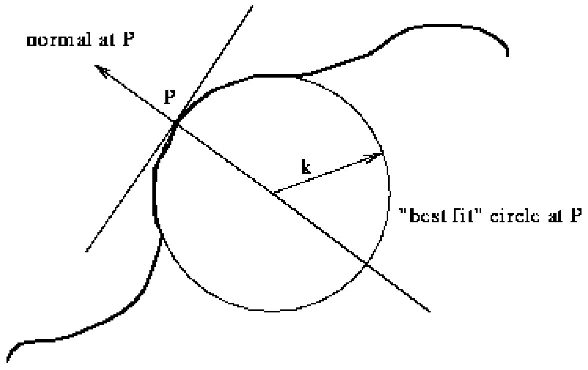

Figure 32: curvature of curve at P is $1/k$

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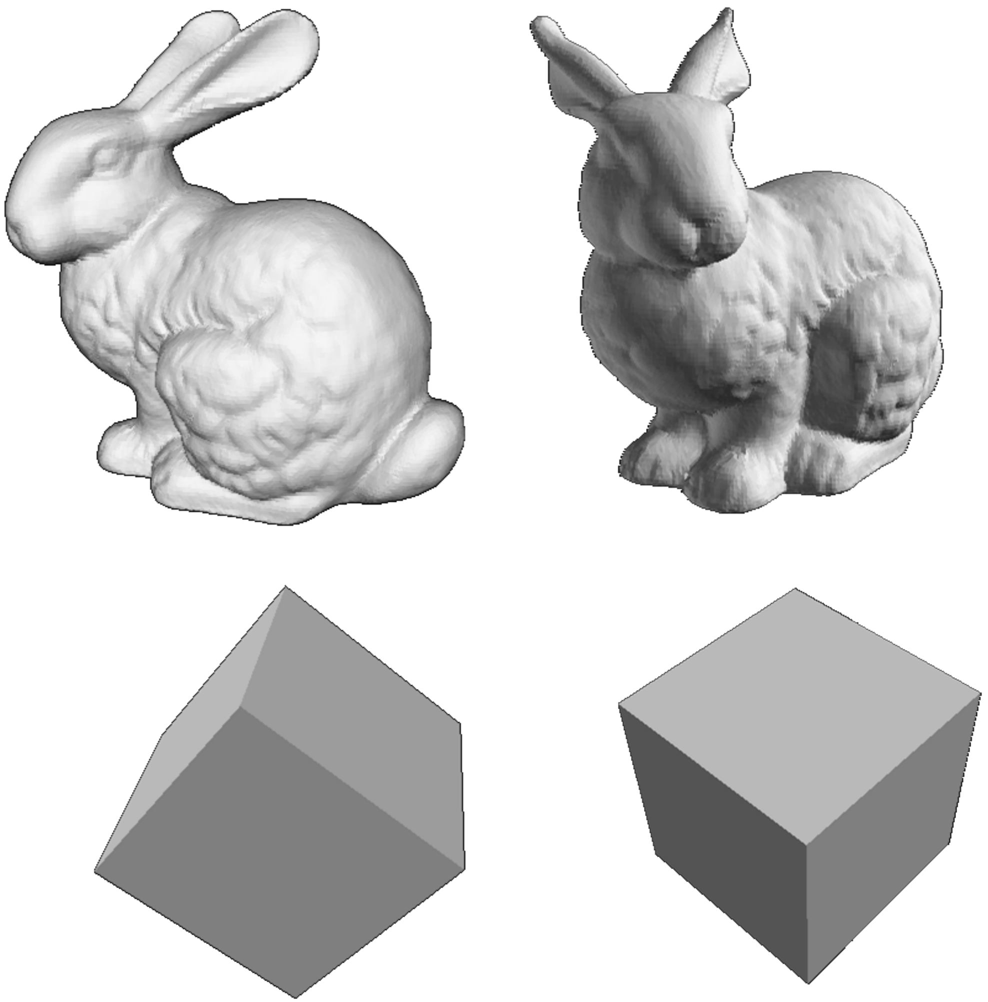

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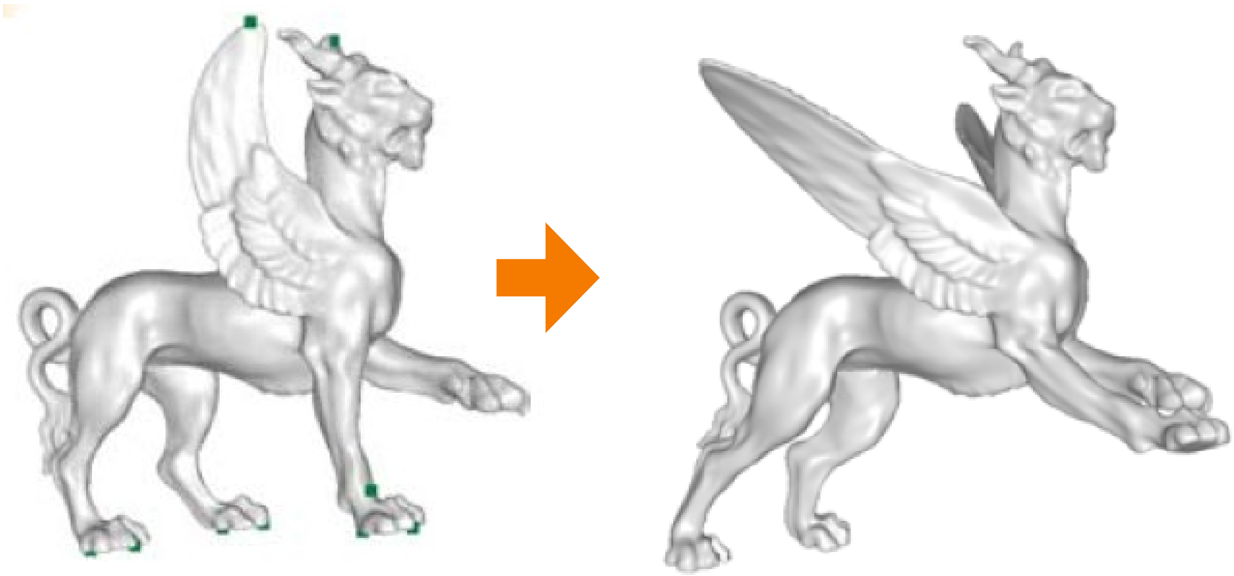

Sheffer

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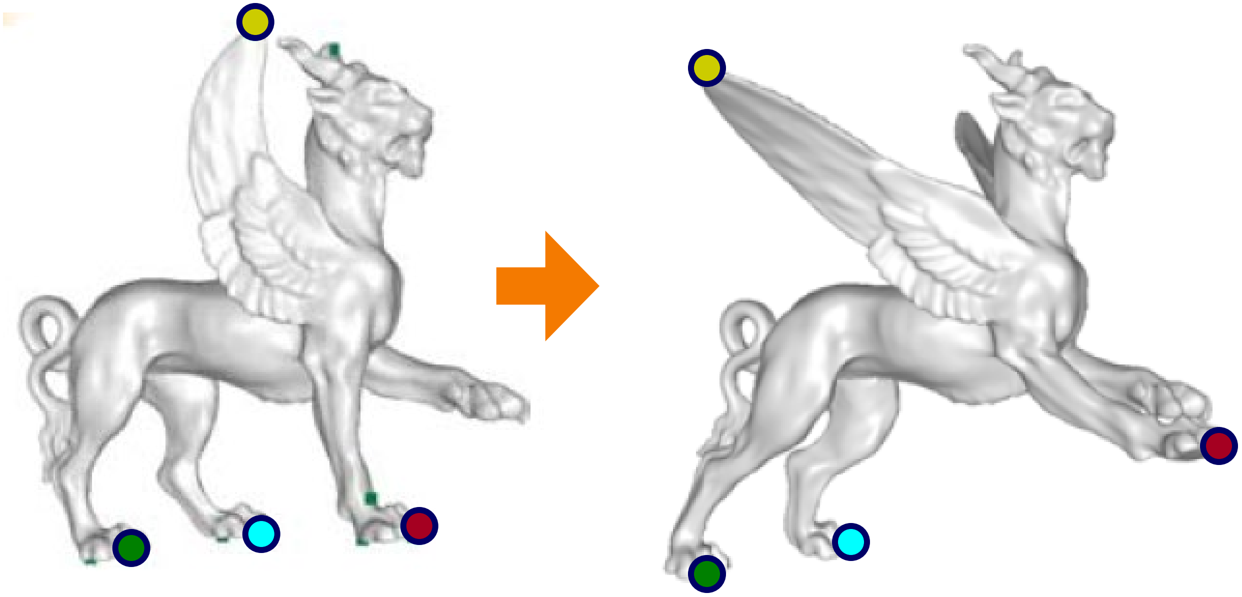

Sheffer

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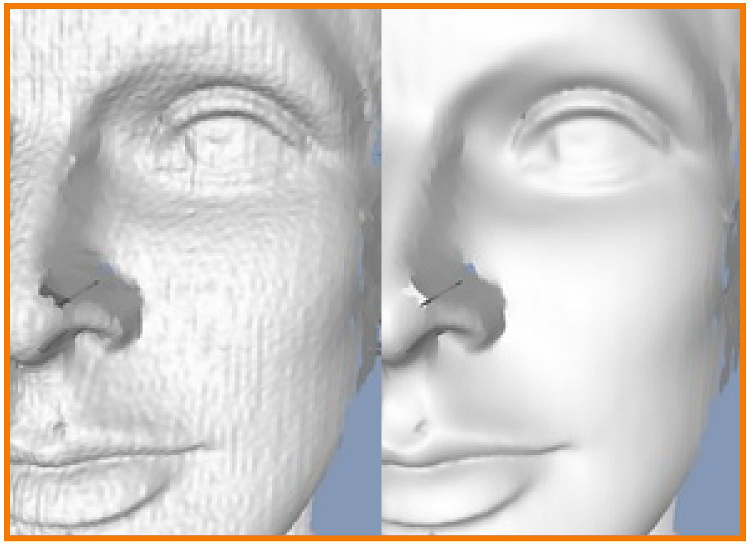
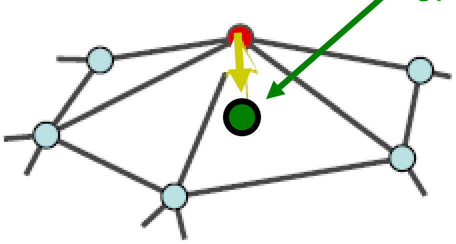


Thouis "Ray" Jones



Olga Sorkine

Polygonal Mesh Processing

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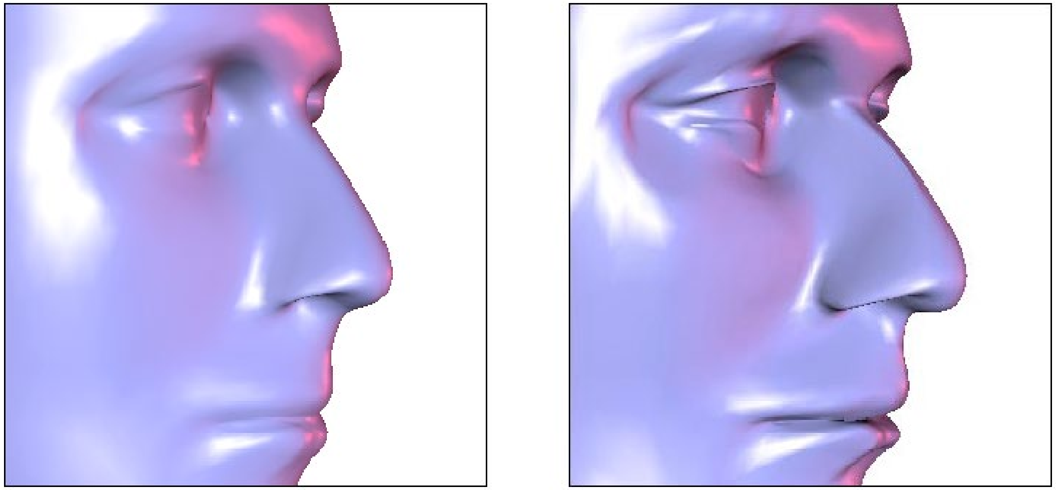

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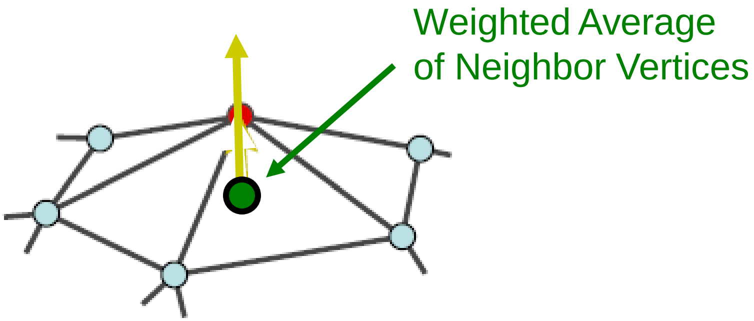

