COS 426
Computer Graphics
Princeton University`

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Feb 11, 2011

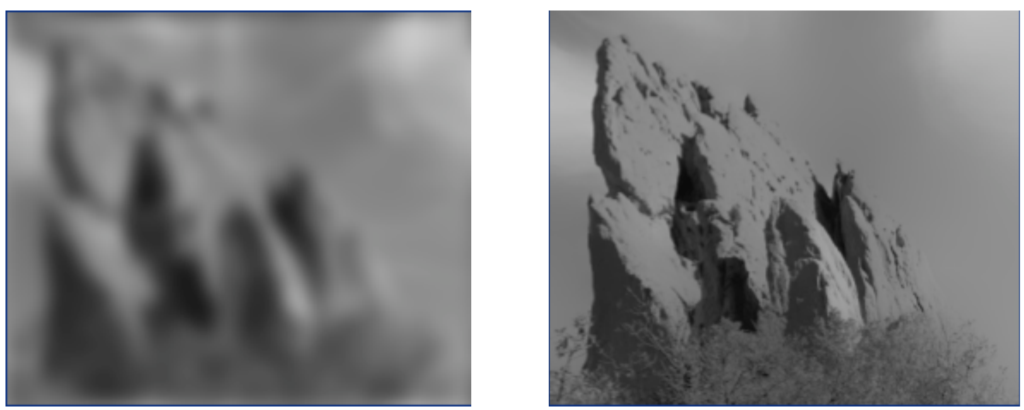
Topics

- **Morphing**

[Beier 1992]

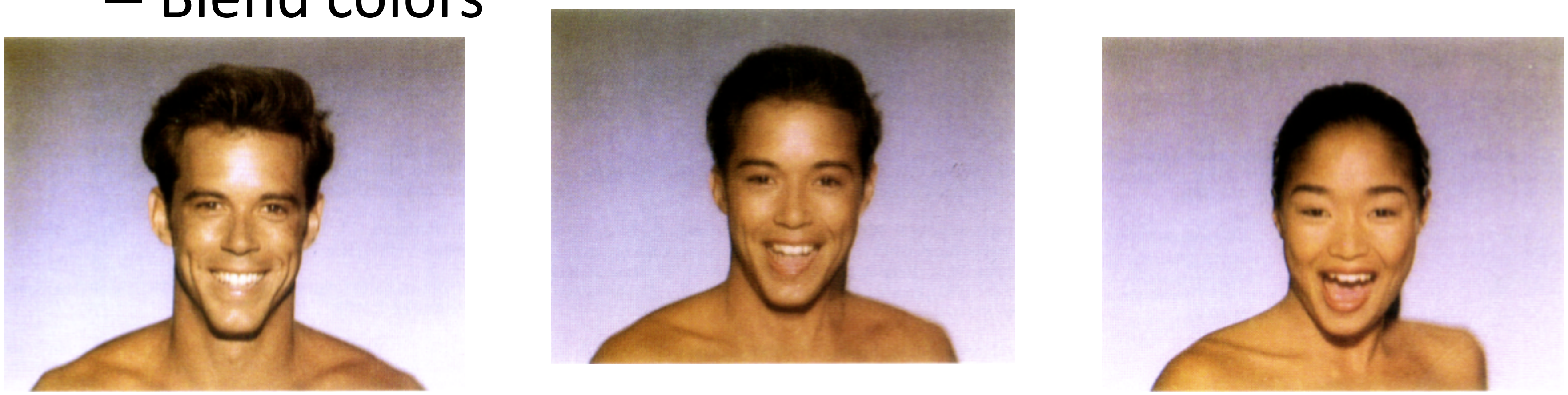
- Bilateral Filtering

[Paris 2008]



Morphing

- **Beier and Neely, 1992:**
  - Align facial features
  - Blend colors



Morphing

- **Beier and Neely, 1992:**
  - **Align facial features**
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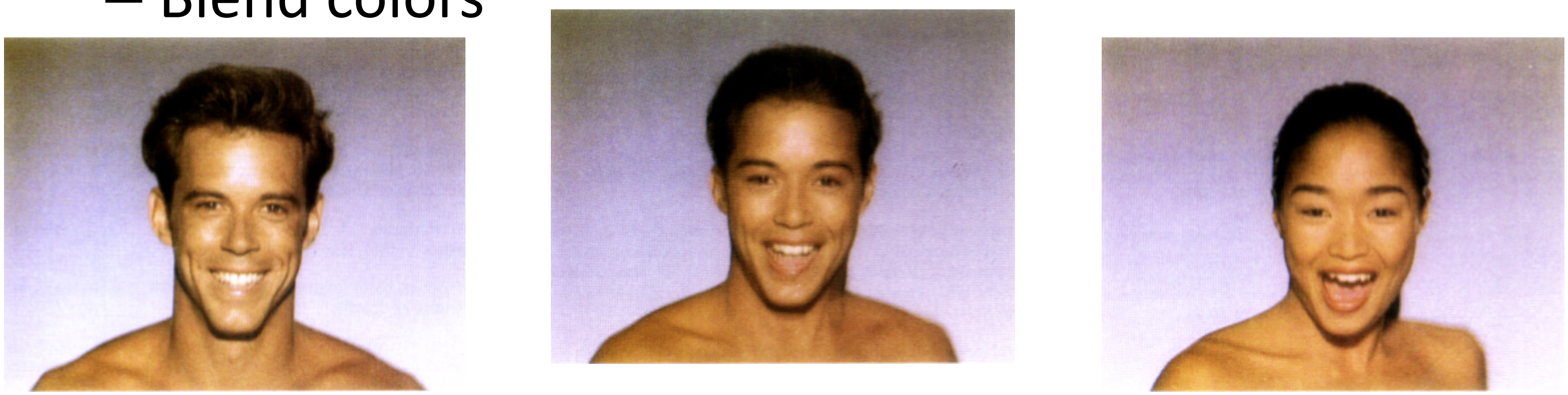

Why align features?

