What computers just *cannot* do.

COS 116, Spring 2012

Adam Finkelstein

“What computers can’t do.”

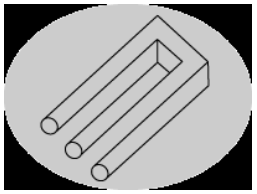
“Prof, what’s with all the negative thinking?!?”

- An obvious motivation:
  Understand the limits of technology

The power of negative thinking….

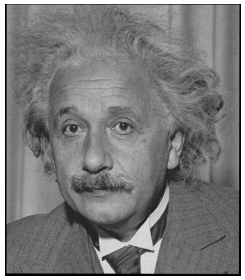
In Science….

Often, impossibility result → deep insight



Examples

- Impossibility of trisecting angle with ruler and compass (Galois) → Group Theory and much of modern math



- Nothing travels faster than light → Relativity and modern physics

In Mathematics…..

"Can mathematicians be replaced by machines?"

[Hilbert, 1900]

Math is axiomatic

Axioms – Set of statements

Derivation rules – finite set of rules for deriving new statements from axioms

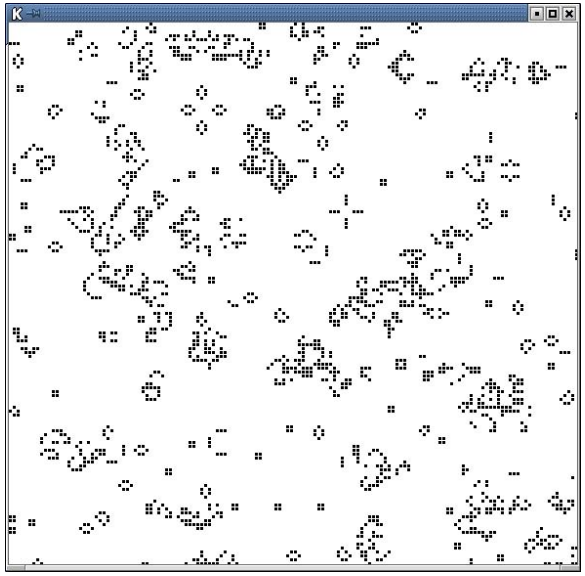
Theorems – Statements that *can* be derived from axioms in a finite number of steps

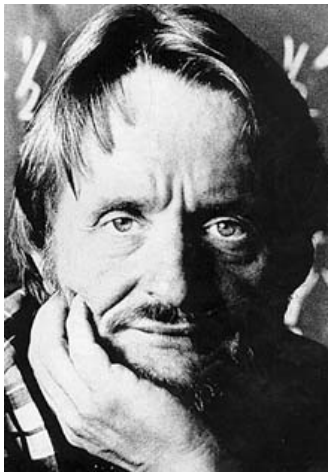
Mathematician – Person who tries to determine whether or not a statement is a theorem.

Understanding complex systems (or even simple systems)….

Can a simple set of mathematical equations “solve” problems like:

“Given starting configuration for the game of life, determine whether or not cell (100,100) is ever occupied by a critter.”

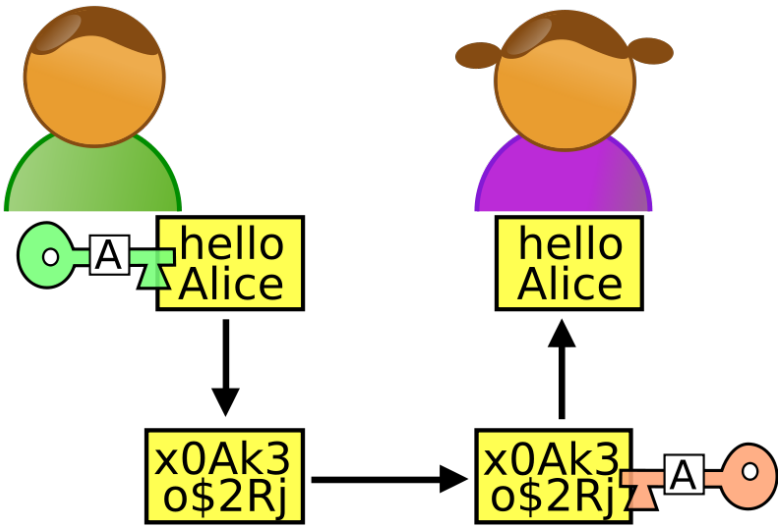




John Conway

In computer science……

CAPTCHA



Cryptography

More Computer Science…

Automated Software Checking?

Windows Vista:

50-million line program

Can computers check whether or not it will ever crash?

Discussion Time

What is a computation?
(need to know this to know what can/cannot be computed)

Next:
How did Turing set about formalizing this age-old notion and what were the features of his model?

What is a computation?