Olga Sorkine

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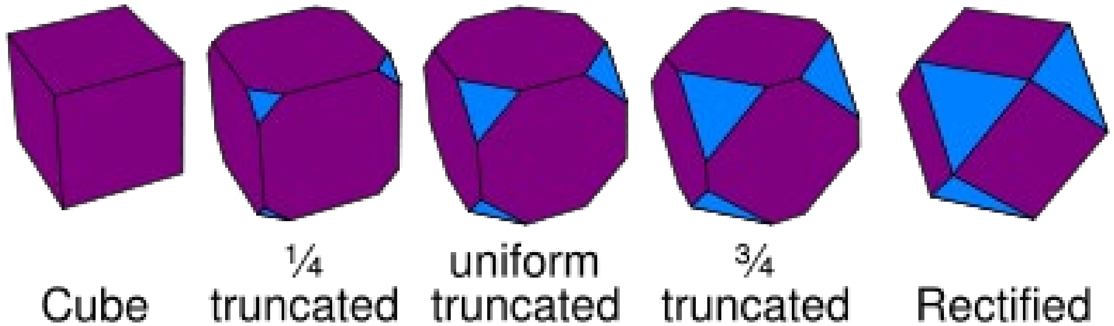
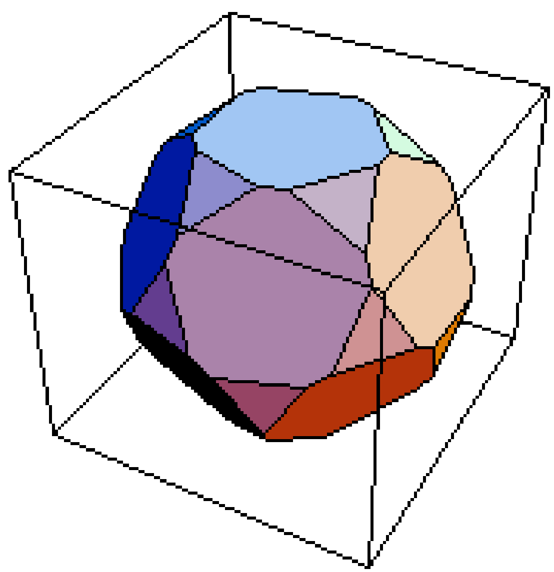
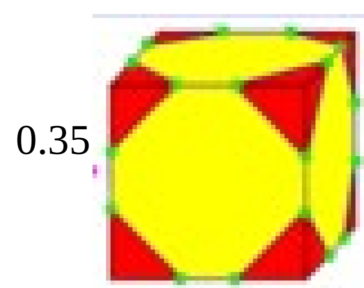
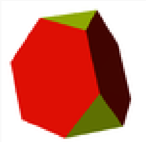
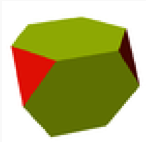
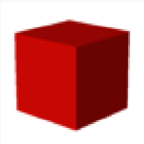
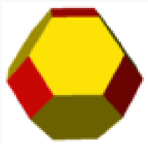
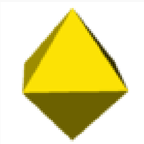
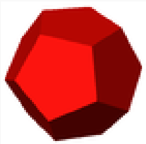
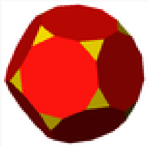
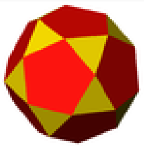
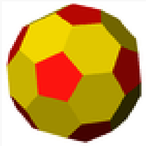
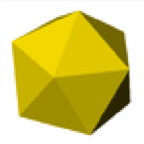






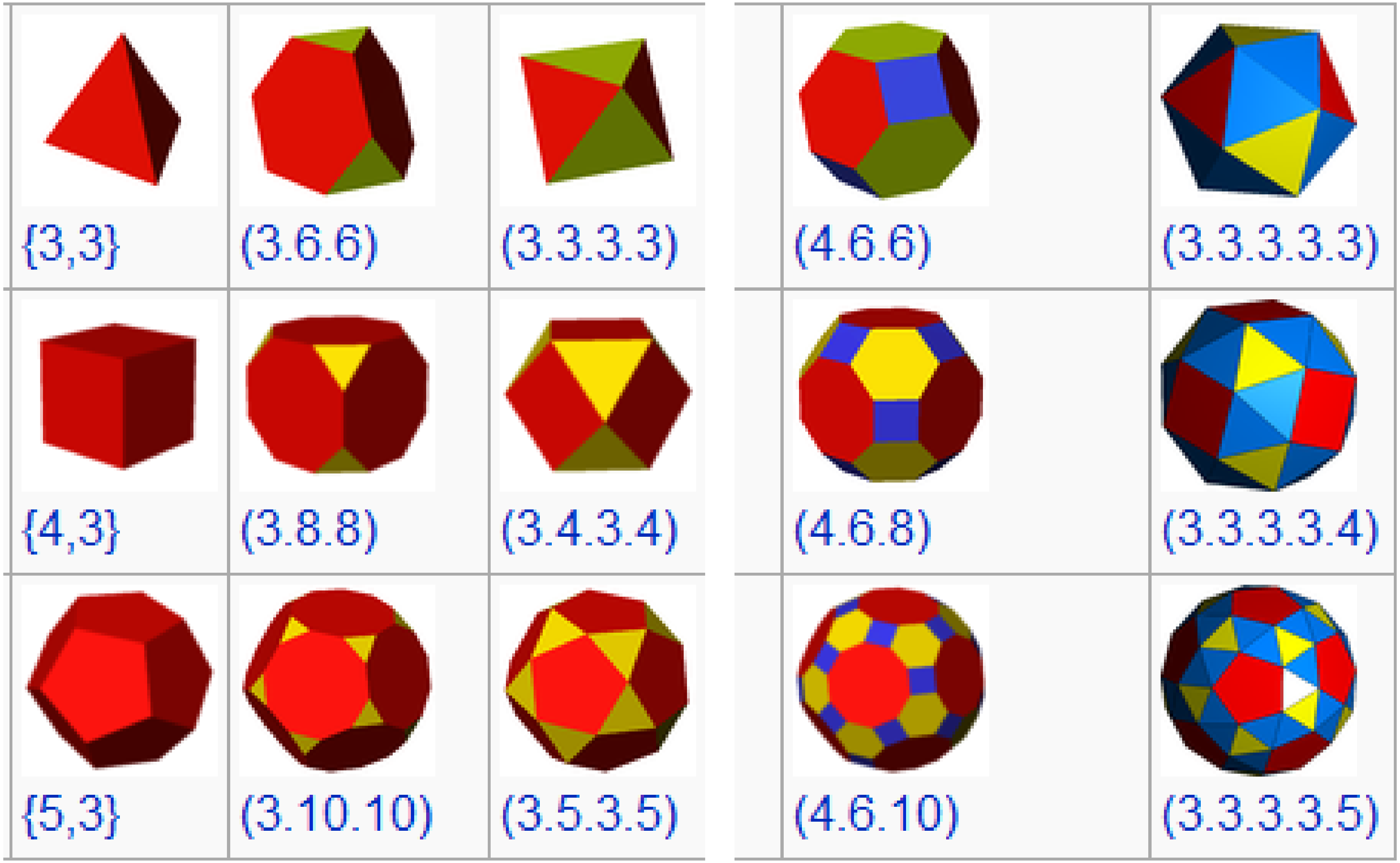
Conway

Polygonal Mesh Processing

Original	Truncation	Rectification	Bitruncation (truncated dual)	Birecification (dual)
 Tetrahedron	 Truncated tetrahedron	 Octahedron	 Truncated tetrahedron	 Tetrahedron
 Cube	 Truncated cube	 Cuboctahedron	 Truncated octahedron	 Octahedron
 Dodecahedron	 Truncated dodecahedron	 Icosidodecahedron	 Truncated icosahedron	 Icosahedron

Wikipedia

Polygonal Mesh Processing



Wikipedia

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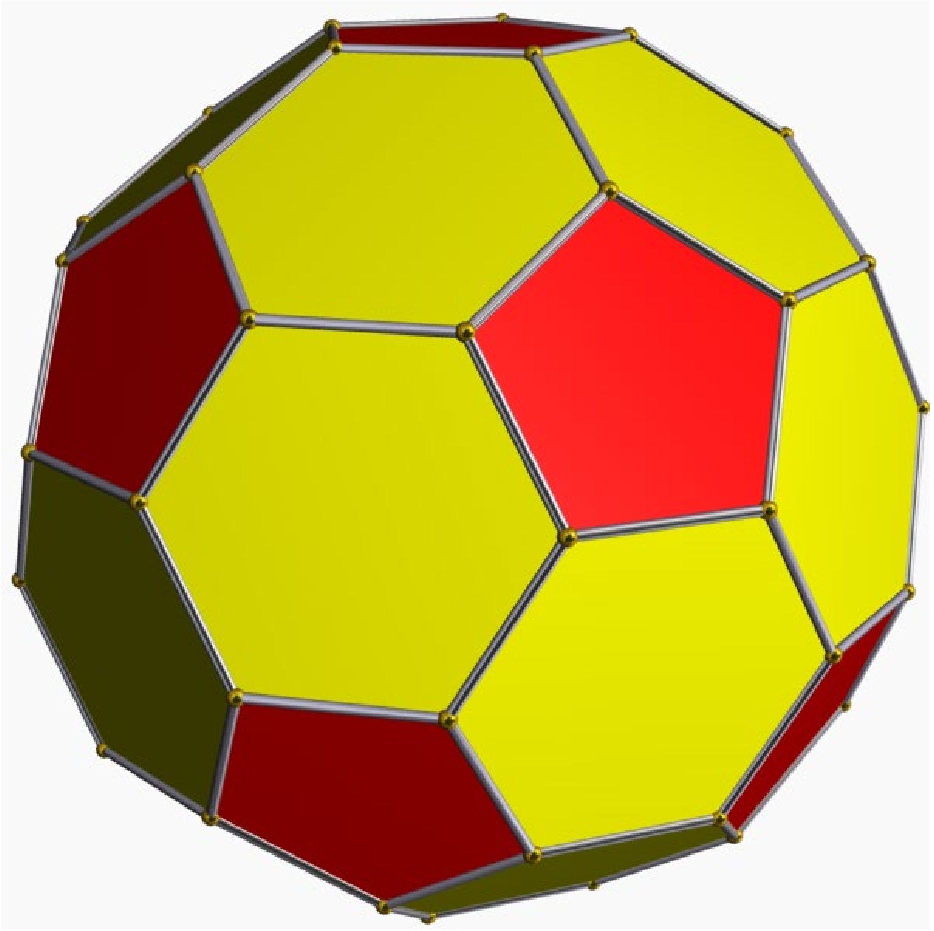

