



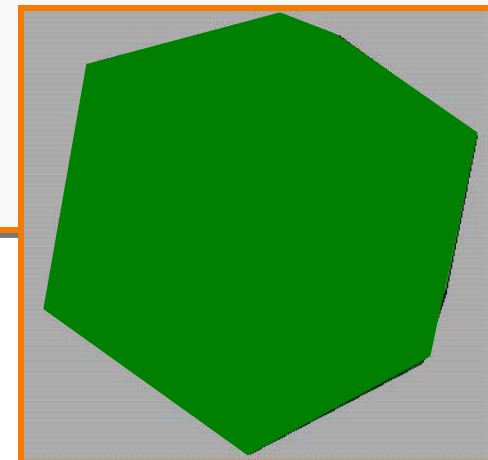
Illumination

COS 426

Ray Casting



```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene, &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

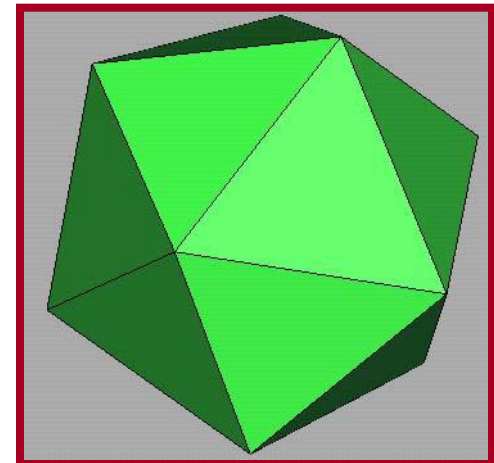


Without Illumination

Ray Casting



```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

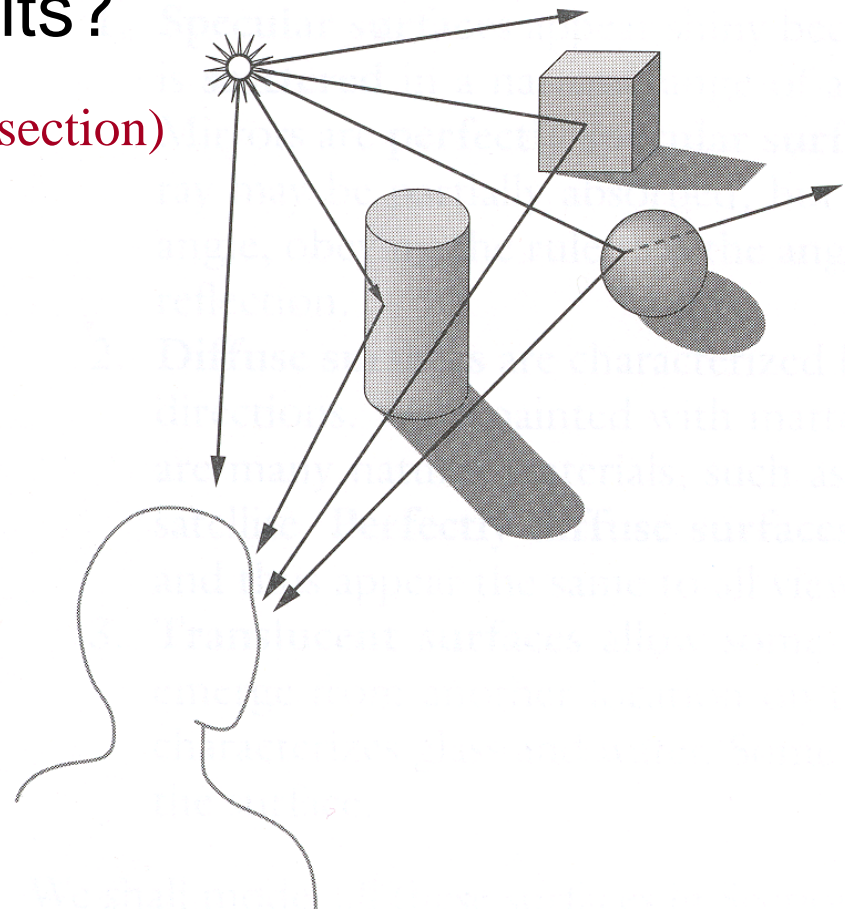


With Illumination

Illumination

- How do we compute radiance for a sample ray once we know what it hits?