(see reading by Davis)

Basic Elements

- **Scratch Pad**
- **Step-by-step description of what to do (“program”); should be finite!**
- **At each step:**
  - **Can only scan a fixed number of symbols**
  - **Can only write a fixed number of symbols**

Turing's model

- 1 dimensional unlimited scratchpad ("infinite tape")
- Only symbols are 0 or 1 (tape has a finite number of 1s)
- Can only scan/write one symbol per step
- Program looks like →

1. PRINT 0
2. GO LEFT
3. GO TO STEP 1 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

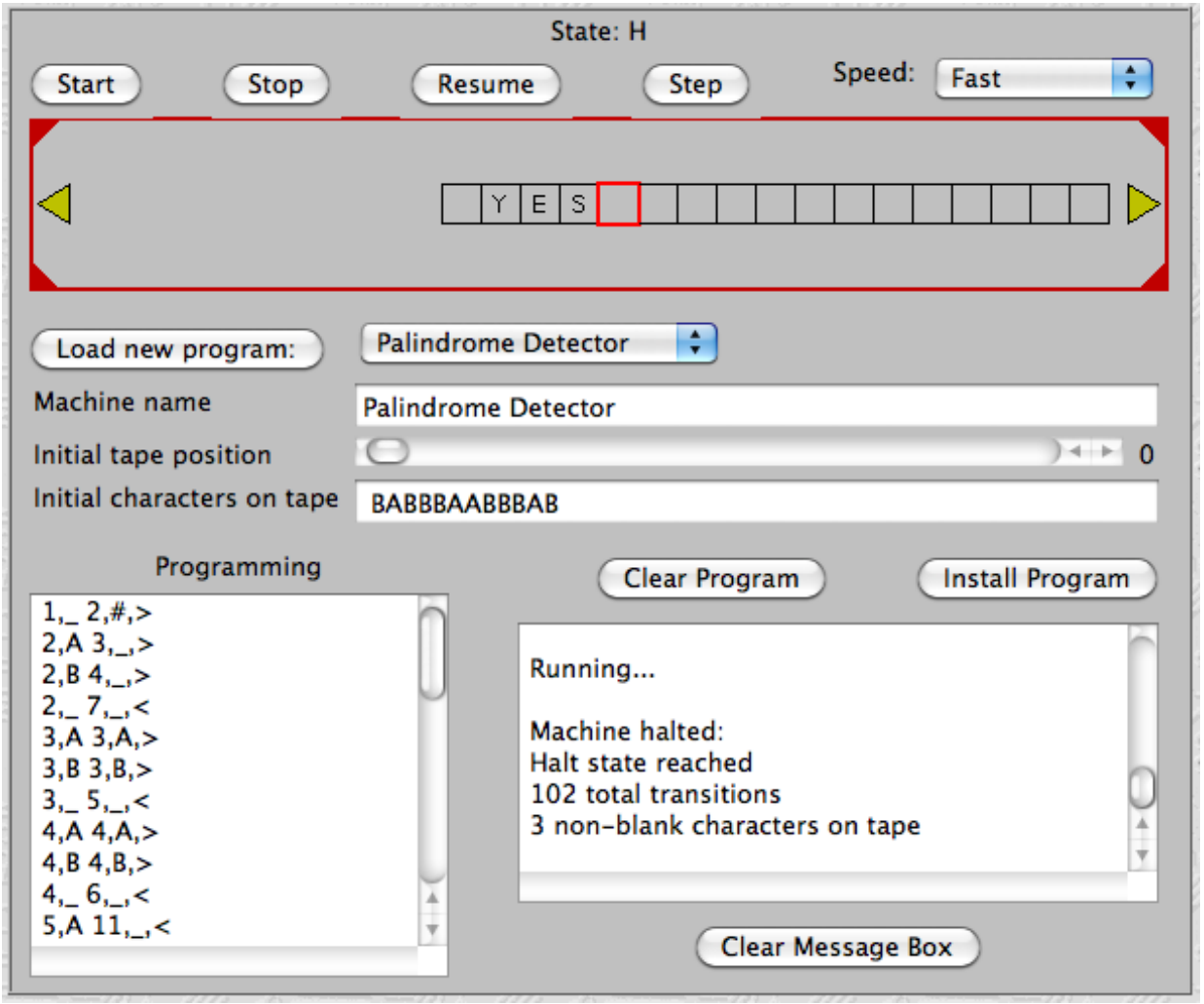
The Doubling Program

Example:
What does this program do?

1. PRINT 0
2. GO RIGHT
3. GO TO STEP 1 if 1 SCANNED
4. GO TO STEP 2 if 0 SCANNED

http://ironphoenix.org/tril/tm/

Let's try another...



Discussion Time

Can this computational model do every computation that pseudocode can?

How do we implement arithmetic instructions, arrays, loops?

Surprising facts about this simple model

- It can do everything that pseudocode can do

Hence it can “simulate” any other physical system, and in particular simulate any other physically realizable “computer.”

[CHURCH-TURING THESIS”]

THIS MODEL CAPTURES THE NOTION OF “COMPUTATION” ----TURING

Recall: Numbers and letters can be written in binary.

A program can also be represented by a string of bits!