Wikipedia

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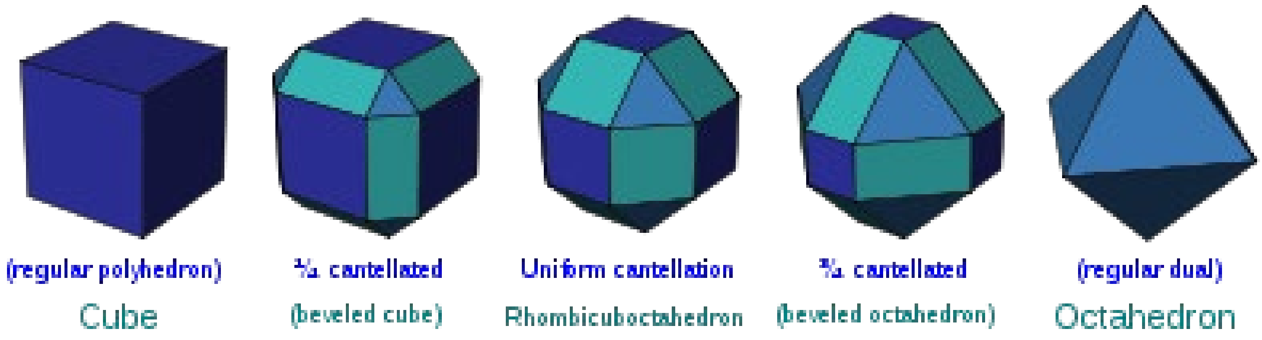
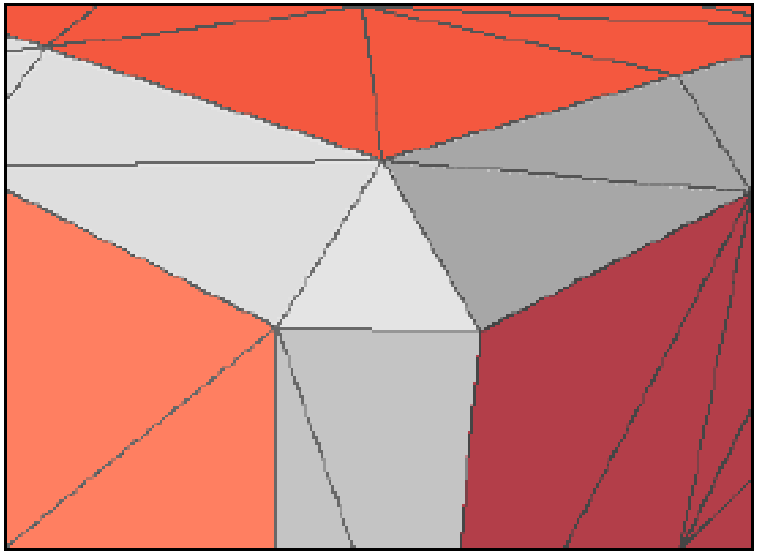
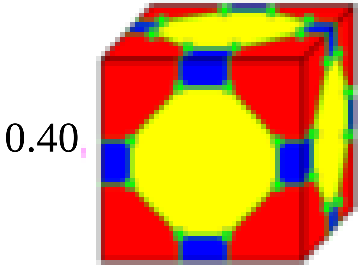




Jarek Rossignac



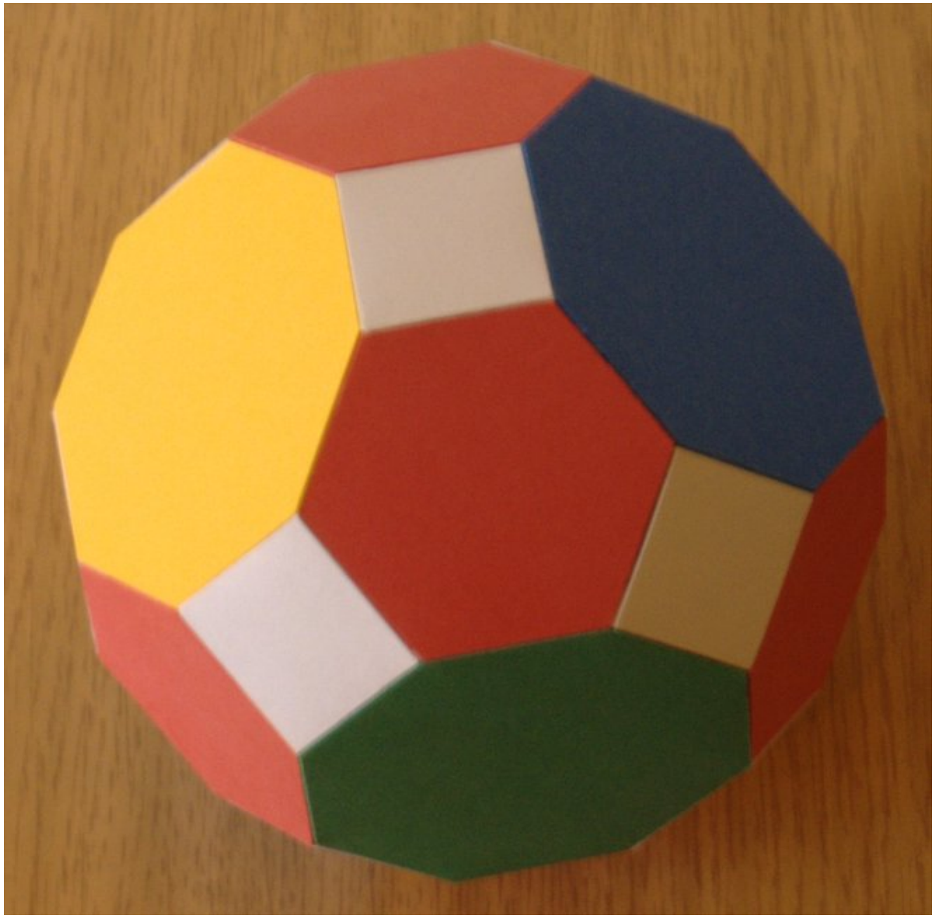
Conway

Polygonal Mesh Processing

Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- ➢ Bevel



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Polygonal Mesh Processing

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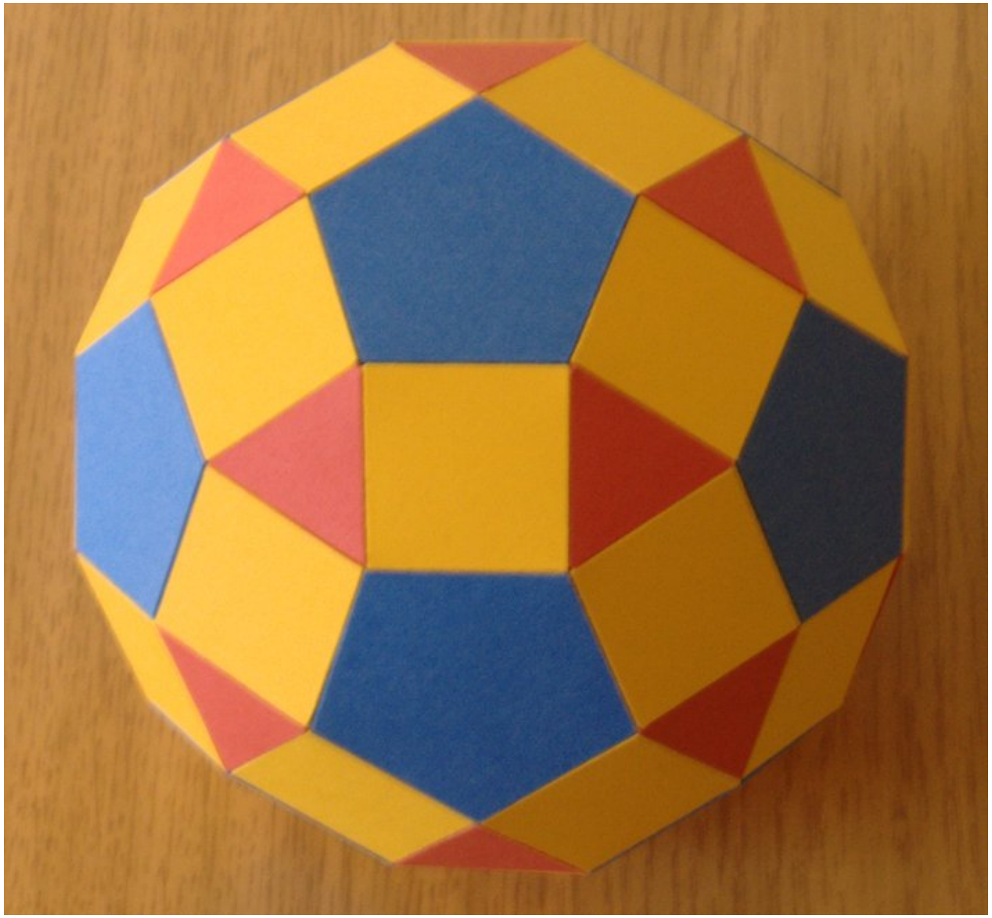

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Polygonal Mesh Processing

- Remeshing
  - Subdivide
  - Resample
  - Simplify
- Topological fixup
  - Fill holes
  - Fix cracks
  - Fix self-intersections
- Boolean operations
  - Crop
  - Subtract

Polygonal Mesh Processing

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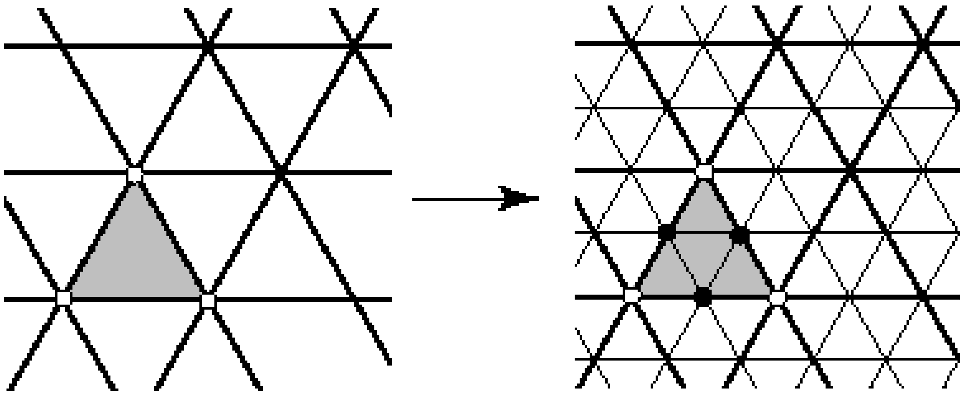

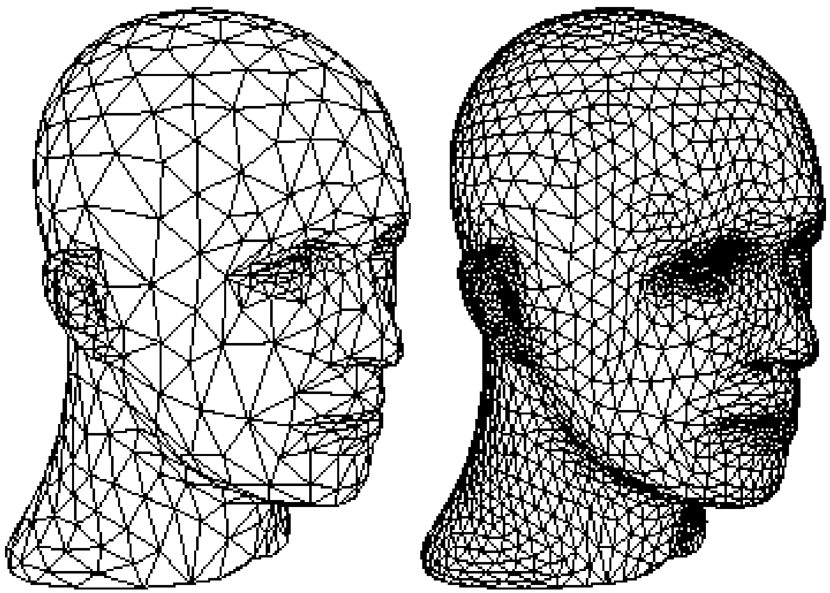

Zorin & Schroeder

Polygonal Mesh Processing

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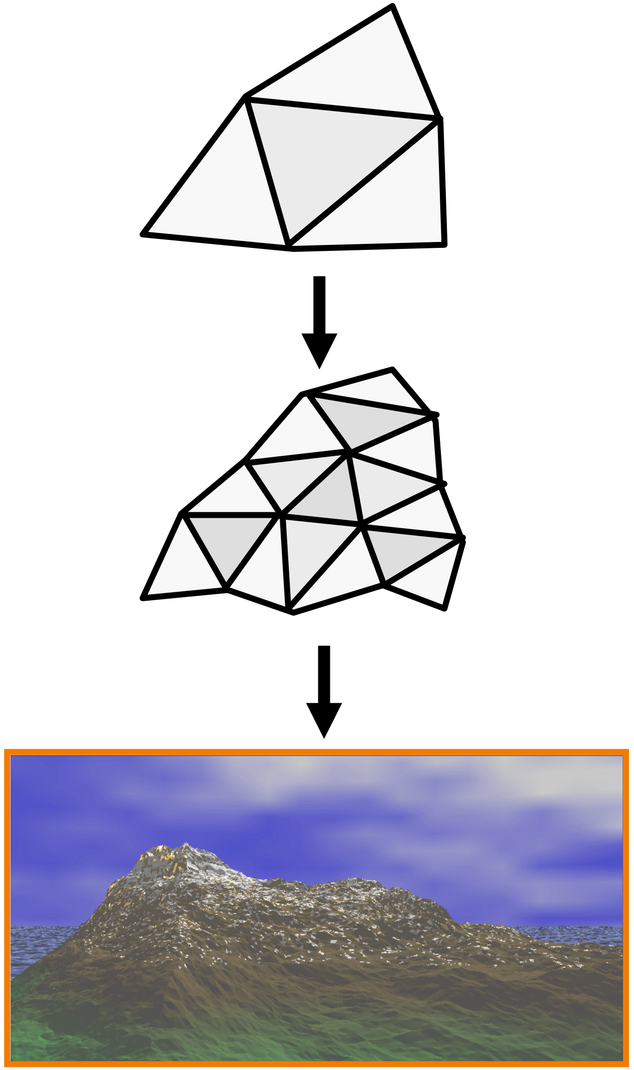