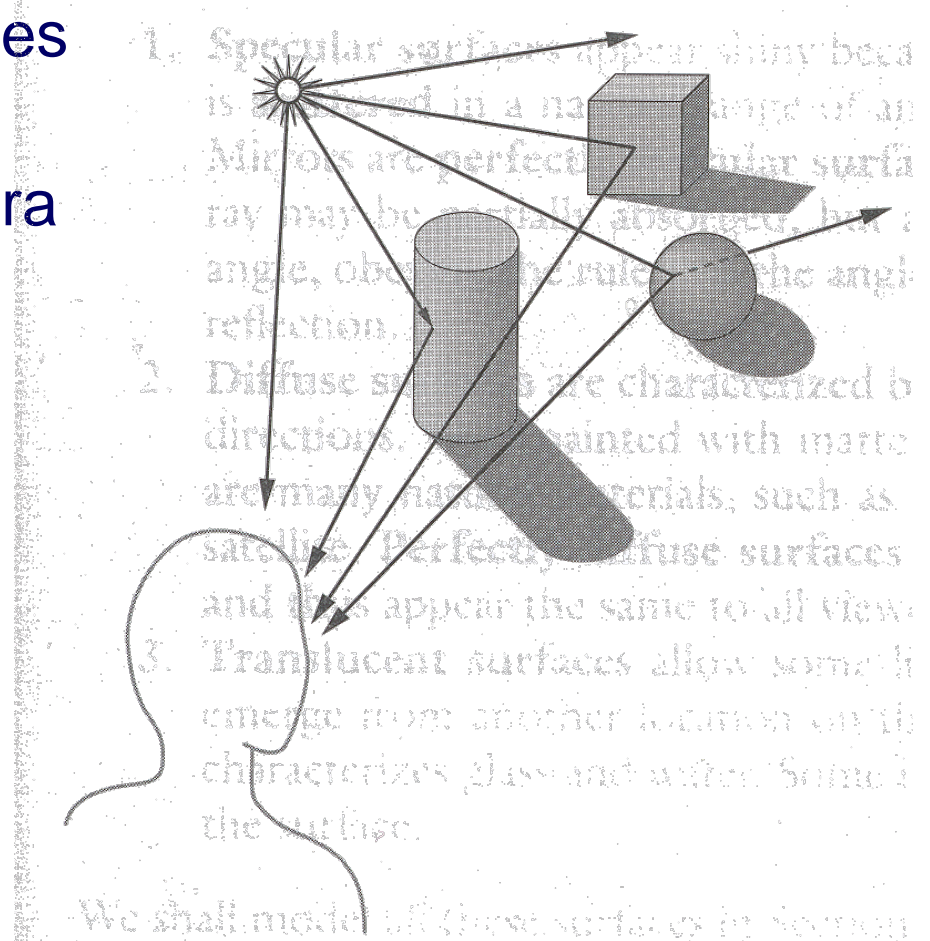
ComputeRadiance(scene, ray, intersection)



Goal

- Must derive computer models for ...
 - Emission at light sources
 - Scattering at surfaces
 - Reception at the camera