Morphing

- **Beier and Neely, 1992:**
  - **Align facial features**
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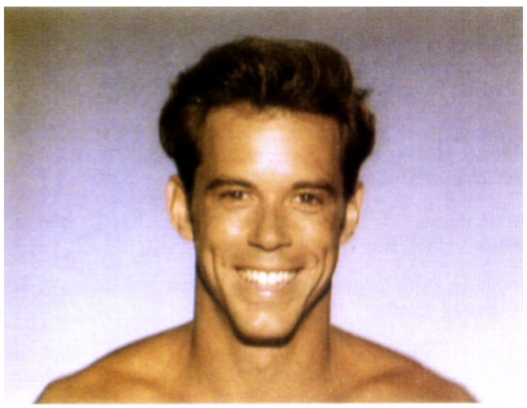
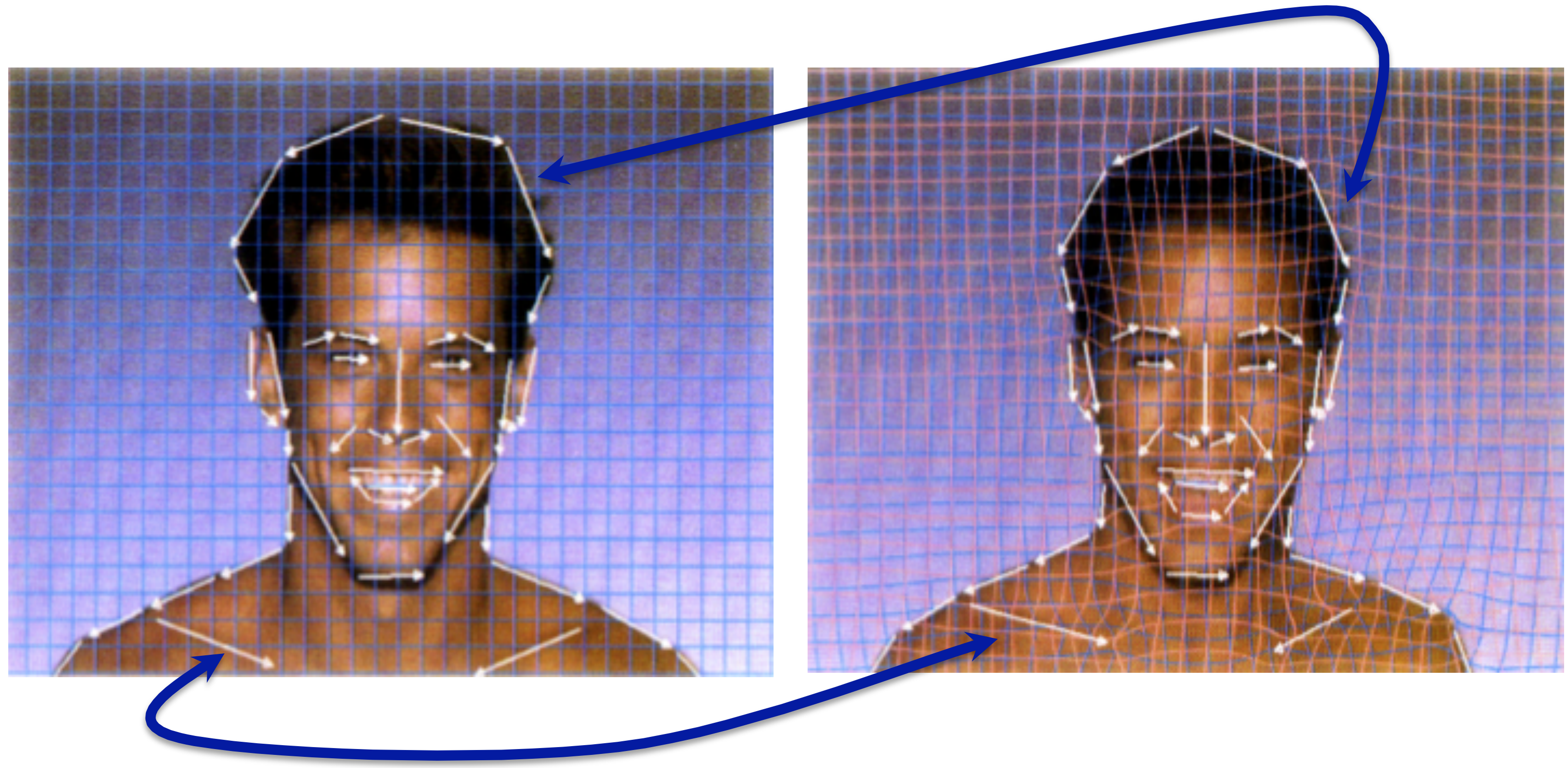


Why align features?