Dirk Balfanz, Igor Guskov, Sanjeev Kumar, & Rudro Samanta,

Polygonal Mesh Processing

Remeshing

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Topological fixup

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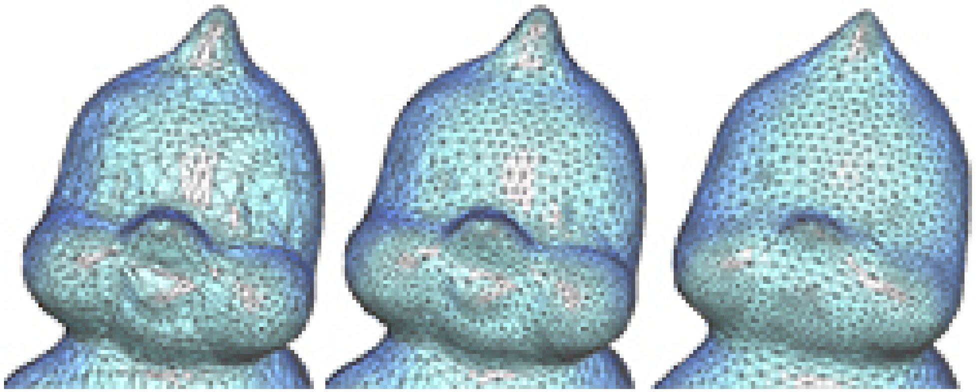

Original Resampled

Sorkine

Polygonal Mesh Processing

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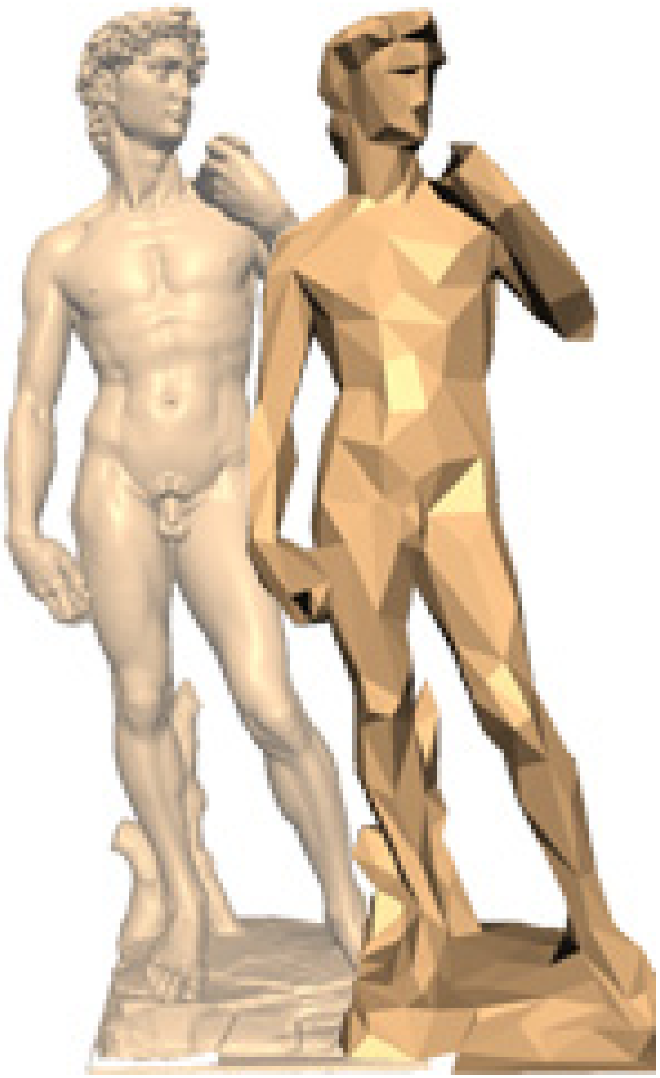

Garland

Polygonal Mesh Processing

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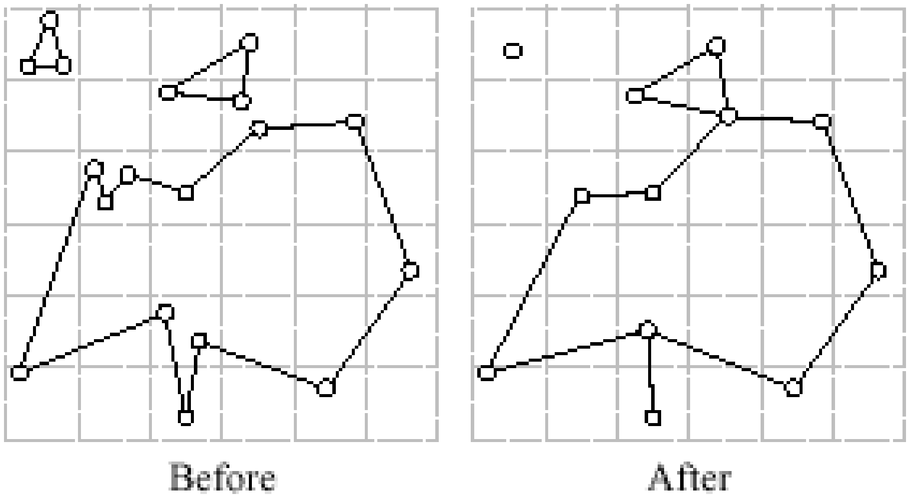

Vertex Clustering

Rossignac

Polygonal Mesh Processing

Remeshing
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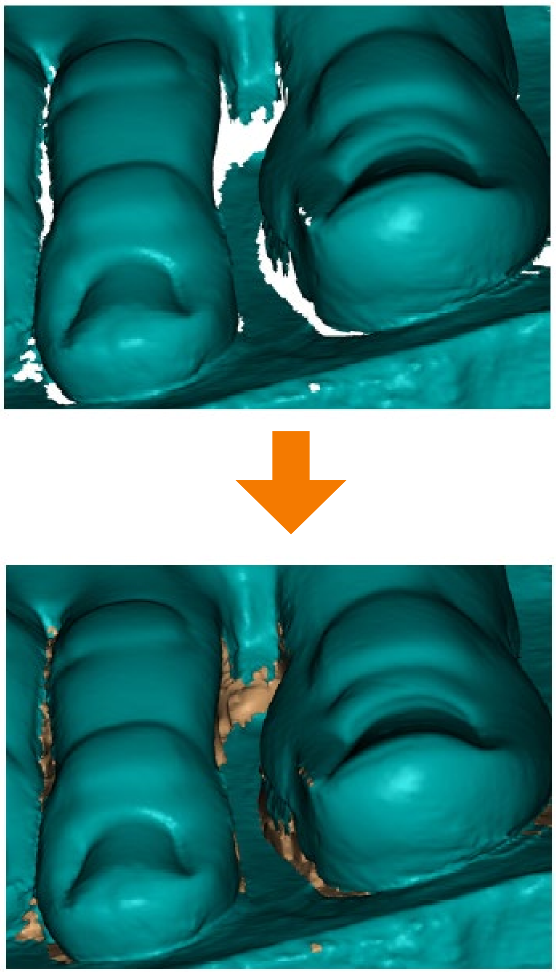

Polygonal Mesh Processing

Remeshing
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Topological fixup
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Boolean operations
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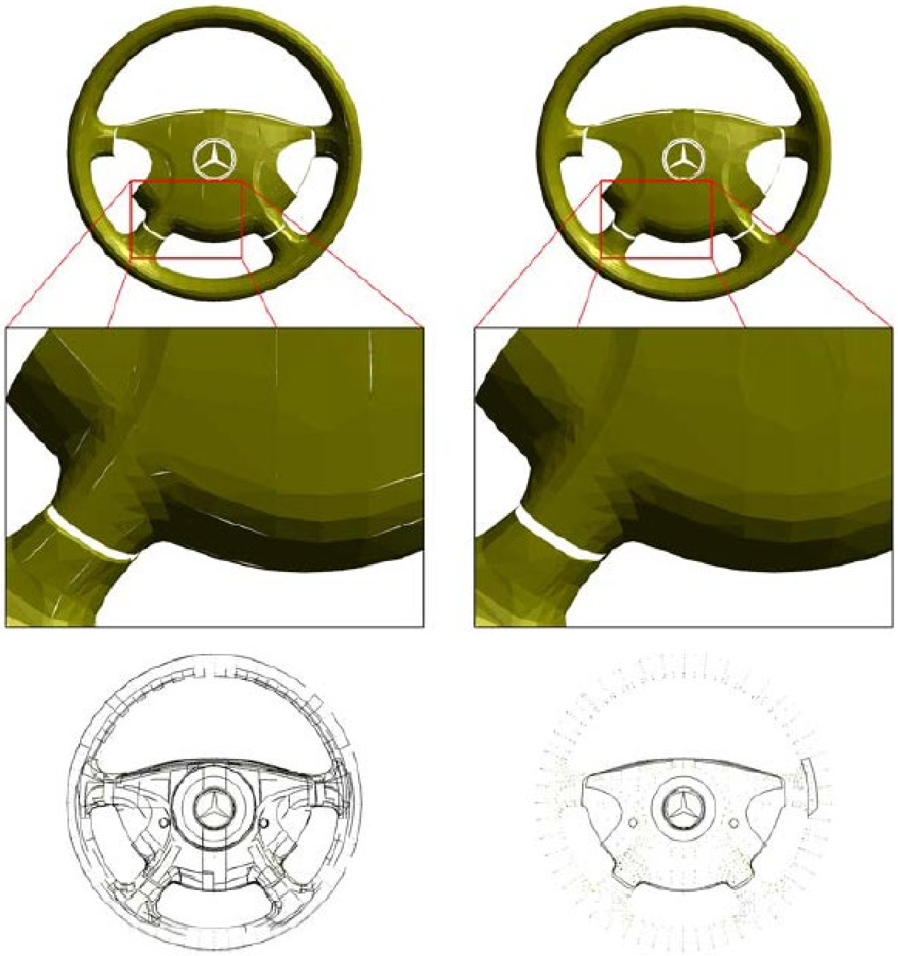

Borodin

Polygonal Mesh Processing

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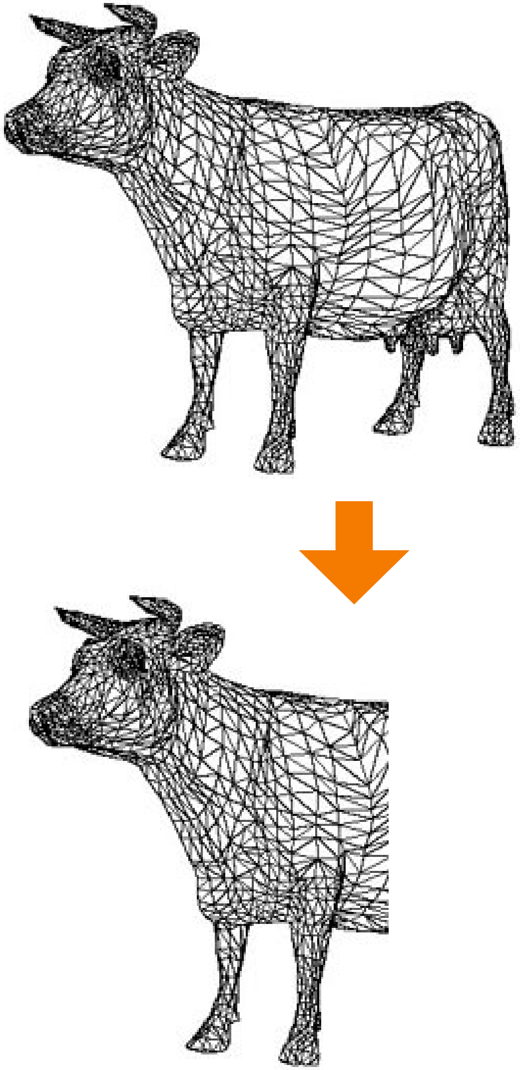