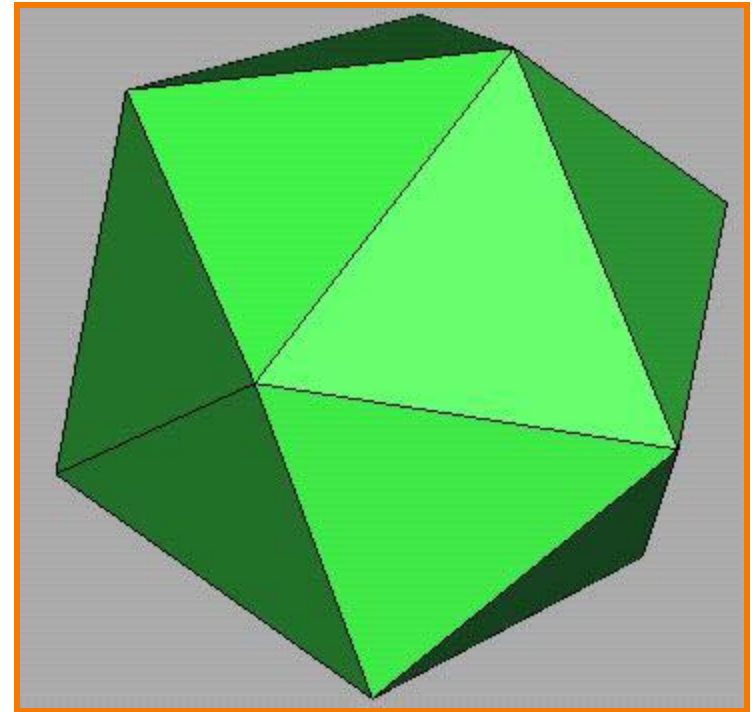
- Desirable features ...
 - Concise
 - Efficient to compute
 - “Accurate”



Overview



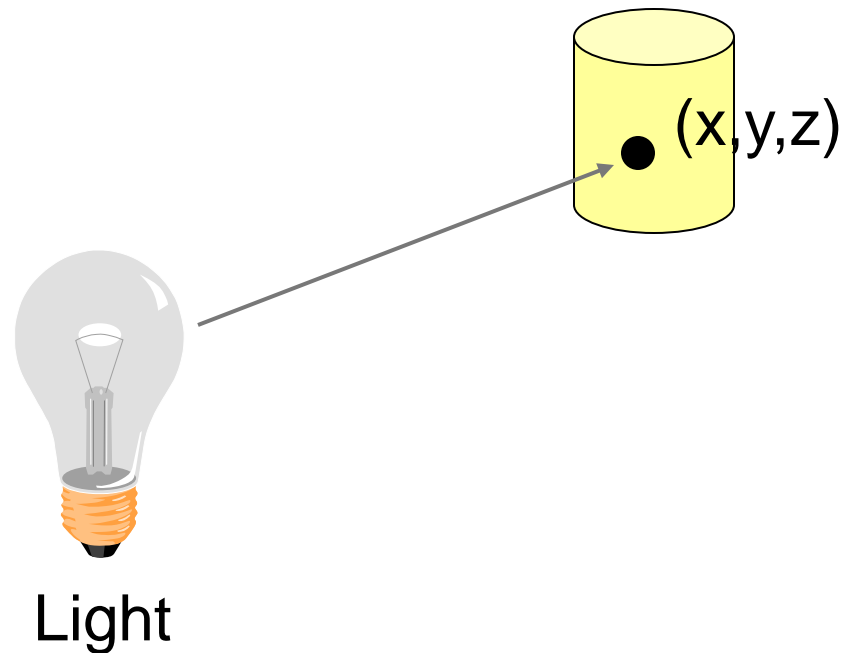
- Direct Illumination
 - Emission at light sources
 - Scattering at surfaces
- Global illumination
 - Shadows
 - Refractions
 - Inter-object reflections



Direct Illumination

Emission at Light Sources

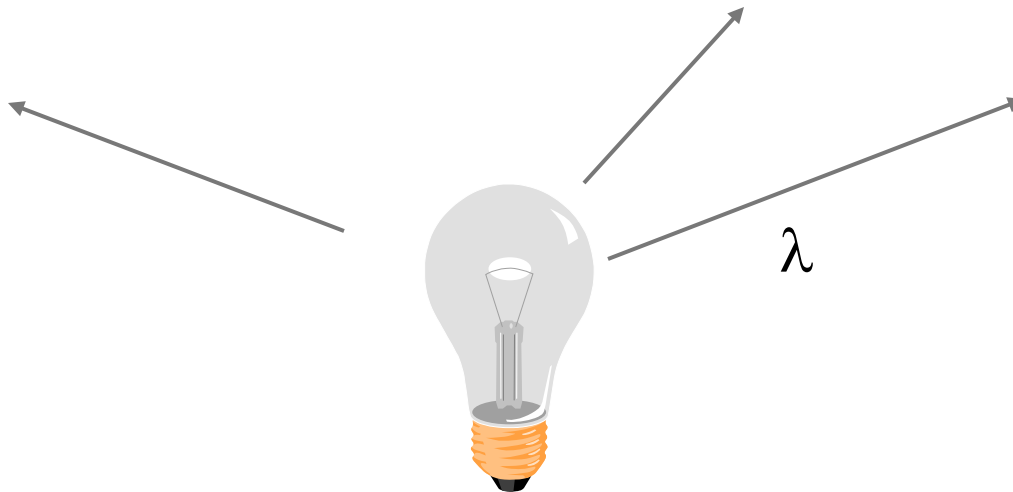
- $I_L(x, y, z, \theta, \phi, \lambda) \dots$
 - describes the intensity of energy,
 - leaving a light source, ...
 - arriving at location (x, y, z) , ...
 - from direction (θ, ϕ) , ...
 - with wavelength λ



