“It ain’t no good if it ain’t snappy enough.” (Efficient Computations)

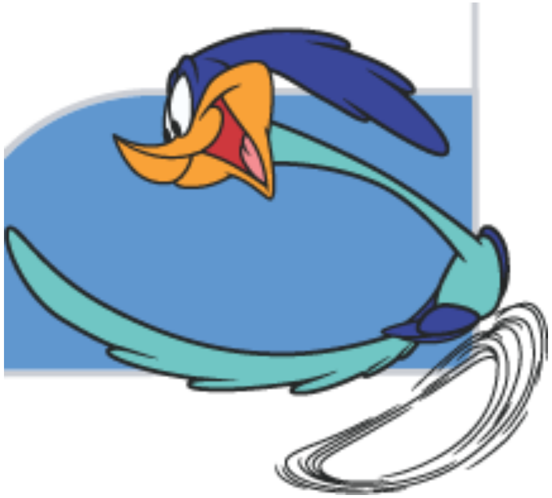
COS 116, Spring 2012
Adam Finkelstein

Today’s focus: efficiency in computation

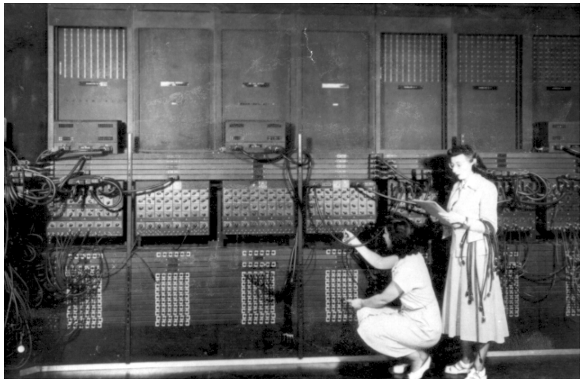
“If it is worth doing, it is worth doing well, and fast.”

Recall: our model of “computation”: pseudocode

Question:
How do we measure the "speed" of an algorithm?



- Ideally, should be independent of:
  - machine
  - technology



“Running time” of an algorithm

- Definition: the number of “elementary operations” performed by the algorithm

- Elementary operations: +, -, *, /, assignment, evaluation of conditionals

  (discussed also in pseudocode handout)

“Speed” of computer: number of elementary operations it can perform per second (Simplified definition)

- Do *not* consider this in “running time” of algorithm; technology-dependent.

Example: Find Min

- n items, stored in array A
- Variables are i, $best$
- $best \leftarrow 1$
- Do for $i = 2$ to n

  {

  if ($A[\ i\ ] < A[best]$) then

  { $best \leftarrow i$ }

  }

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- How many operations executed **before** the loop?
  - A: 0 B: 1 C: 2 D: 3

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- How many operations **per iteration** of the loop?
  - A: 0 B: 1 C: 2 D: 3

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- Do for $i = 2$ to n

  {

  if ($A[\ i\ ] < A[best]$) then

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  }

- How many **times** does the loop run?
  - A: n B: n+1 C: n-1 D: 2n

"iterations"

Example: Find Min

- n items, stored in array A
- Variables are i, $best$
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  {

  if ($A[\ i\ ] < A[best]$) then

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  }

1 assignment & 1 comparison
= 2 operations per loop iteration

Uses at most $2(\underbrace{n - 1}_{\text{Number of iterations}}) + 1$ operations (roughly = 2n)

Initialization

Efficiency of Selection Sort

Do for $i = 1$ to $n - 1$

{

Find cheapest bottle among those numbered i to n

Swap that bottle and the i' th bottle.

}

About $2(n - i)$ steps

3 steps

- For the i' th round, takes at most $2(n - i) + 3$
- To figure out running time, need to figure out how to sum
  $(n - i)$ for $i = 1$ to $n - 1$
  …and then double the result.