“Code” for a program

= Binary Representation

Many conventions possible (e.g., ASCII)

Davis’ convention:

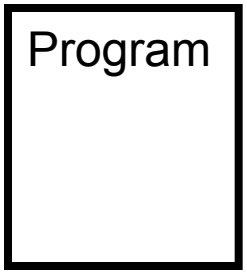
Code	Instruction
000	PRINT 0
001	PRINT 1
010	GO LEFT
011	GO RIGHT
$1010\underbrace{\ldots}_{i}01$	GO TO STEP i IF 0 IS SCANNED
$1101\underbrace{\ldots}_{i}10$	GO TO STEP i IF 1 IS SCANNED
100	STOP

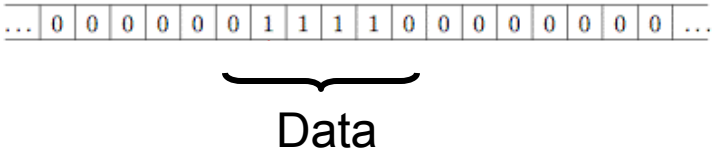
P ⟷ Code (P)

Programs and Data

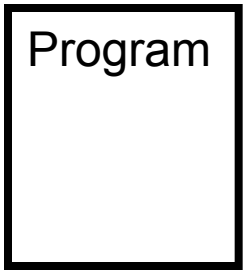
A False Dichotomy!

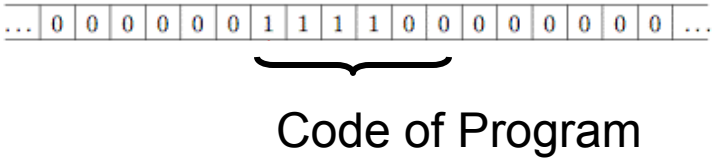
Usual viewpoint -



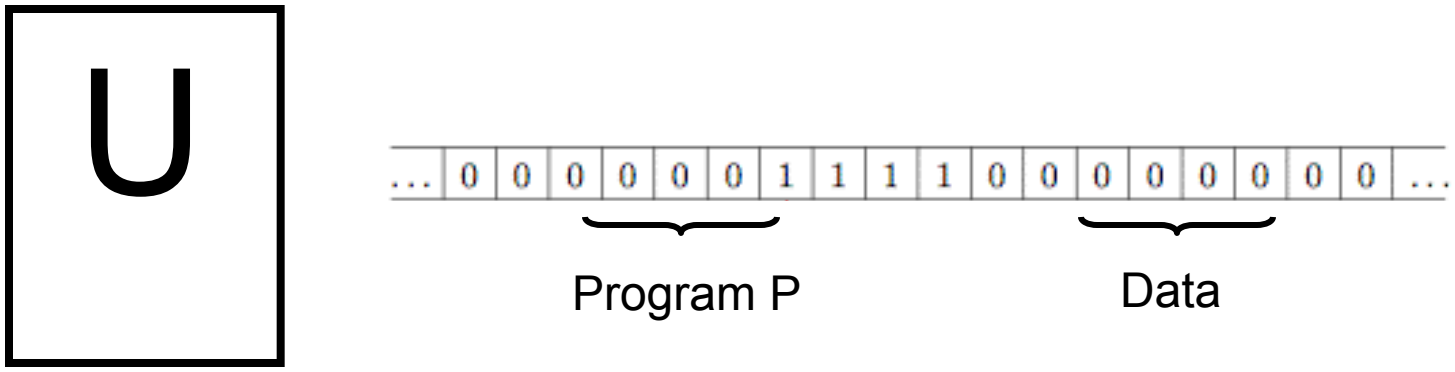


But can have -





Universal Program U

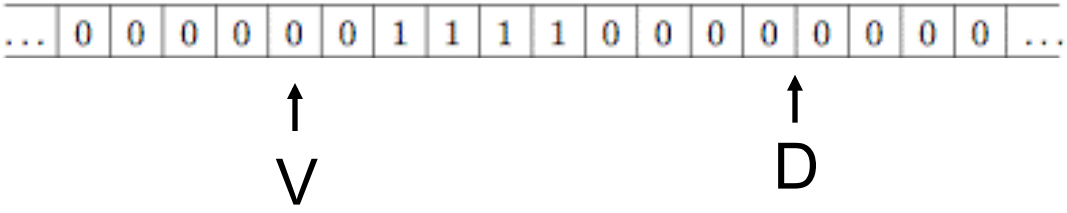


- U "simulates" what P would do on that data

(Sometimes also known as "interpreter")

Automated Bug Checking Revisited

Halting Problem



Let P = program such that code(P) = V.
Does P halt on data D?

IDEAS???

Trivial Idea: Simulate P using universal program U.
If P halts, will eventually detect.

Problem: But if P never halts, neither does the simulation.

Next Week: Halting Problem is unsolvable by another program

Read this proof in the Davis article, and try to understand.

Ponder the meaning of "Proof by contradiction."
How convincing is such a proof?

"When something's not right, it's wrong…" -Bob Dylan

Homework for next week will be posted this afternoon.
Includes: Write a Turing-Post program that
prints the bit sequence 101 infinitely often,
as well as its binary code

Thurs: Digital audio (guest: Prof. Fiebrink)