Empirical Models



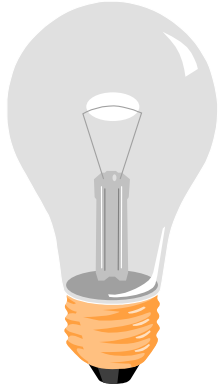
- Ideally measure irradiant energy for “all” situations
 - Too much storage
 - Difficult in practice



OpenGL Light Source Models



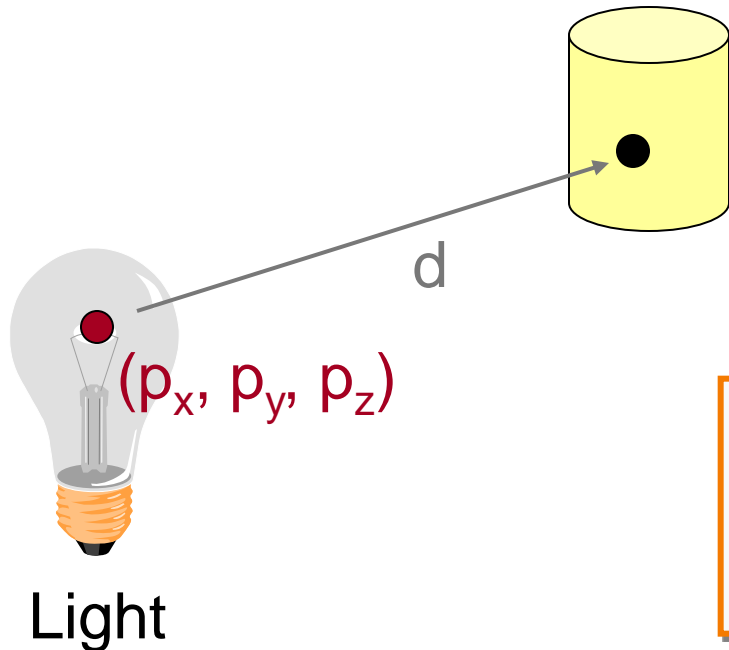
- Simple mathematical models:
 - Point light
 - Spot light
 - Directional light



Point Light Source



- Models omni-directional point source
 - intensity (I_0),
 - position (p_x, p_y, p_z),
 - coefficients (c_a, l_a, q_a) for attenuation with distance (d)