Polygonal Mesh Processing

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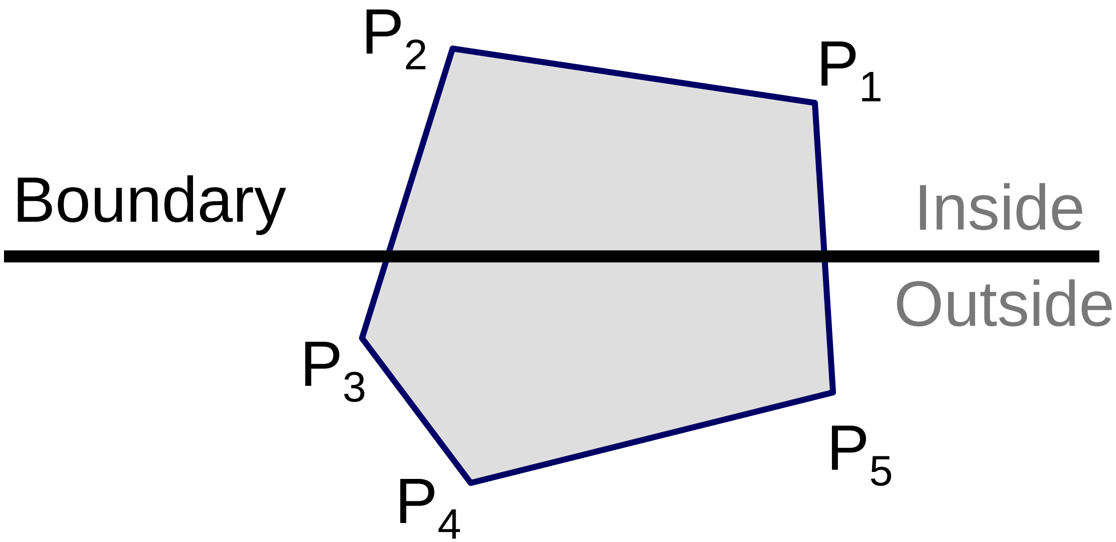

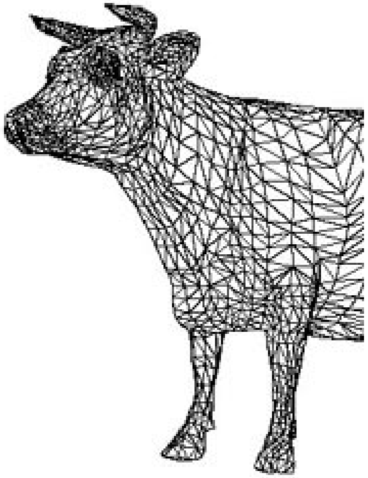

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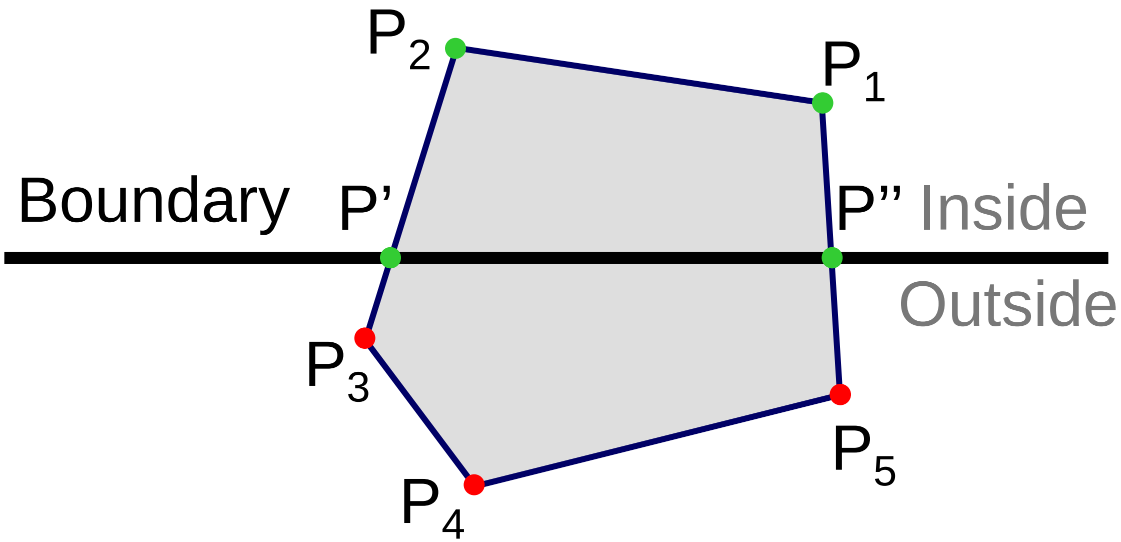

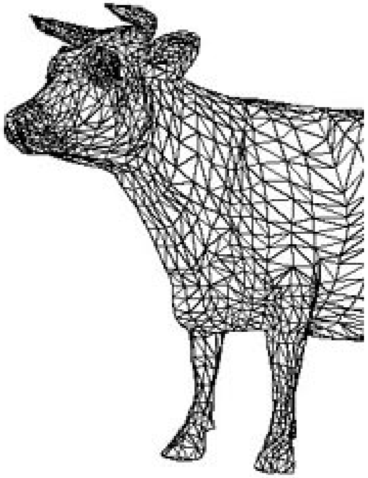

Polygonal Mesh Processing

Remeshing
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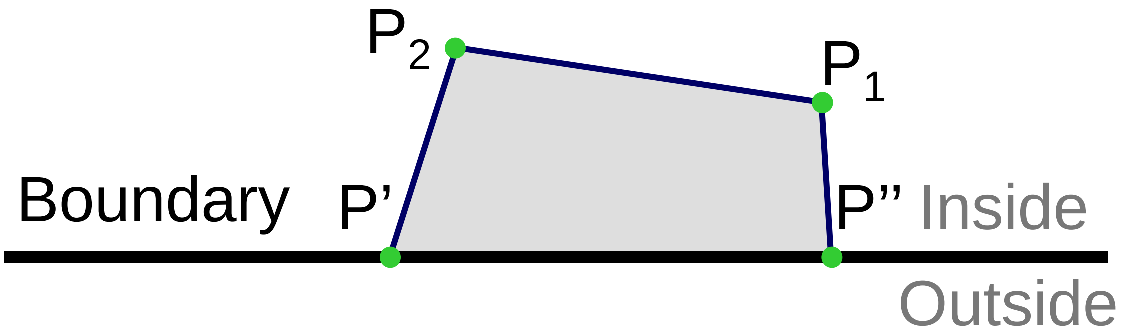
- Simplify



Topological fixup
- Fill holes
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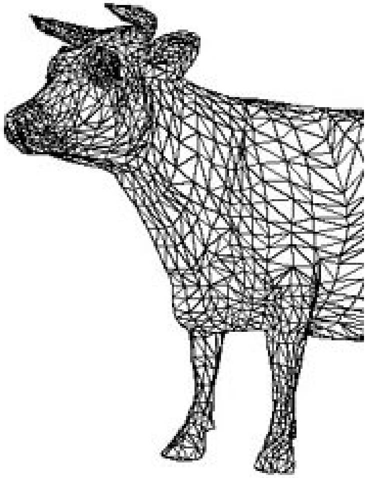

Polygonal Mesh Processing

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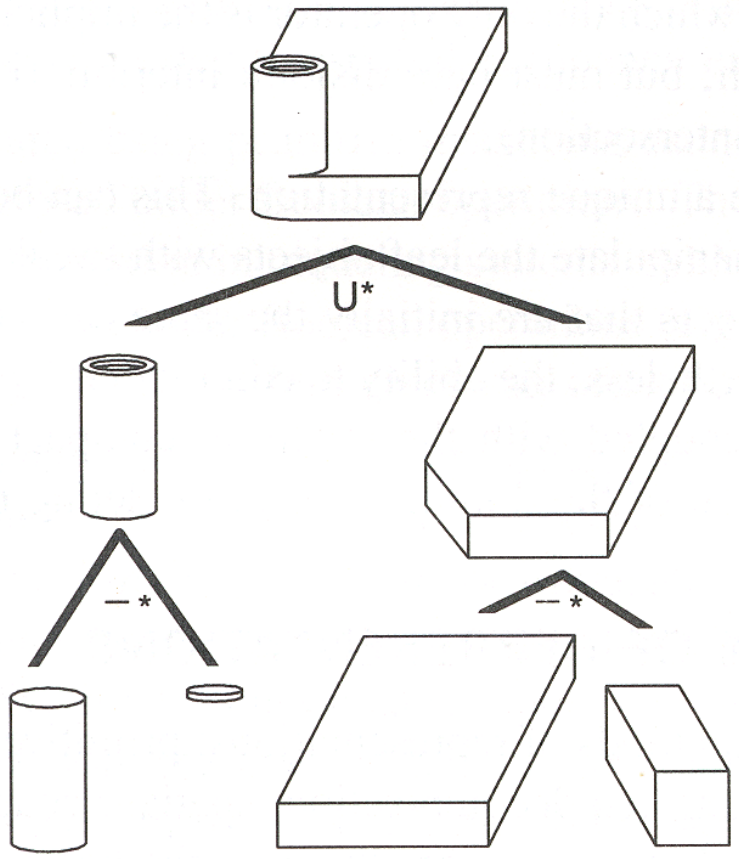


FvDFH Figure 12.27

Polygonal Mesh Processing

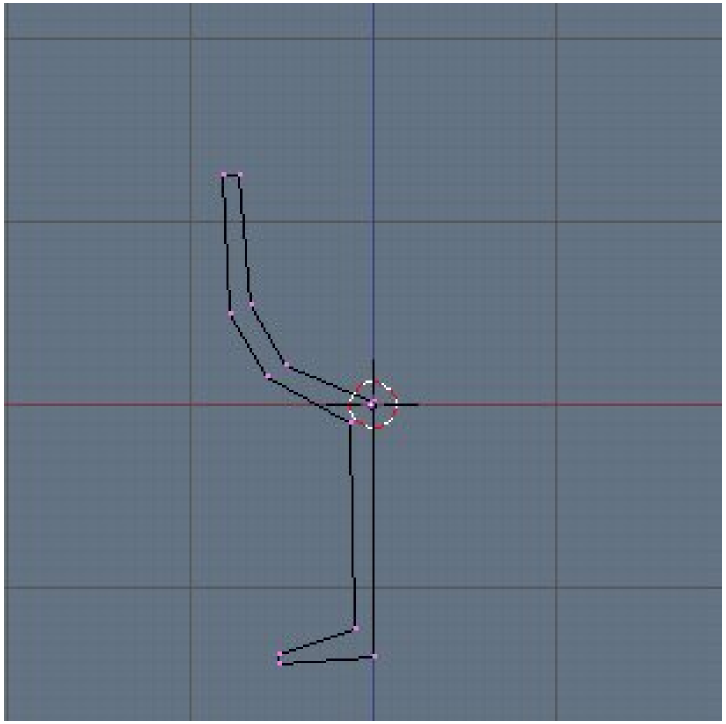
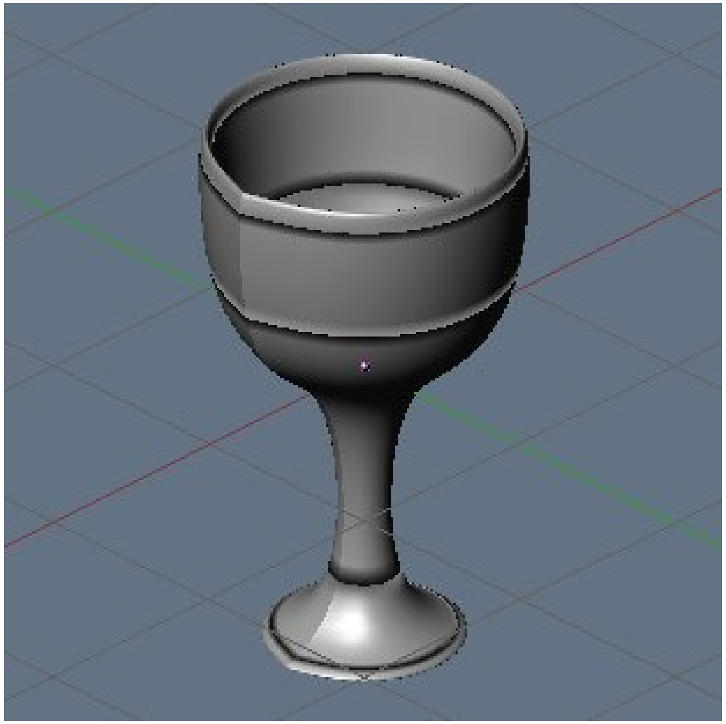
Procedural generation