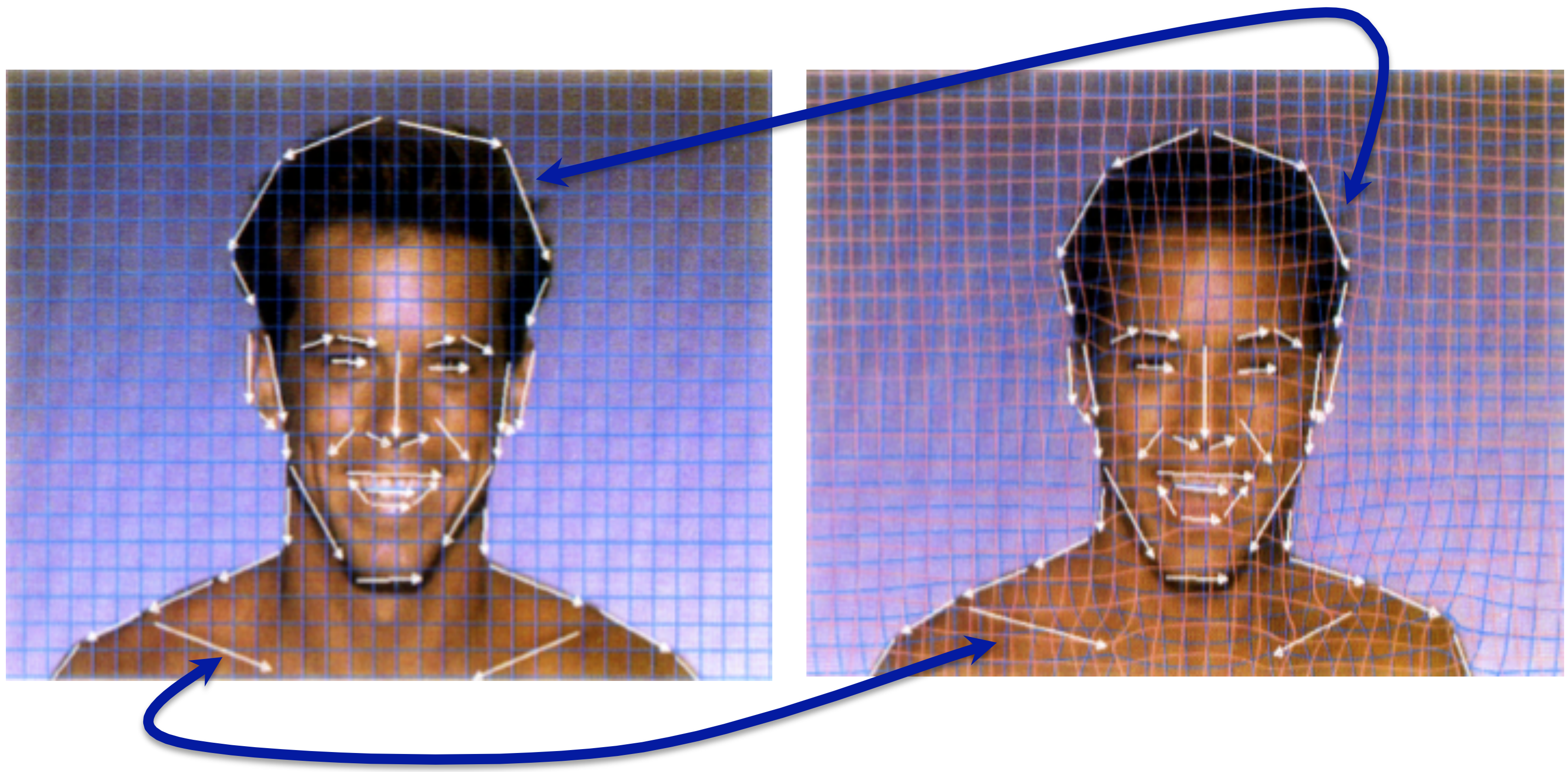
Morphing: Align Features

- Associate primitives: e.g. lines



Morphing: Align Features

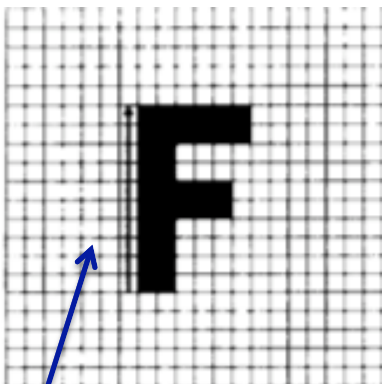
- Associate primitives: e.g. lines



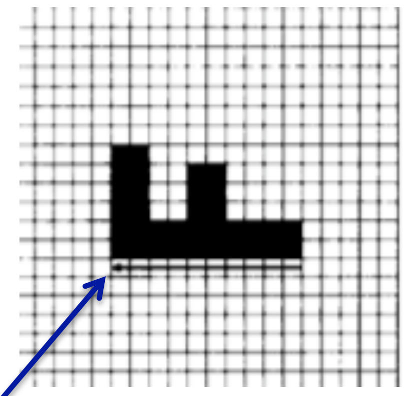
- Move primitives so that they align (at some intermediate location) -> warp accordingly

Morphing: Align Features

- A simple case: 1 image, 1 primitive:



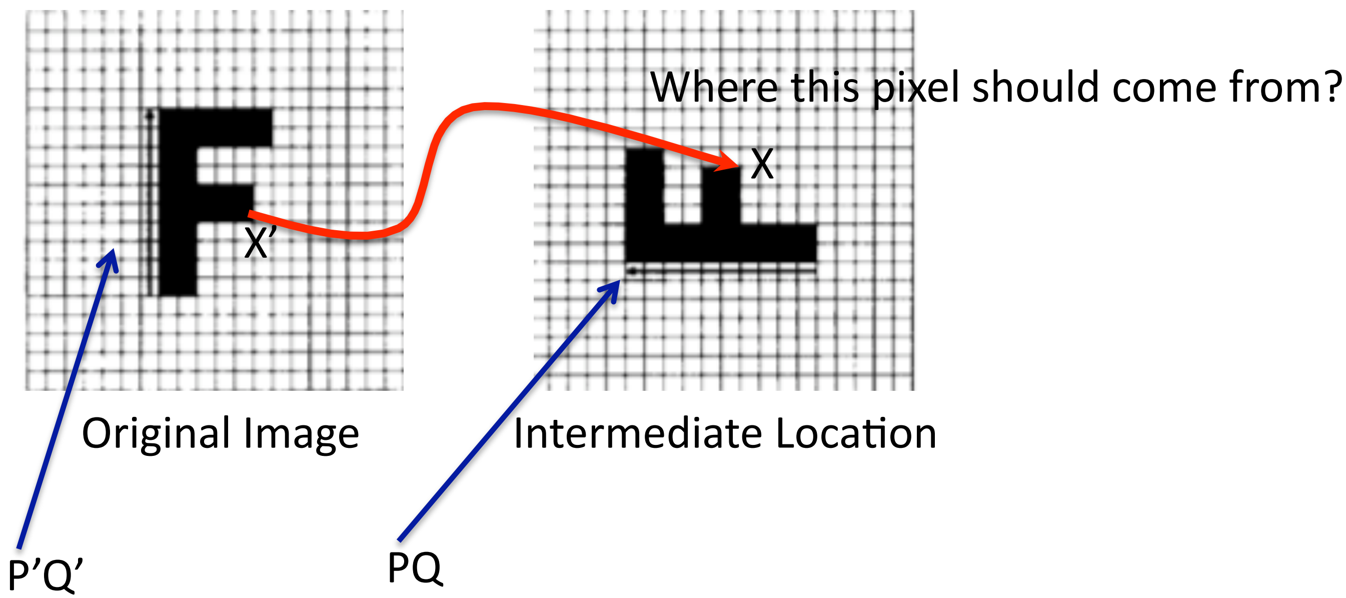
Original Image



Intermediate Location

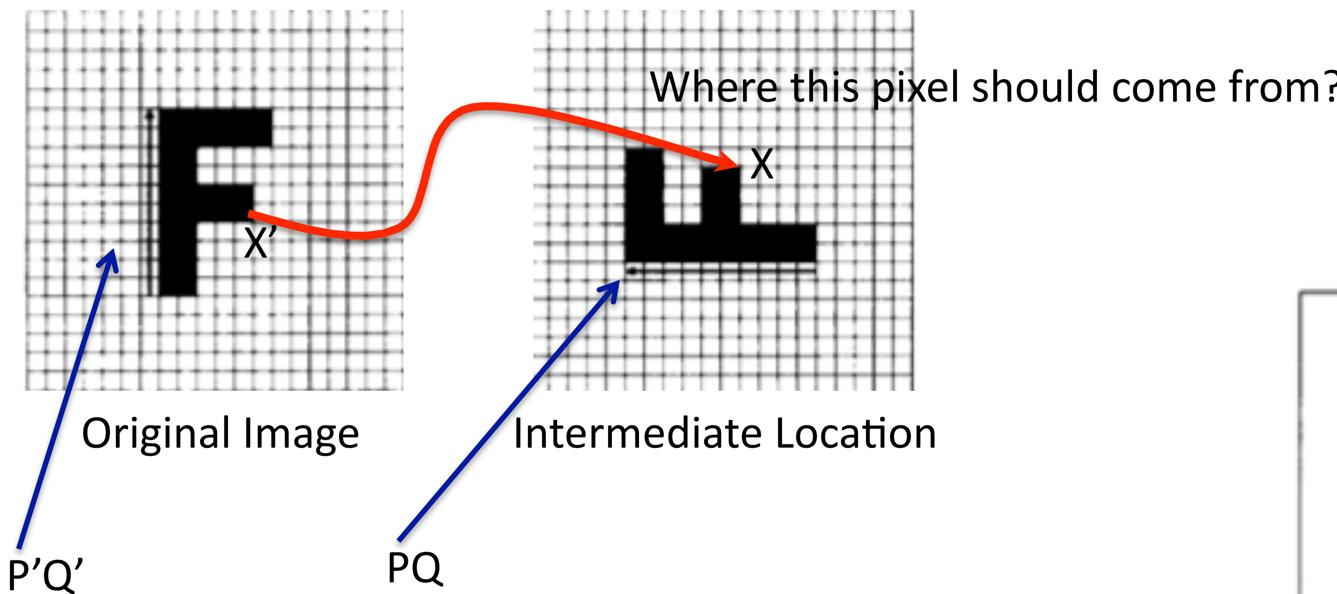
Morphing: Align Features

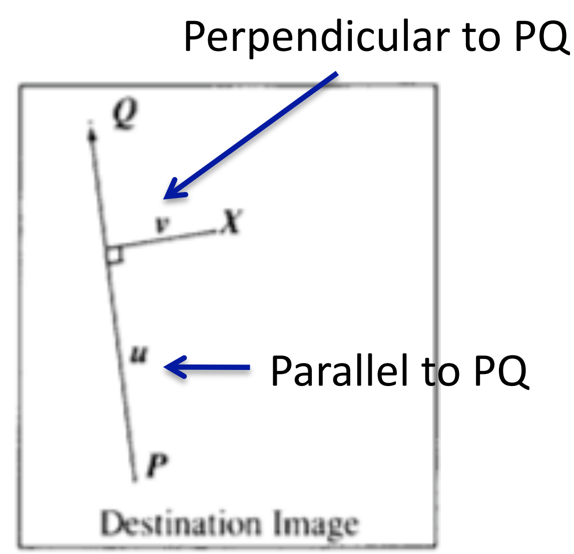
- A simple case: 1 image, 1 primitive:



Morphing: Align Features

- A simple case: 1 image, 1 primitive:



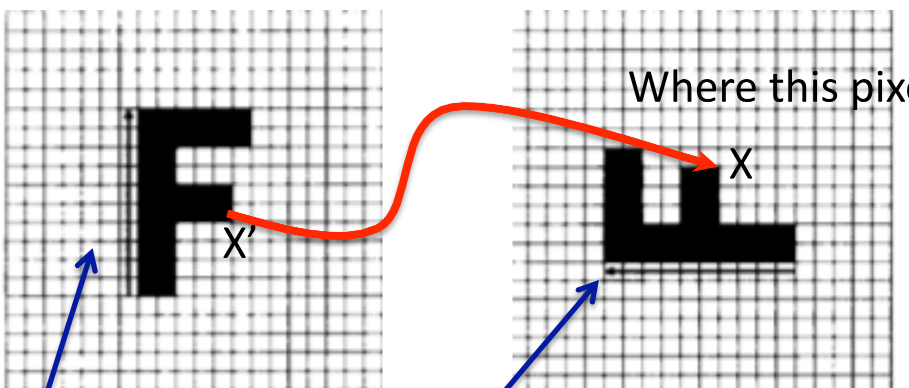


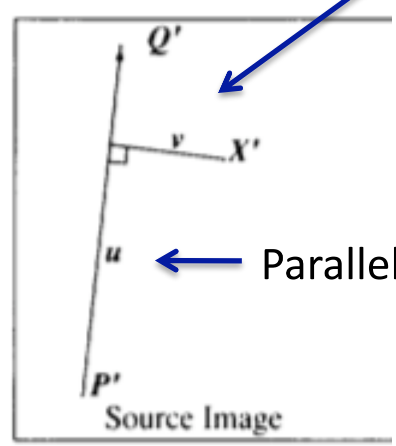
1. Find local coordinates: u,v

   $X = u \bullet PQ + v \bullet PQ^{\perp}$

Morphing: Align Features

- A simple case: 1 image, 1 primitive:

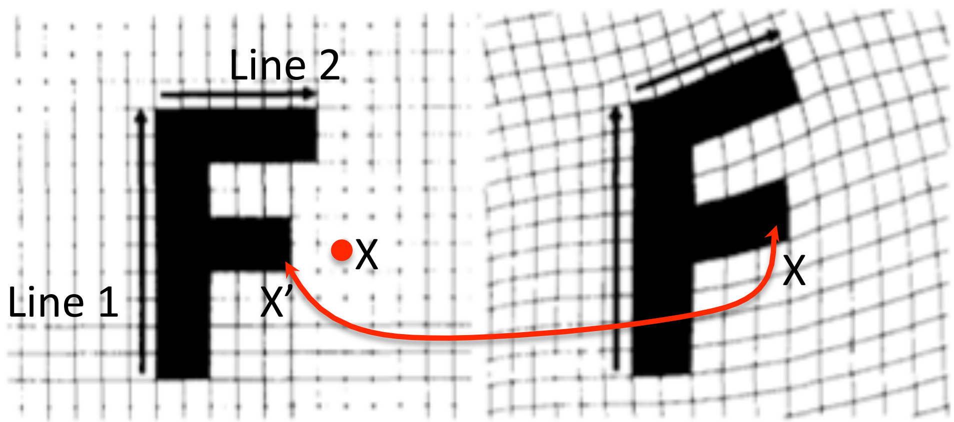




1. Find local coordinates: u,v
   $X = P + u \bullet PQ + v \bullet PQ^{\perp}$
2. Location in original image: $X' = P' + u \bullet P'Q' + v \bullet P'Q'^{\perp}$

Morphing: Align Features

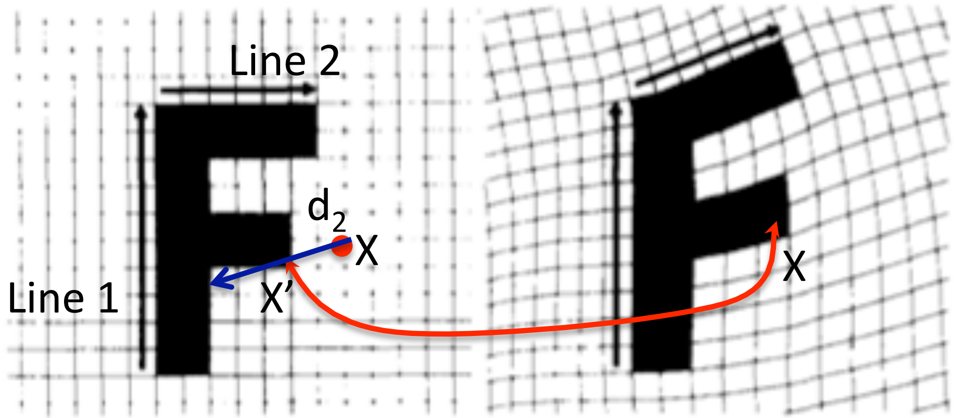
- Multiple lines?