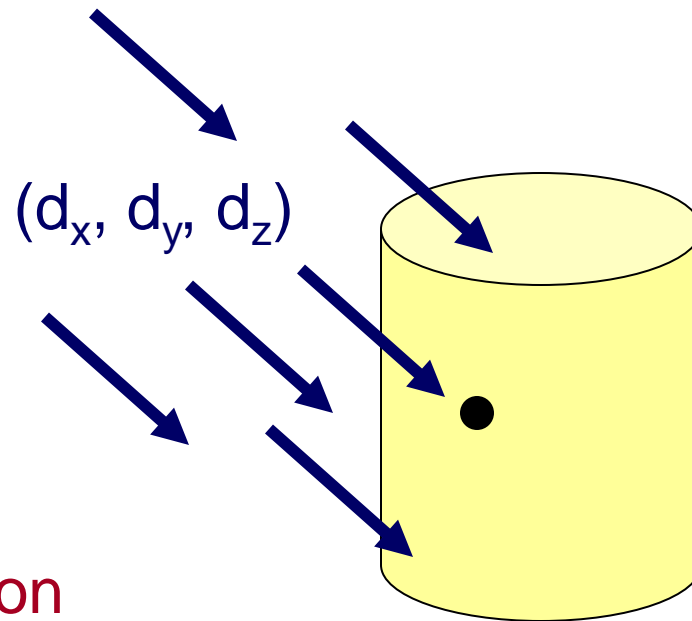


$$I_L = \frac{I_0}{c_a + l_a d + q_a d^2}$$

Directional Light Source



- Models point light source at infinity
 - intensity (I_0),
 - direction (d_x, d_y, d_z)



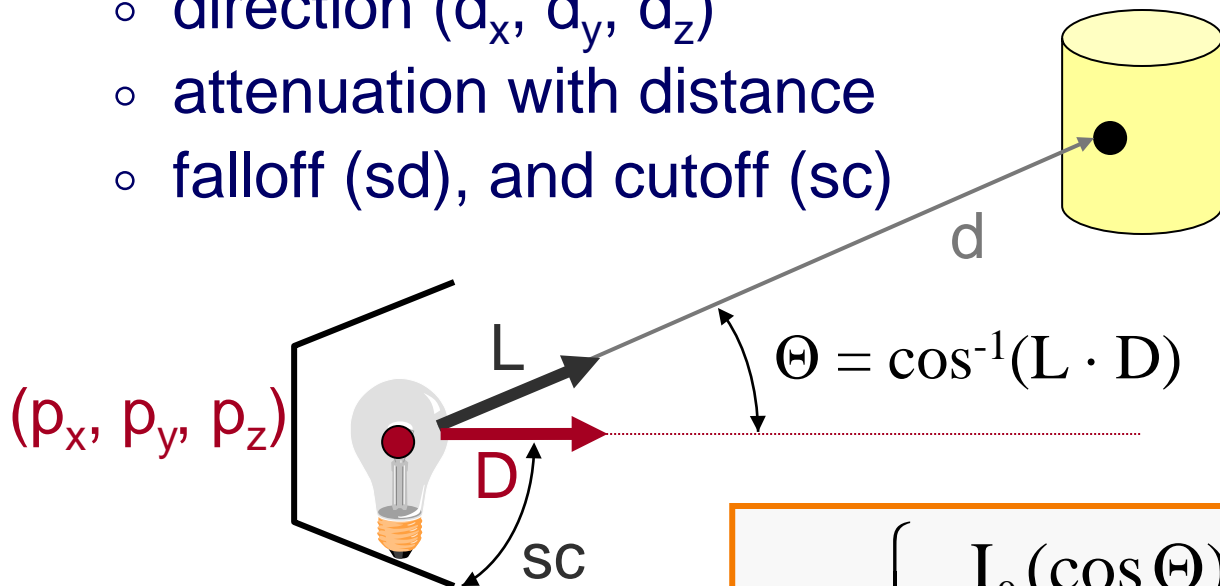
No attenuation
with distance

$$I_L = I_0$$

Spot Light Source



- Models point light source with direction
 - intensity (I_0),
 - position (p_x, p_y, p_z),
 - direction (d_x, d_y, d_z)
 - attenuation with distance
 - falloff (sd), and cutoff (sc)