Gauss's trick : Sum of $(n - i)$ for $i = 1$ to $n - 1$

$$
\begin{array}{rcccccccccc}
 & S = & 1 & + & 2 & + \ldots + & (n-2) & + & (n-1) \\
+ & S = & (n-1) & + & (n-2) & + \ldots + & 2 & + & 1 \\
\hline
 & 2S = & n & + & n & + \ldots + & n & + & n
\end{array}
$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{n - 1 \text{ times}}$$

$$2S = n(n - 1)$$

- So total time for selection sort is $\leq n(n - 1) + 3n$

Discussion Time

"20 Questions":

I have a number between 1 and a million in mind.
Guess it by asking me yes/no questions,
and keep the number of questions small.

Question 1: "Is the number bigger than half a million?" No

Question 2: "Is the number bigger than a quarter million?" No

Strategy: Each question halves the range of possible answers.

Pseudocode: Guessing number from1 to n

```
Lower ← 1
Upper ← n
Found ← 0
Do while (Found=0)
 {
   Guess ←Round( (Lower + Upper)/2 )
   If (Guess = True Number)
            {
            Found ← 1
            Print(Guess)
            }
    If (Guess < True Number)
            {
             Lower ← Guess
            }
    else
            {
            Upper← Guess
            }
}
```

Binary Search

How many times does the loop run??

Brief detour: Logarithms (CS view)

- $\log_2 n = K$ means $2^{K\text{-}1} < n \leq 2^K$
- In words: K is the number of times you need to divide n by 2 in order to get a number ≤ 1

John Napier

n	16	1024	1048576	8388608
$\log_2 n$	4	10	20	23

Running times encountered in this lecture

	n= 8	n= 1024	n= 1048576	n=8388608
$\log_2 n$	3	10	20	23
n	8	1024	1048576	8388608
n^2	64	1048576	1099511627776	70368744177664

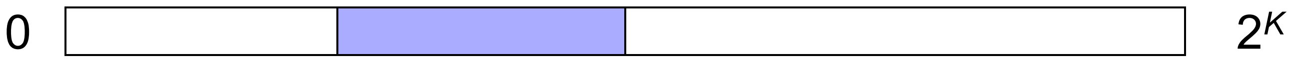
Efficiency really makes a difference!

Next….

"There are only 10 types of people in the world – those who know binary and those who don't."

Binary search and binary representation of numbers

- Say we know $0 \le \text{number} < 2^K$



Is $2^K / 2 \le$ number $< 2^K$?

No

Yes

Is $2^K / 4 \le$ number $< 2^K / 2$?

No

Yes

Is $2^K \times 3/8 \le$ number $< 2^K / 2$?

No

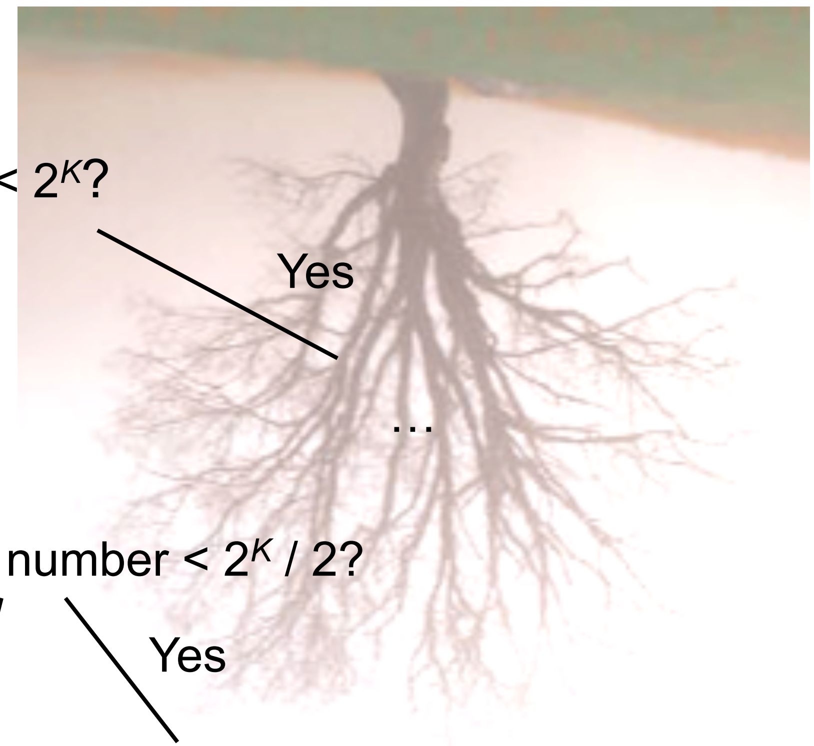
Yes

…

…

Binary representations (cont' d)

- In general, each number can be uniquely identified by a sequence of yes/no answers to these questions.
- Correspond to paths down this "tree":



Is $2^K / 2 \le$ number $< 2^K$?