  - Find $X' = X + w_1 \bullet d_1 + w_2 \bullet d_2$

Morphing: Align Features

- Multiple lines?

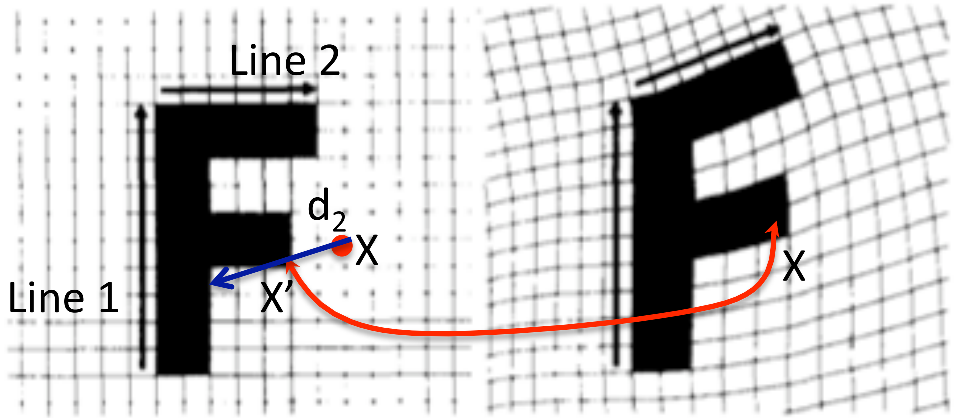


  - Find $X' = X + w_1 \bullet \underbrace{d_1}_{=0} + w_2 \bullet \underbrace{d_2}_{\neq 0}$

Line 1 did not move

Morphing: Align Features

- Multiple lines?

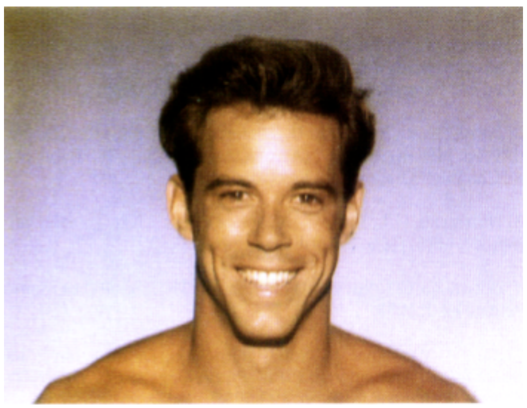


  – Find $X' = X + \underset{\approx .6}{\underline{w_1}} \bullet \underset{=0}{\underline{d_1}} + \underset{\approx .4}{\underline{w_2}} \bullet \underset{\neq 0}{\underline{d_2}}$

Line 1 is longer and closer

Morphing

- **Beier and Neely, 1992:**
  - Align facial features
  - **Blend colors**



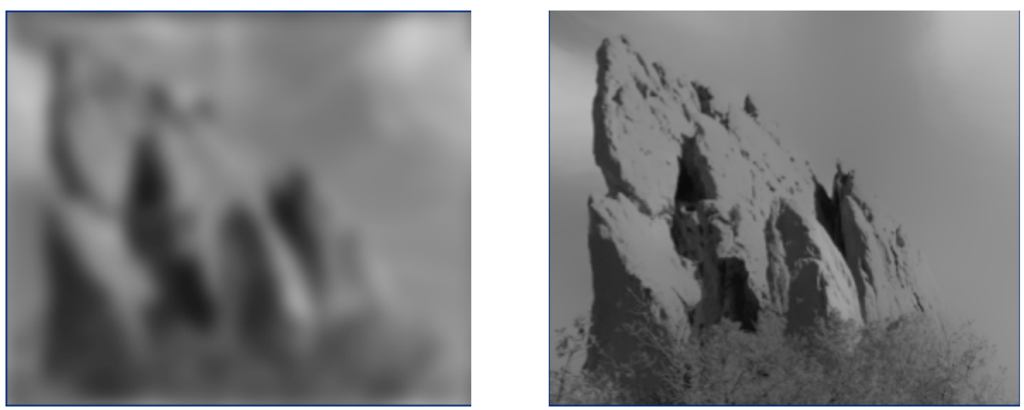
Topics

- Morphing

[Beier 1992]

- **Bilateral Filtering**

[Paris 2008]



Bilateral Filtering

Input

Taken from
SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Gaussian Blur

Bilateral Filtering

Bilateral Filtering

Input

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SIGGRAPH 2008 Course
http://people.csail.mit.edu/sparis/bf_course/

Edge-preserving

Gaussian Blur

Bilateral Filtering

Bilateral Filtering

- How?

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Bilateral Filtering

- How?

$$\underline{h(x)} = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value
at pixel x

Bilateral Filtering

- How?

$$\underline{h(x)} = \cancel{k_d^{-1}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Bilateral Filtering

- How?

$$\underbrace{h(x)}_{\text{Filtered value at pixel x}} = \underbrace{\cancel{k_d^{-1}}}_{\text{Later}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}_{\text{Go over every pixel } \xi \text{ in image}} f(\xi)c(\xi - x)s(\xi - x)\underline{d\xi}$$

Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

Later

Go over every pixel ξ in image

Value at a pixel

Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Later

Value at a pixel

Go over every pixel ξ in image

Is pixel close to x?