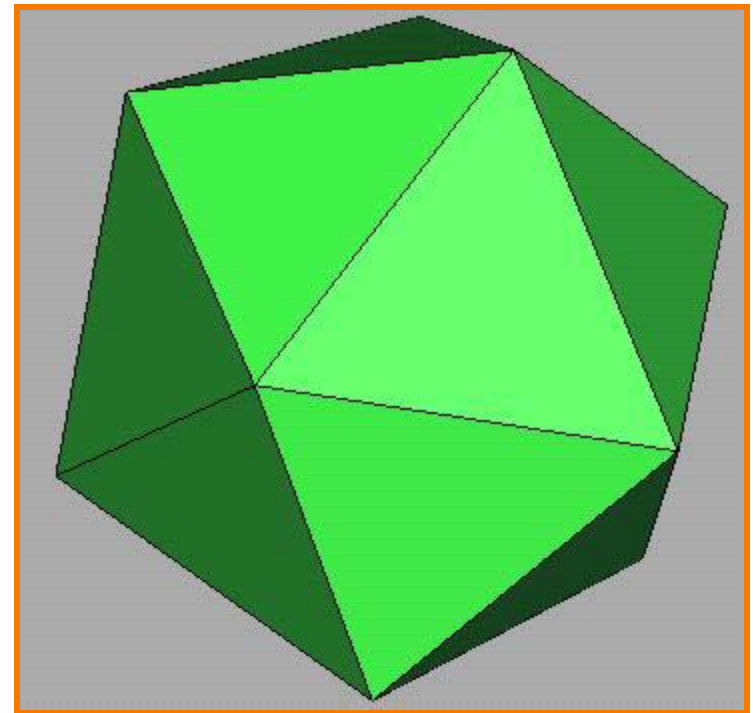


$$I_L = \begin{cases} \frac{I_0 (\cos \Theta)^{sd}}{c_a + l_a d + q_a d^2} & \text{if } \Theta \leq sc, \\ 0 & \text{otherwise} \end{cases}$$

Overview



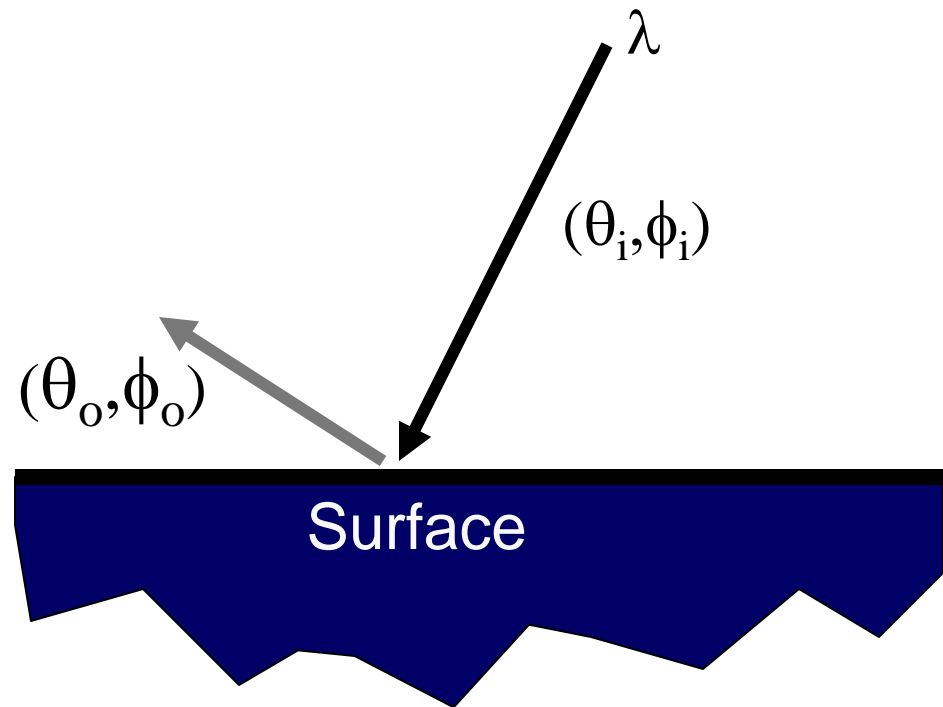
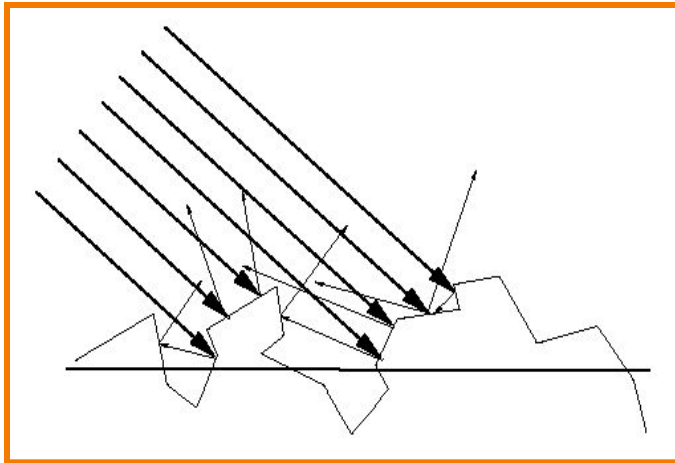
- Direct Illumination
 - Emission at light sources
 - Scattering at surfaces
- Global illumination
 - Shadows
 - Refractions
 - Inter-object reflections