- Surface of revolution
- Sweep

Polygonal Mesh Processing

Procedural generation

- ➢ Surface of revolution
- ◦ Sweep

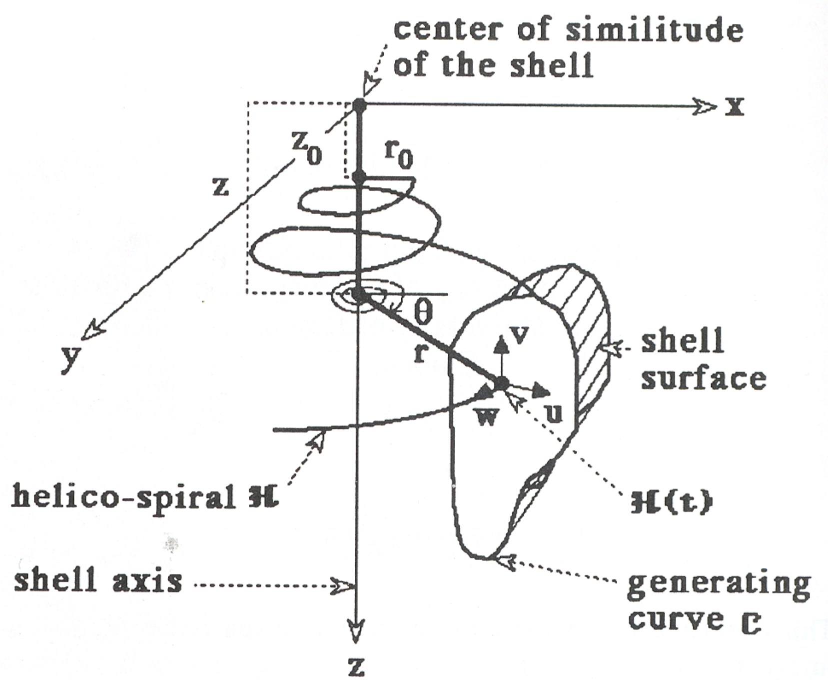



sphynx.co.uk

Polygonal Mesh Processing

Procedural generation

- Surface of revolution
- Sweep



Fowler et al., 1992

Polygonal Mesh Processing

Procedural generation

- Surface of revolution
- Sweep

Fowler et al., 1992

Polygonal Mesh Processing

Most operations use a few low-level operations:

- Subdivide face
- Subdivide edge
- Collapse edge
- Merge vertices
- Remove vertex

Polygonal Mesh Processing

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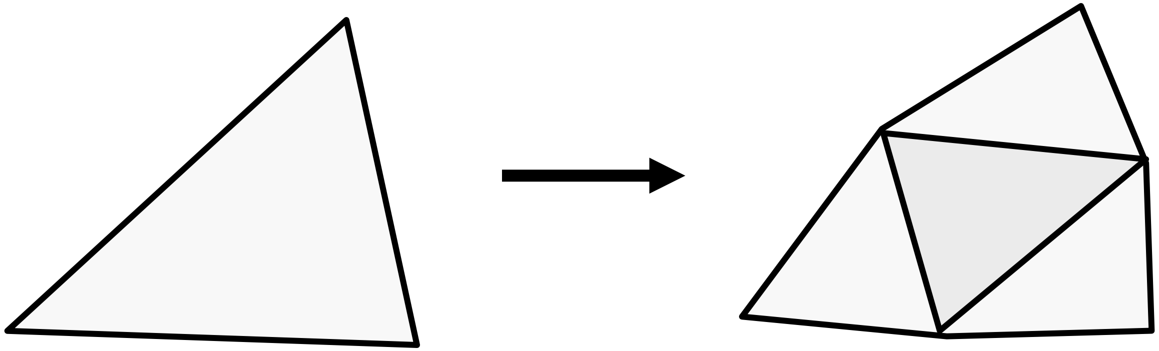

Subdivide face

Polygonal Mesh Processing

Most operations use a few low-level operations:

- Subdivide face
- ➢ Subdivide edge
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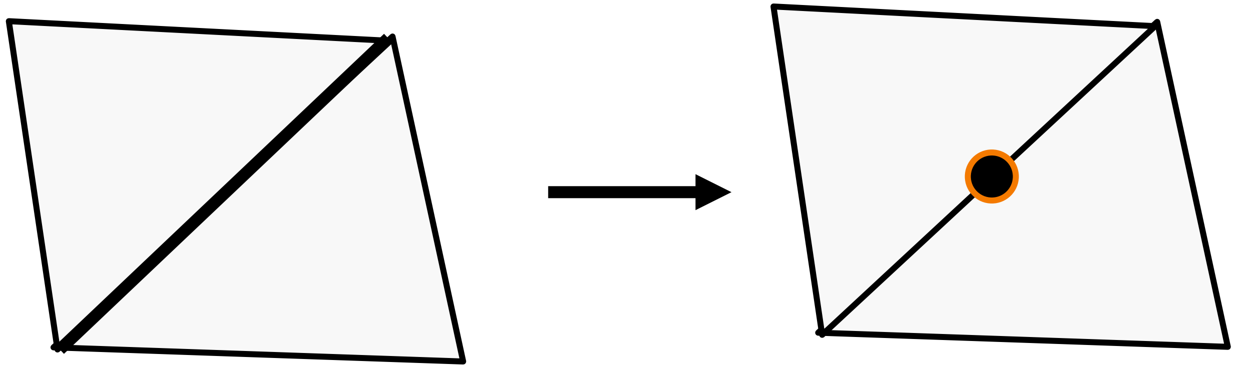
- Remove vertex



Subdivide edge

Polygonal Mesh Processing

Most operations use a few low-level operations:

- Subdivide face
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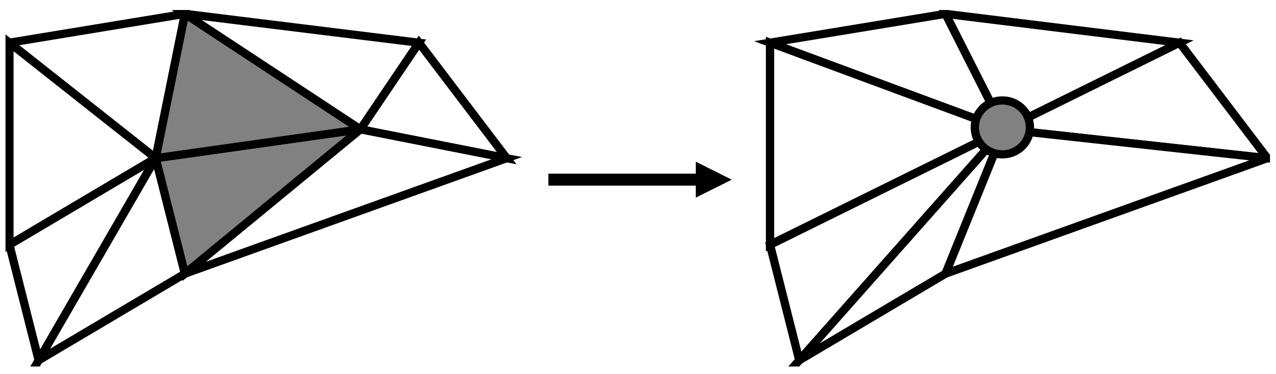
- Collapse edge
- Merge vertices
- Remove vertex

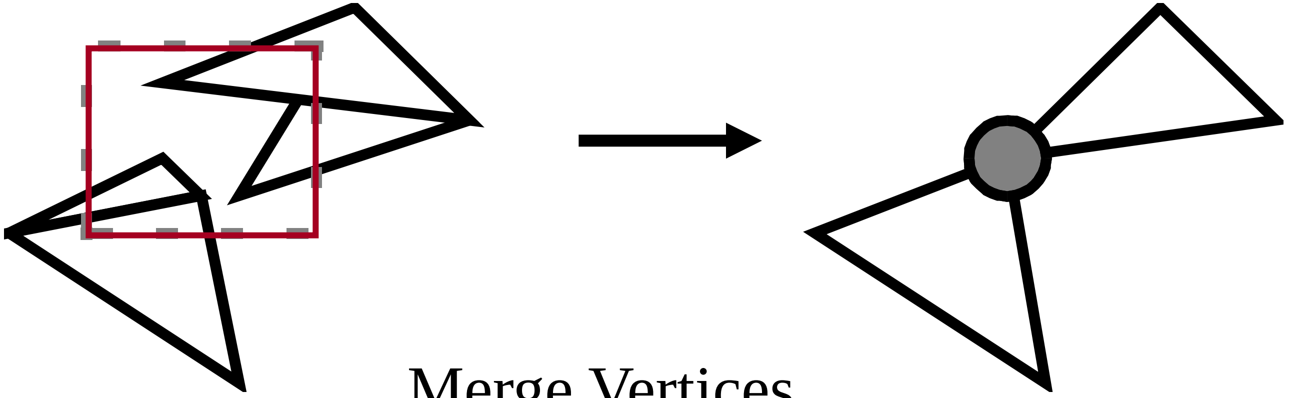


Collapse edge

Polygonal Mesh Processing

Most operations use a few low-level operations:

- Subdivide face
- Subdivide edge
- Collapse edge
- ➢ Merge vertices
- Remove vertex

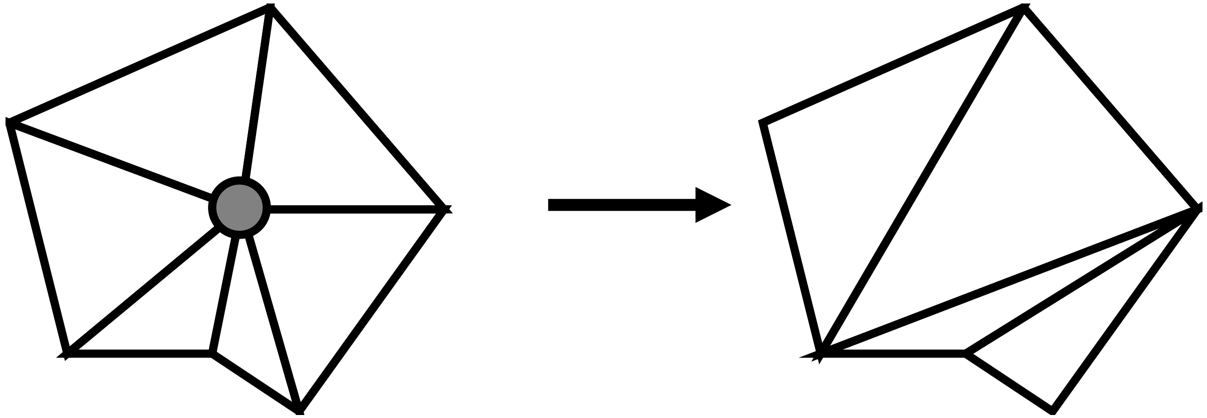


Merge Vertices

Polygonal Mesh Processing

Most operations use a few low-level operations:

- Subdivide face
- Subdivide edge
- Collapse edge
- Merge vertices
- ➢ Remove vertex



Remove Vertex

Outline

Acquisition

Processing

Representation

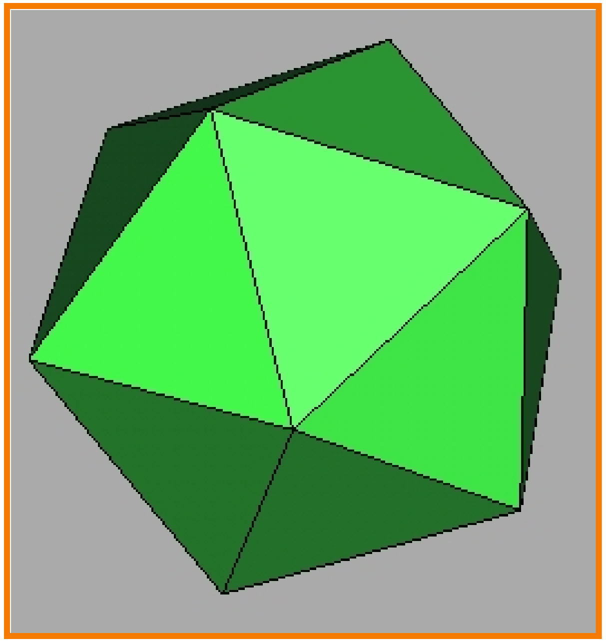
Polygon Mesh Representation

Data structures determine algorithms

- Data structure must support key operations of algorithm efficiently

Examples:

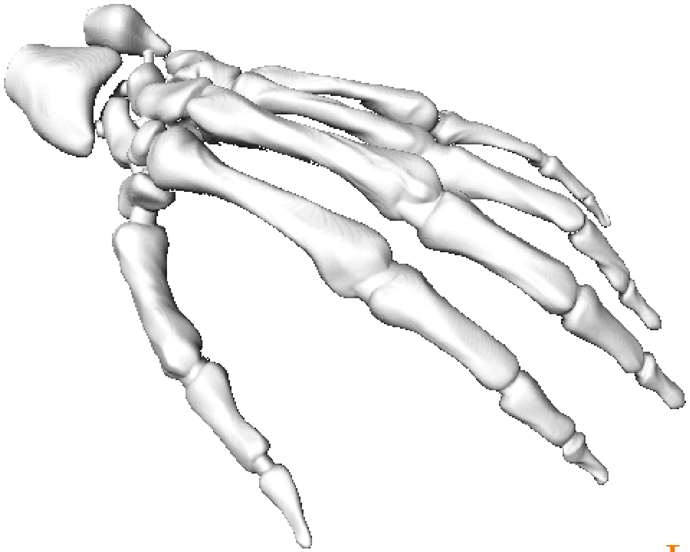
- Drawing a mesh
- Removing a vertex
- Smoothing a region
- Intersecting polyhedra

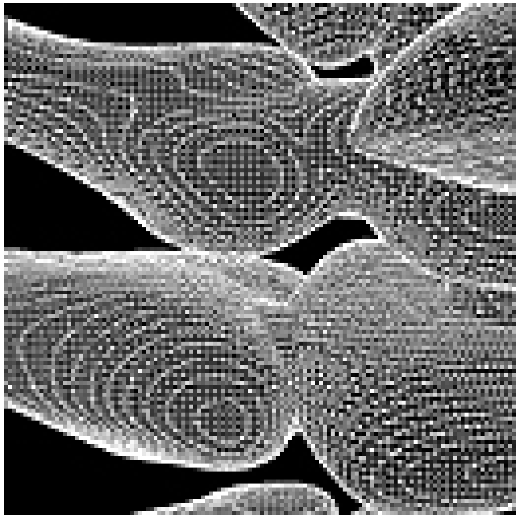


Different data structures for different algorithms

Polygon Mesh Representation

Important properties of mesh representation?