Bilateral Filtering

- How?

$$\underbrace{h(x)}_{\text{Filtered value at pixel x}} = \underbrace{\cancel{k_d^{-1}}}_{\text{Later}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}_{\text{Go over every pixel } \xi \text{ in image}} \underbrace{f(\xi)}_{\text{Value at a pixel}} \underbrace{c(\xi - x)}_{\text{Is pixel close to } x\text{?}} \underbrace{s(\xi - x)}_{\text{Is pixel similar to } x\text{?}} \underline{d\xi}$$

Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

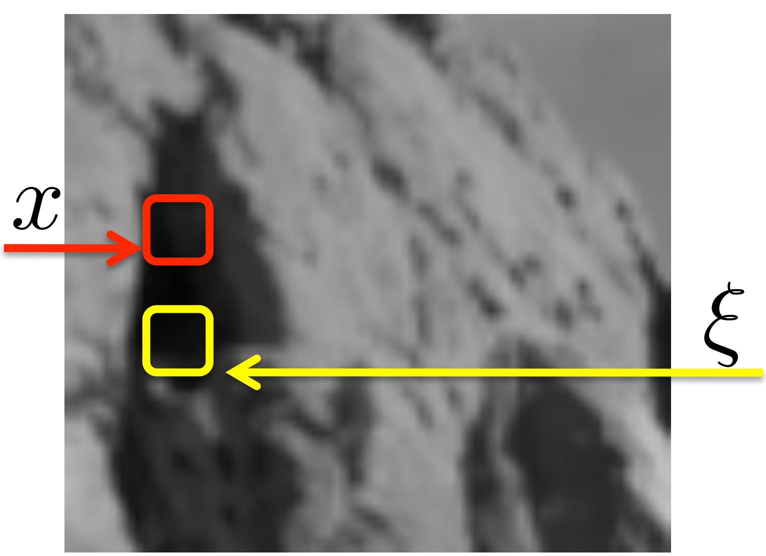
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x?

Is pixel close to x?



Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\,c(\xi - x)\,s(\xi - x)\,d\xi$$

Filtered value at pixel x

Later

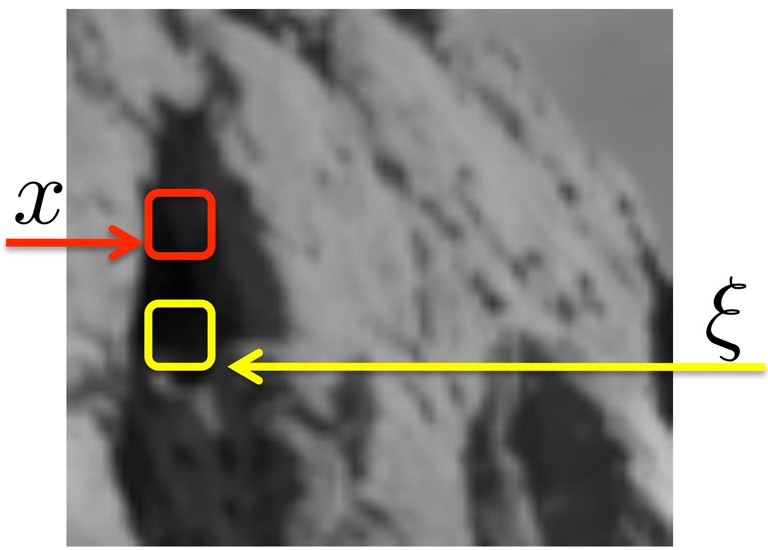
Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x?

Is pixel close to x?

???



Bilateral Filtering

- How?

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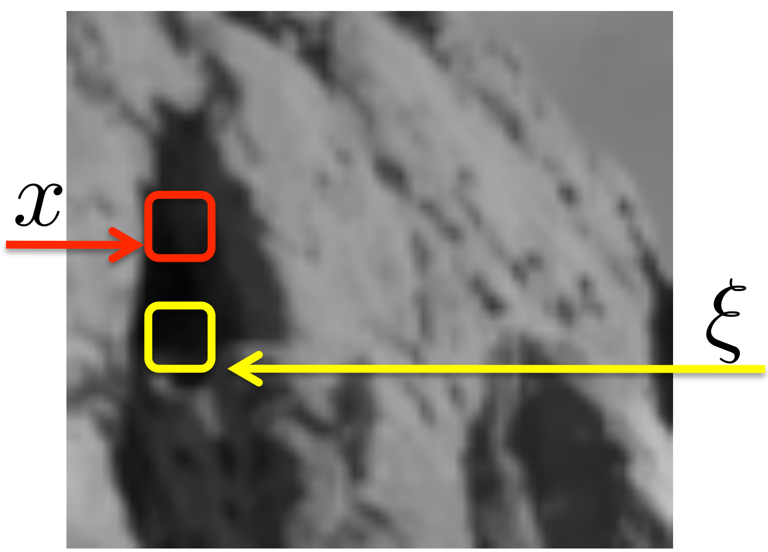
0

???



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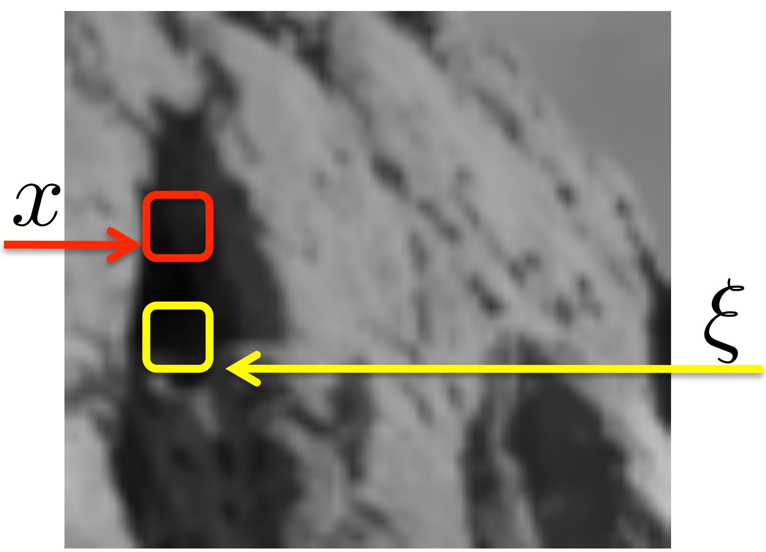

0

≈1

???

Bilateral Filtering

- How?