No → Is $2^K / 4 \le$ number $< 2^K / 2$?

- No → Is $2^K / 8 \le$ number $< 2^K / 4$?
  - No → …
  - Yes → …
- Yes → Is $2^K \times 3/8 \le$ number $< 2^K / 2$?
  - No → …
  - Yes → …

Yes → …

Binary representation of *n*
(the more standard definition)

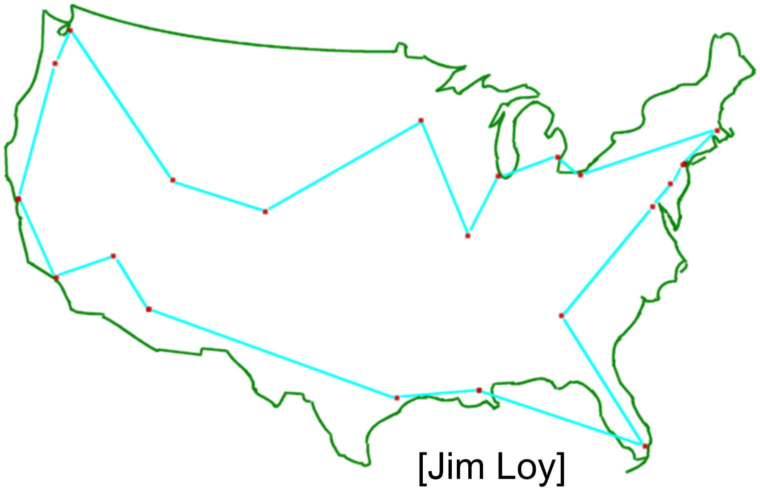
$$n = 2^k \mathrm{b}_k + 2^{k-1} b_{k-1} + \ldots + 2\, b_2 + b_1$$

where the b' s are either 0 or 1)

The binary representation of n is:

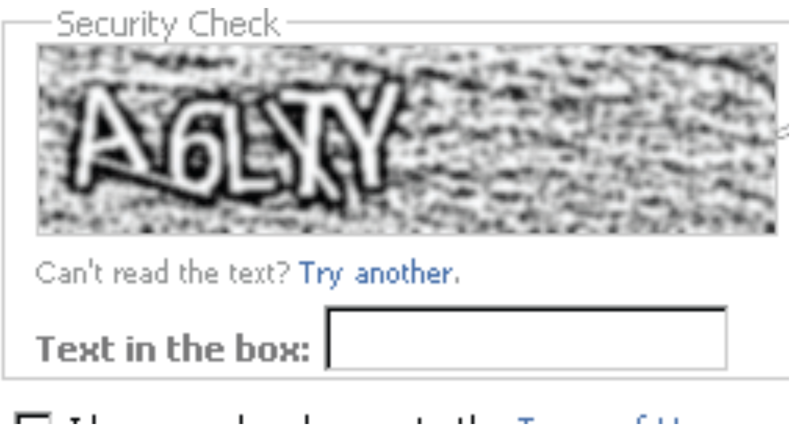
$$\lfloor n \rfloor_2 = b_k\, b_{k-1} \ldots b_2\, b_1$$

Efficiency of Effort: A lens on the world



[Jim Loy]

- QWERTY keyboard
- “UPS Truck Driver’s Problem” (a.k.a. Traveling Salesman Problem or TSP)
- CAPTCHA’s
- Quantum computing



Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

SIAM J. Computing 26(5) 1997

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Can n particles do 2^n "operations" in a single step?
Or is Quantum Mechanics not quite correct?

Computational efficiency has a bearing on physical theories.