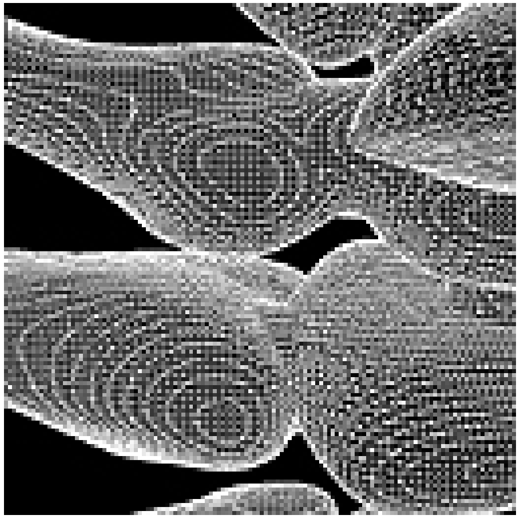


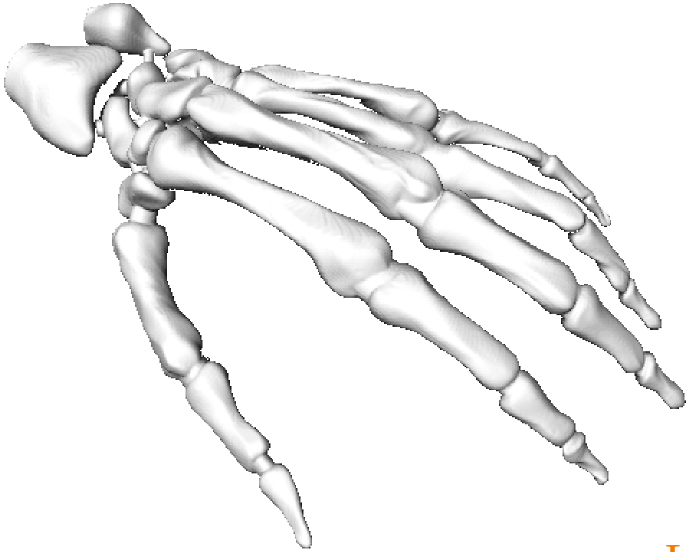
Large Geometric Model Repository
Georgia Tech

Polygon Mesh Representation

Important properties of mesh representation?

- Efficient traversal of topology
- Efficient use of memory
- Efficient updates



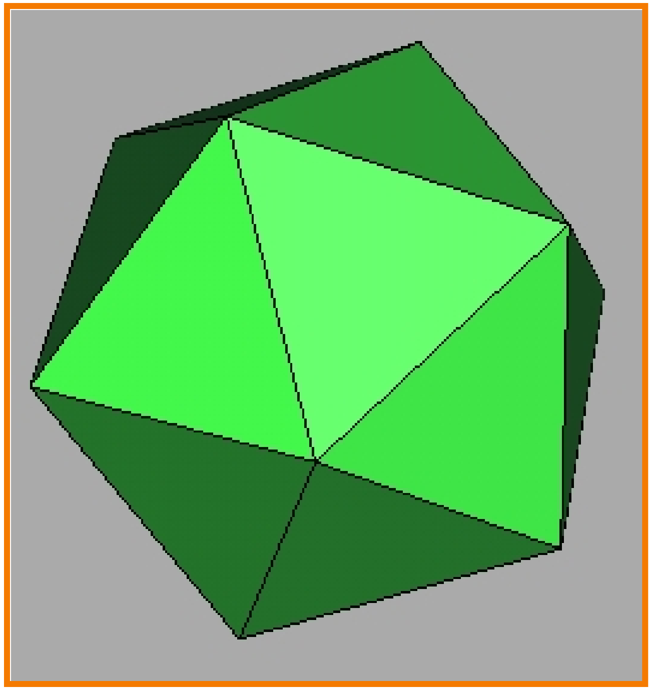


Large Geometric Model Repository
Georgia Tech

Polygon Mesh Representation

Possible data structures

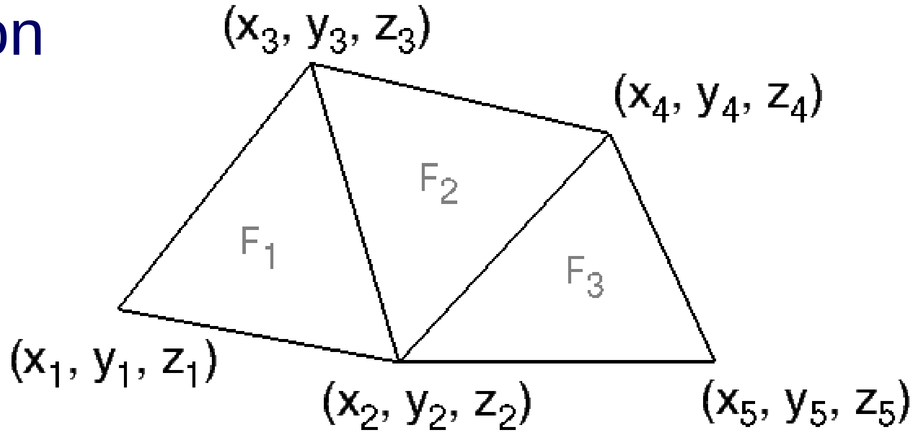
- List of independent faces
- Vertex and face tables
- Adjacency lists
- Winged edge
- Half edge
- etc.

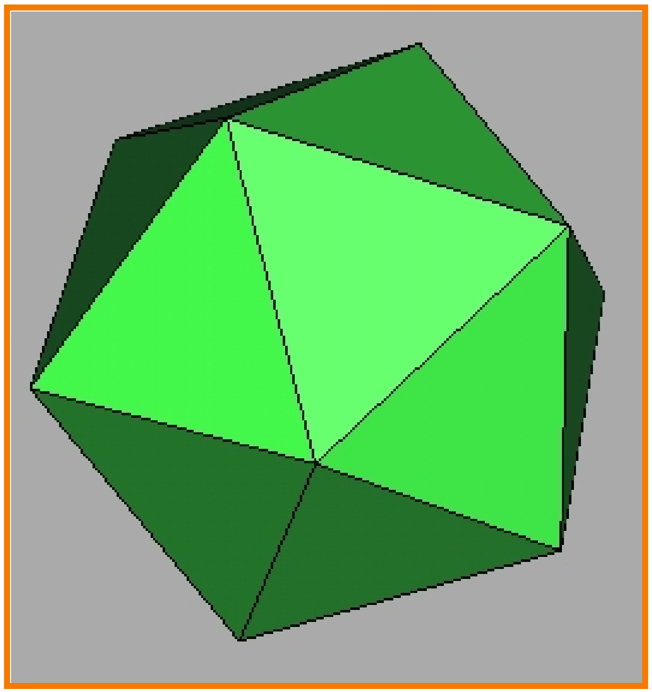


Independent Faces

Each face lists vertex coordinates

- Redundant vertices
- No adjacency information



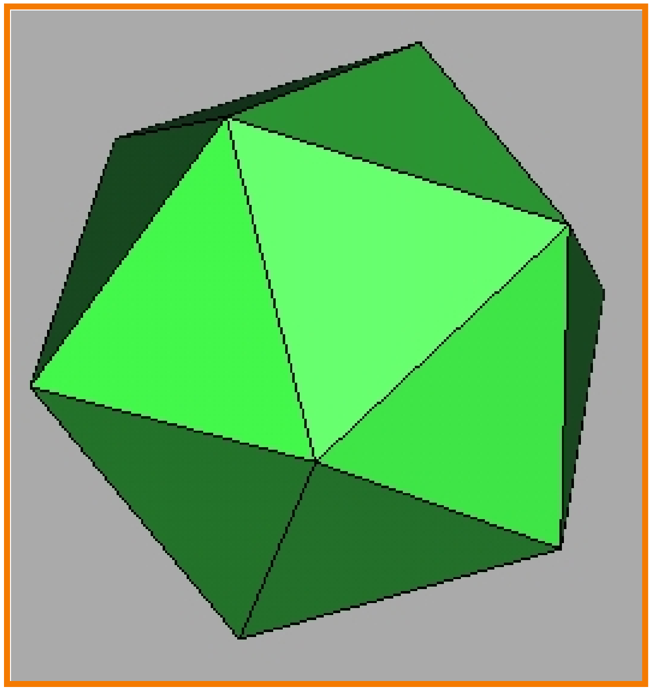


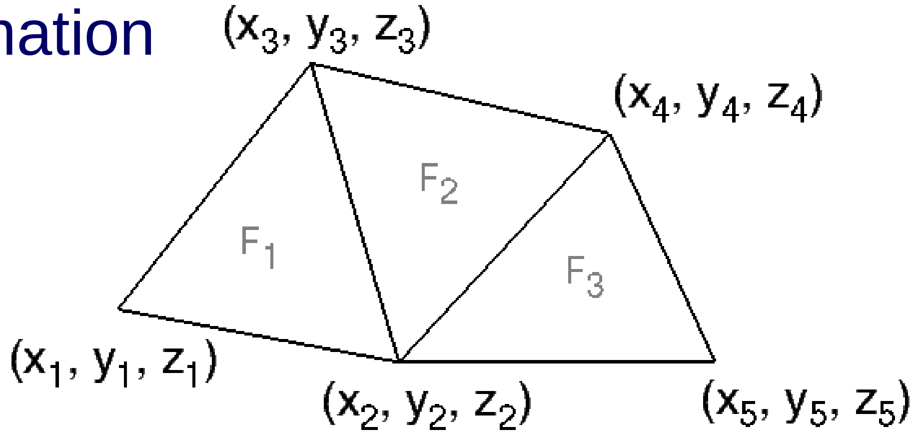
FACE TABLE	
F_1	(x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3)
F_2	(x_2, y_2, z_2) (x_4, y_4, z_4) (x_3, y_3, z_3)
F_3	(x_2, y_2, z_2) (x_5, y_5, z_5) (x_4, y_4, z_4)

Vertex and Face Tables

Each face lists vertex references

- Shared vertices
- Still no adjacency information





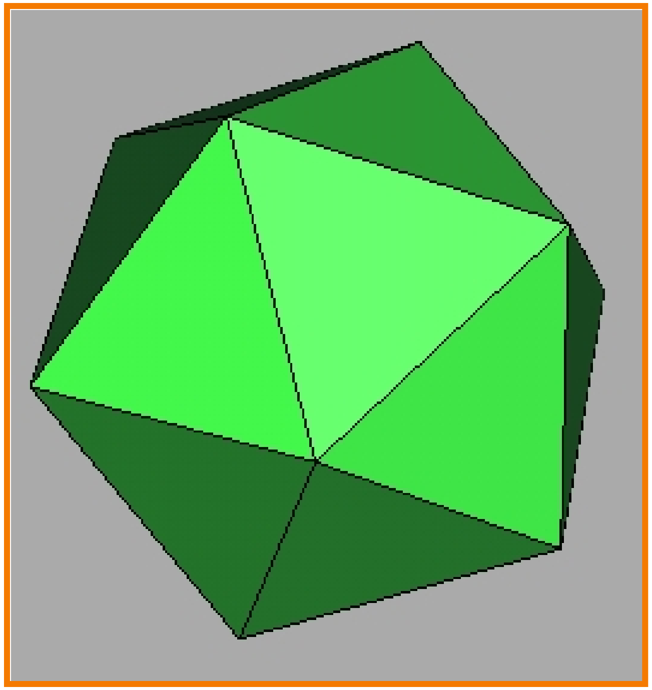
VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