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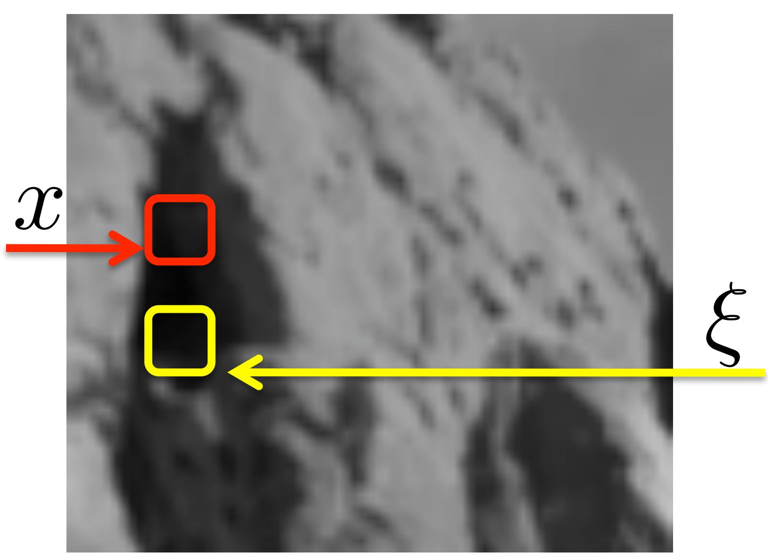
0

≈1

≈1



Bilateral Filtering

- How?

$$h(x) = \cancel{k_d^{-1}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi - x)s(\xi - x)d\xi$$

Filtered value at pixel x

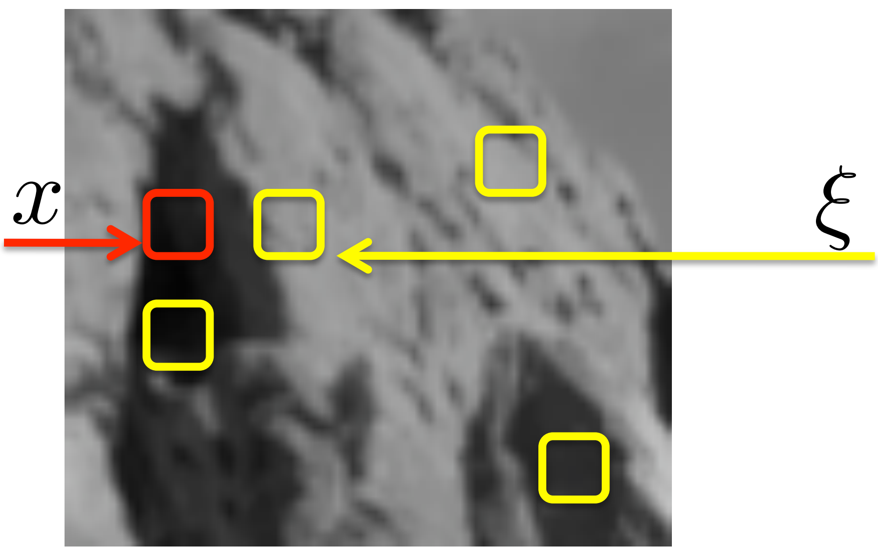
Later

Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x?

Is pixel close to x?



Bilateral Filtering

- How?

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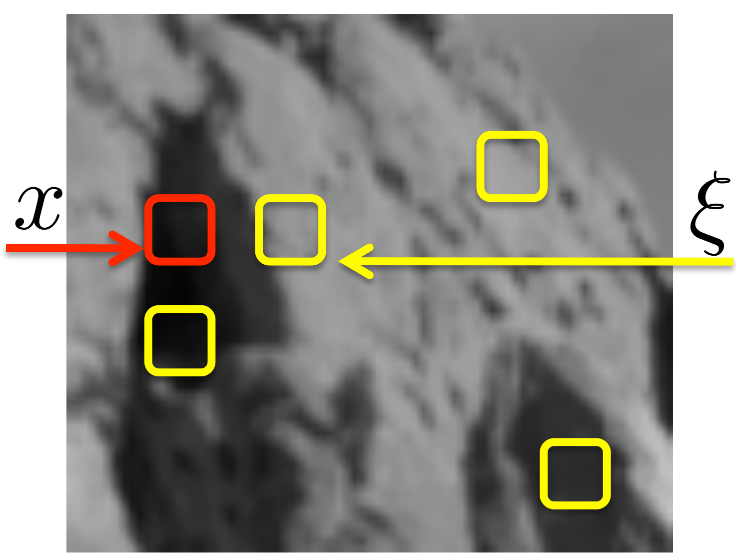
≈1

≈1

≈0

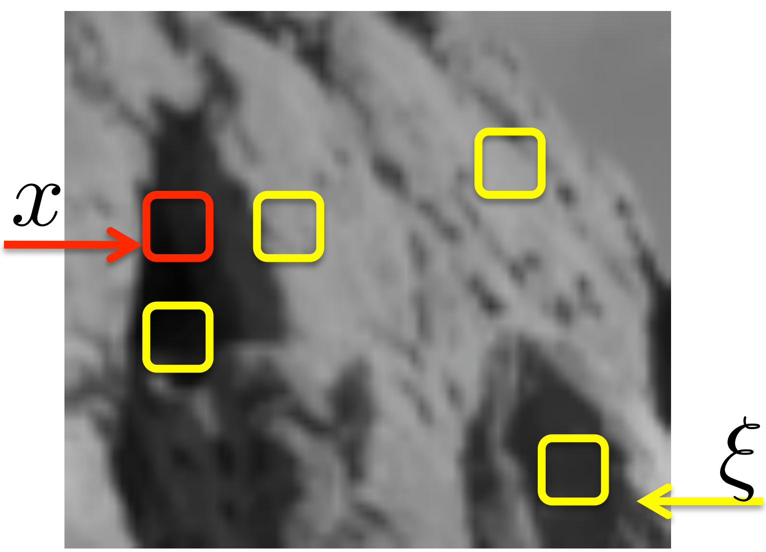
EDGE!



Bilateral Filtering

- How?

$$\underbrace{h(x)}_{\text{Filtered value at pixel x}} = \underset{\text{Later}}{\cancel{k_d^{-1}}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}_{\text{Go over every pixel } \xi \text{ in image}} \underbrace{f(\xi)}_{\text{Value at a pixel}} \underbrace{c(\xi - x)}_{\text{Is pixel close to } x\text{?}} \underbrace{s(\xi - x)}_{\text{Is pixel similar to } x\text{?}} \underline{d\xi}$$



Bilateral Filtering

- How?

