COS 426
Computer Graphics
Princeton University`
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Feb 11, 2011
Topics

• Morphing

[Beier 1992]

• Bilateral Filtering

[Paris 2008]
Morphing

• Beier and Neely, 1992:
  – Align facial features
  – Blend colors
Morphing

- Beier and Neely, 1992:
  - Align facial features
  - Blend colors

Why align features?
Morphing

- Beier and Neely, 1992:
  - Align facial features
  - Blend colors

Why align features?
Morphing: Align Features

• Associate primitives: e.g. lines
Morphing: Align Features

• Associate primitives: e.g. lines

• Move primitives so that they align (at some intermediate location) -> warp accordingly
Morphing: Align Features

• A simple case: 1 image, 1 primitive:
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

Where this pixel should come from?

Original Image

Intermediate Location

P'Q'
PQ
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

1. Find local coordinates: $u,v$
   $X = u \cdot PQ + v \cdot PQ^\perp$
Morphing: Align Features

• A simple case: 1 image, 1 primitive:

1. Find local coordinates: \( u, v \)
   \[ X = P + u \cdot PQ + v \cdot PQ^\perp \]

2. Location in original image: \( X' = P' + u \cdot P'Q' + v \cdot P'Q'^\perp \)
Morphing: Align Features

• Multiple lines?

– Find $X' = X + w_1 \cdot d_1 + w_2 \cdot d_2$
Morphing: Align Features

• Multiple lines?

\[ X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \]

- Find \( X' = X + w_1 \cdot d_1 + w_2 \cdot d_2 \)

\[ = 0 \quad \neq 0 \]

Line 1 did not move
Morphing: Align Features

• Multiple lines?

\[
X' = X + w_1 \cdot d_1 + w_2 \cdot d_2
\]

\[\approx 0.6 \approx 0 \approx 0.4 \neq 0\]

Line 1 is longer and closer
Morphing

• Beier and Neely, 1992:
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Topics

• Morphing

[Beier 1992]

• Bilateral Filtering

[Paris 2008]
Bilateral Filtering

Taken from
SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Input

Gaussian Blur

Bilateral Filtering
Bilateral Filtering

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Edge-preserving

Input

Gaussian Blur

Bilateral Filtering
Bilateral Filtering

• How?
Bilateral Filtering

- How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]
Bilateral Filtering

- How?

\[ h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]

Filtered value at pixel x
Bilateral Filtering

- How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]

Filtered value at pixel x
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{Ka} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

- Filtered value at pixel \( x \)
- Later
- Go over every pixel \( \xi \) in image
- Value at a pixel
- Is pixel close to \( x \)?
- Is pixel similar to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel x

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) \, d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

- How?

\[ h(x) = \frac{1}{Kd} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\( \approx 1 \)

\( ? \)

\( 0 \)
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\( x \)

\( \xi \)
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\( \approx 1 \)

\( \approx 1 \)

\( \approx 0 \)

EDGE!
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Later

Is pixel close to \( x \)?

Is pixel similar to \( x \)?
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\[ \approx 0 \quad \text{(FAR!)} \]

\[ \approx 0 \]

\[ \approx 1 \]
Bilateral Filtering

• How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel \( x \)

Later

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?
Bilateral Filtering

- How?

\[ h(x) = \frac{1}{K} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\( \approx 1 \)

\( \approx 0 \)

FAR!

\( \approx 0 \)

EDGE!
Bilateral Filtering

• How?

\[ h(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Later

Filtered value at pixel \( x \)

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel close to \( x \)?

Is pixel similar to \( x \)?

\[ c(\xi - x) = e^{-\frac{1}{2} \left( \frac{\|\xi - x\|}{\sigma_d} \right)} \]

\[ s(\xi - x) = e^{-\frac{1}{2} \left( \frac{\|f(\xi) - f(x)\|}{\sigma_r} \right)} \]
Bilateral Filtering

• How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Go over every pixel $\xi$ in image

Value at a pixel

Is pixel close to $x$?

Is pixel similar to $x$?

Normalization:

$$k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) d\xi$$
Bilateral Filtering

• In Practice?

\[ h(x) = \frac{1}{kd} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi \]

Filtered value at pixel x

Go over every pixel \( \xi \) in image

Value at a pixel

Is pixel similar to \( x \)?

Is pixel close to \( x \)?

Normalization
Questions?

- Morphing

[Beier 1992]

- Bilateral Filtering

[Paris 2008]