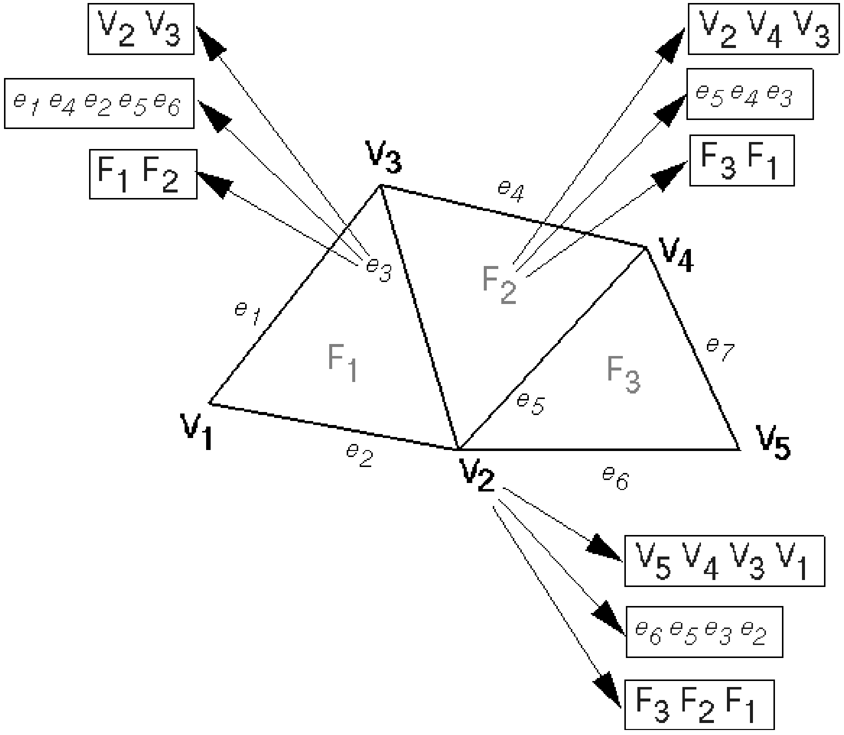
FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

Adjacency Lists

Store all vertex, edge, and face adjacencies

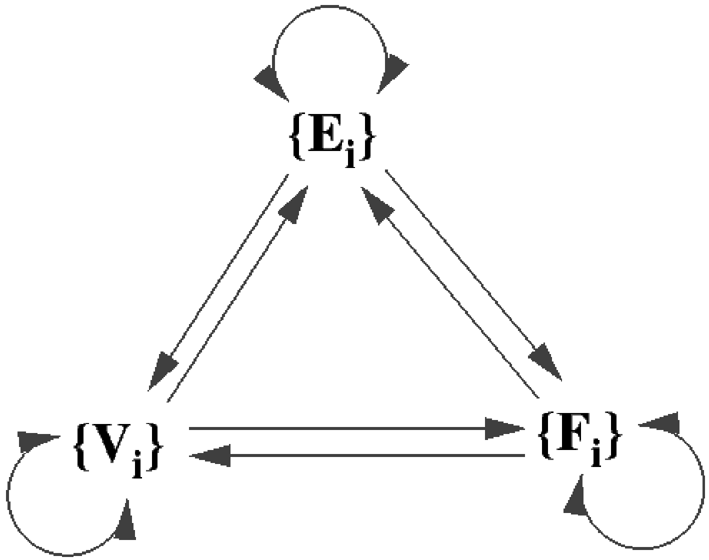
- Efficient adjacency traversal
- Extra storage

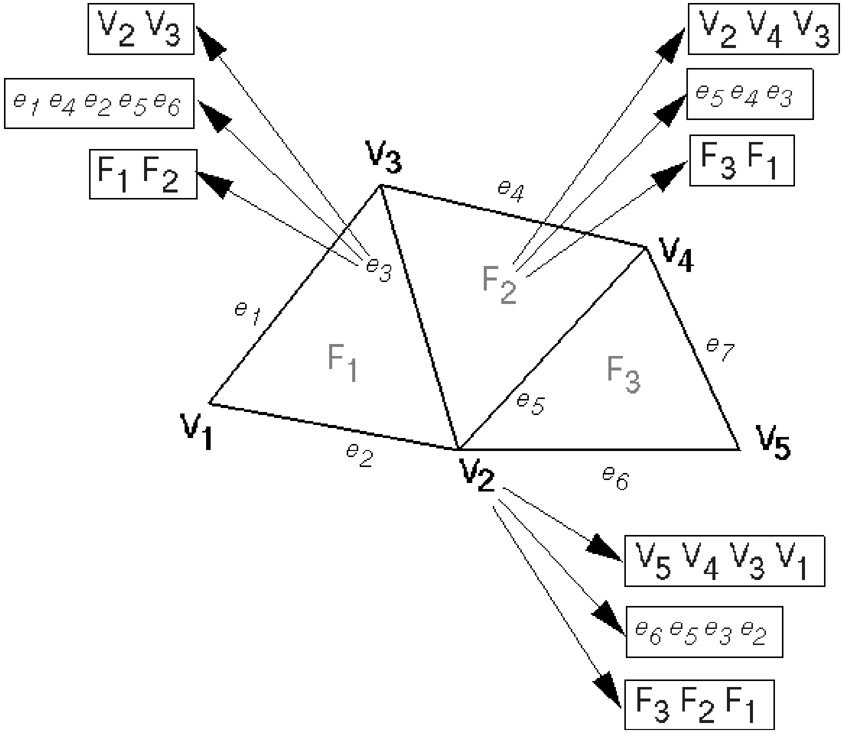




Partial Adjacency Lists

Can we store only some adjacency relationships and derive others?

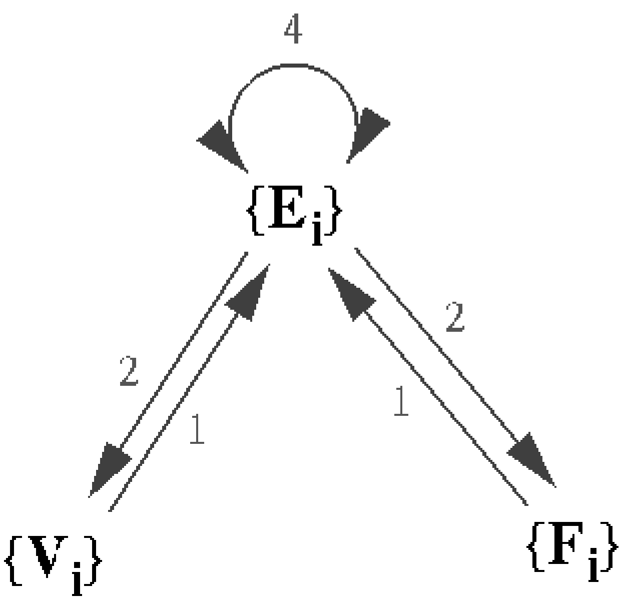


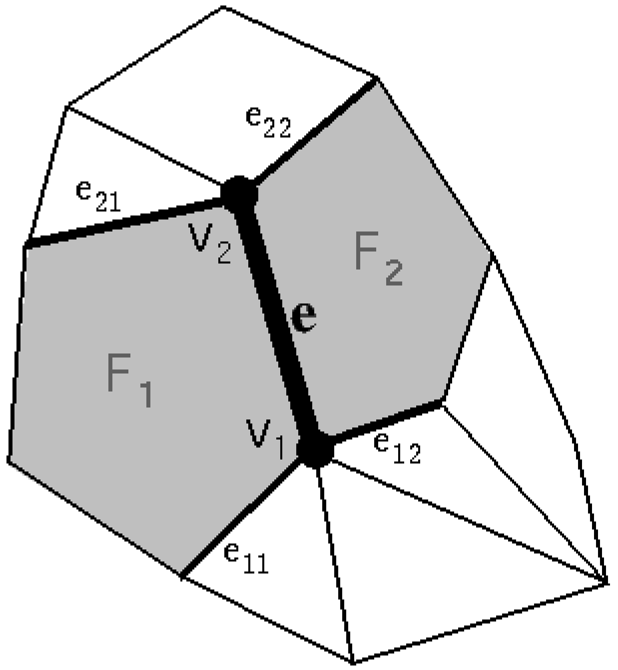


Winged Edge

Adjacency encoded in edges

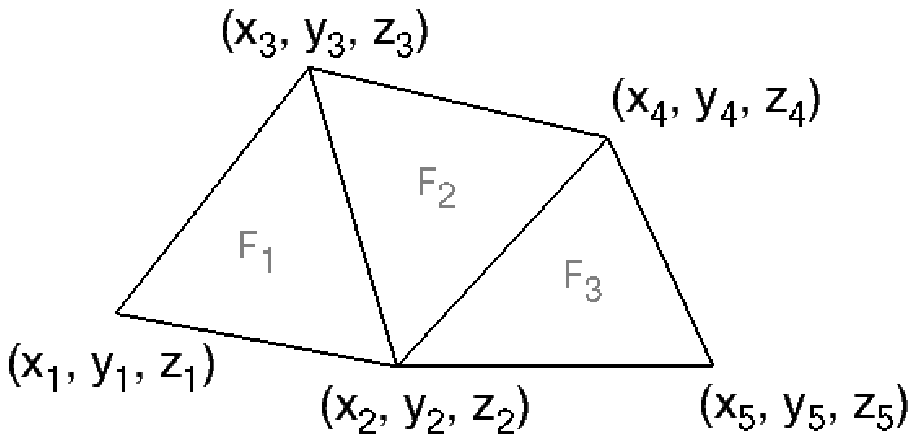
- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons





Winged Edge

Example:



VERTEX TABLE

V_1	X_1	Y_1	Z_1	e_1
V_2	X_2	Y_2	Z_2	e_6
V_3	X_3	Y_3	Z_3	e_3
V_4	X_4	Y_4	Z_4	e_5
V_5	X_5	Y_5	Z_5	e_6

EDGE TABLE

					11	12	21	22
e_1	V_1	V_3		F_1	e_2	e_2	e_4	e_3
e_2	V_1	V_2	F_1		e_1	e_1	e_3	e_6
e_3	V_2	V_3	F_1	F_2	e_2	e_5	e_1	e_4
e_4	V_3	V_4		F_2	e_1	e_3	e_7	e_5
e_5	V_2	V_4	F_2	F_3	e_3	e_6	e_4	e_7
e_6	V_2	V_5	F_3		e_5	e_2	e_7	e_7
e_7	V_4	V_5		F_3	e_4	e_5	e_6	e_6

FACE TABLE

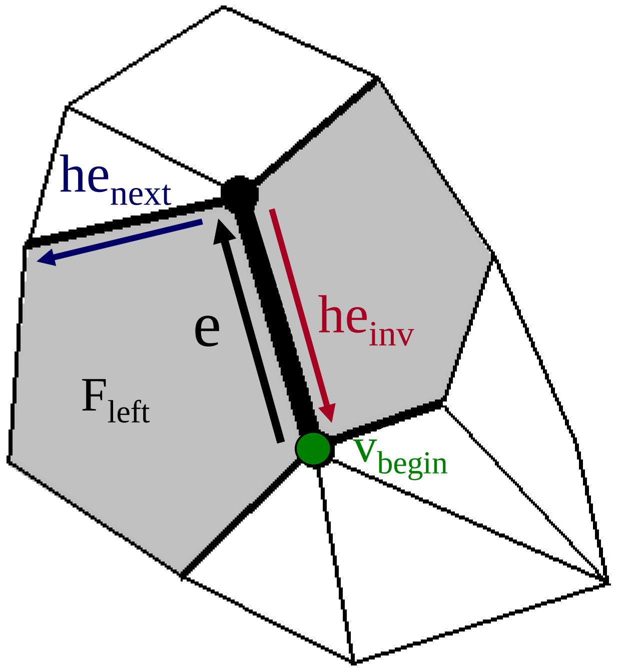
F_1	e_1
F_2	e_3
F_3	e_5

Half Edge

Adjacency encoded in edges

- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons

Similar to winged-edge, except adjacency encoded in half-edges



Summary

Polygonal meshes

- Most common surface representation
- Fast rendering

Processing operations

- Must consider irregular vertex sampling
- Must handle/avoid topological degeneracies

Representation

- Which adjacency relationships to store depend on which operations must be efficient