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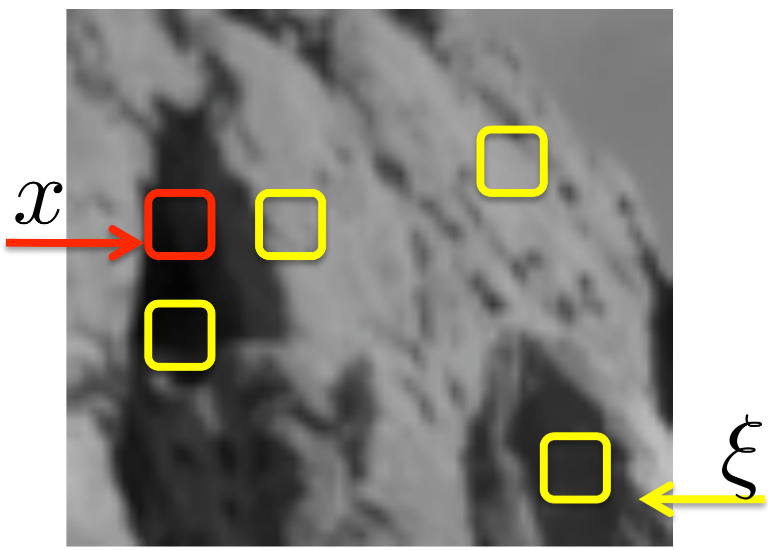


≈0

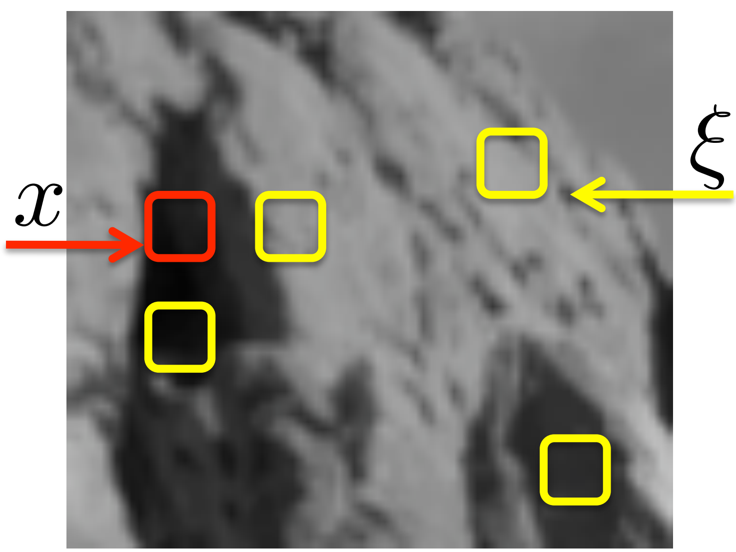
≈0
FAR!

≈1

Bilateral Filtering

- How?

$$\underbrace{h(x)}_{\substack{\text{Filtered value}\\ \text{at pixel x}}} = \underbrace{\cancel{k_d^{-1}}}_{\text{Later}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}_{\text{Go over every pixel } \xi \text{ in image}} \underbrace{f(\xi)}_{\text{Value at a pixel}}\underbrace{c(\xi - x)}_{\text{Is pixel close to } x\text{?}}\underbrace{s(\xi - x)}_{\text{Is pixel similar to } x\text{?}}\underbrace{d\xi}$$



Bilateral Filtering

- How?

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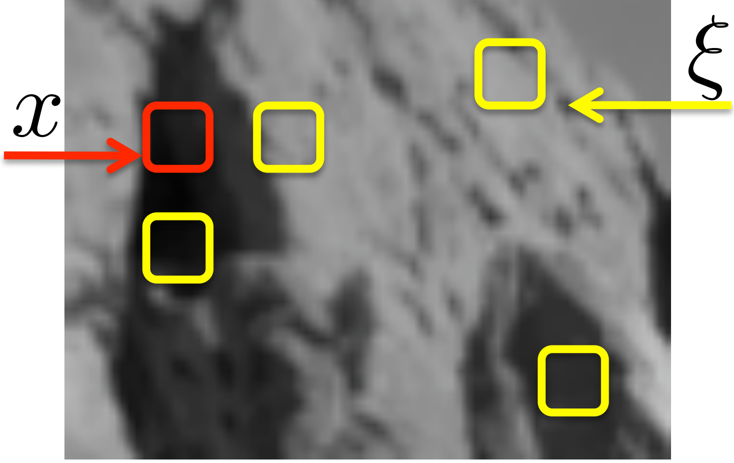
≈1

≈0
FAR!

≈0
EDGE!



Bilateral Filtering

- How?

$$\underbrace{h(x)}_{\text{Filtered value at pixel x}} = \underbrace{\cancel{k_d^{-1}}}_{\text{Later}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}_{\text{Go over every pixel } \xi \text{ in image}} \underbrace{f(\xi)}_{\text{Value at a pixel}} \underbrace{c(\xi - x)}_{\text{Is pixel close to } x\text{?}} \underbrace{s(\xi - x)}_{\text{Is pixel similar to } x\text{?}} \underline{d\xi}$$

$$c(\xi - x) = e^{-\frac{1}{2}\left(\frac{\|\xi - x\|}{\sigma_d}\right)}$$

$$s(\xi - x) = e^{-\frac{1}{2}\left(\frac{\|f(\xi) - f(x)\|}{\sigma_r}\right)}$$

Bilateral Filtering

- How?

$$h(x) = k_d^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi - x) s(\xi - x) d\xi$$

Filtered value at pixel x

Value at a pixel

Go over every pixel ξ in image

Is pixel similar to x?

Is pixel close to x?

Normalization:

$$k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi - x) s(\xi - x) d\xi$$

Bilateral Filtering

- In Practice?

???

???

$$\underbrace{h(x)}_{\text{Filtered value at pixel x}} = \boxed{k_d^{-1}} \underbrace{\boxed{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}}_{\text{Go over every pixel } \xi \text{ in image}} \underbrace{f(\xi)}_{\text{Value at a pixel}} \underbrace{c(\xi - x)}_{} \underbrace{s(\xi - x)}_{} \underbrace{d\xi}_{}$$

Filtered value at pixel x

Go over every pixel ξ in image

Value at a pixel

Is pixel similar to x?

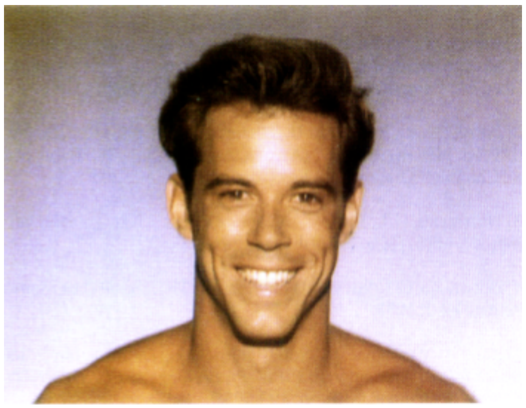
Is pixel close to x?

Normalization

Questions?

- Morphing

[Beier 1992]



- Bilateral Filtering

[Paris 2008]