Direct Illumination

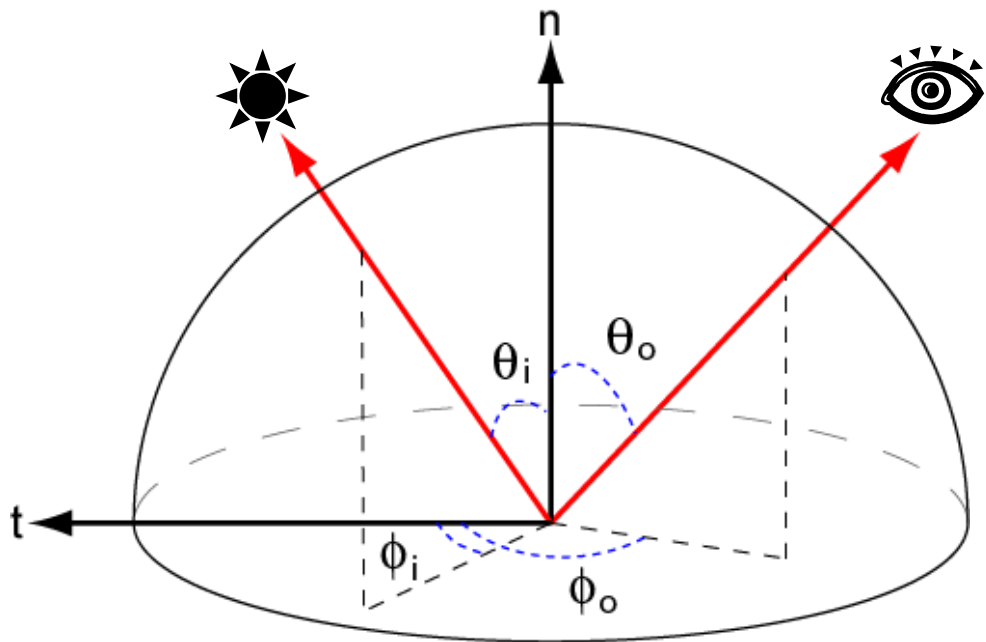
Scattering at Surfaces

- $\text{BRDF}(\theta_i, \phi_i, \theta_o, \phi_o, \lambda)$...
 - describes the fraction of incident energy,
 - arriving from direction (θ_i, ϕ_i) , ...
 - leaving in direction (θ_o, ϕ_o) , ...
 - with wavelength λ



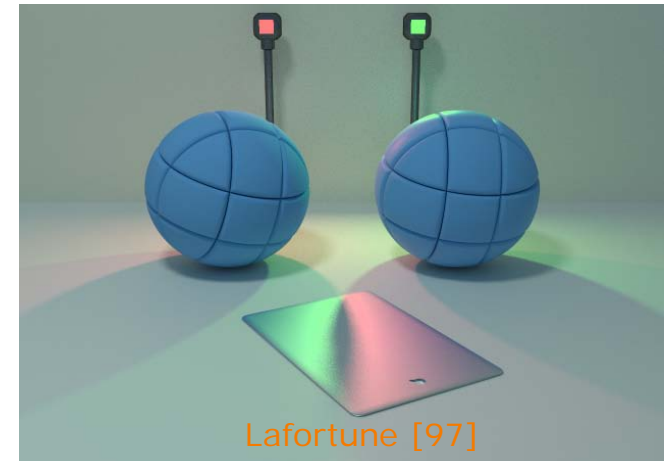
Empirical Models

- Ideally measure BRDF for “all” combinations of angles: $\theta_i, \phi_i, \theta_o, \phi_o$
 - Difficult in practice
 - Too much storage



Parametric Models

- Approximate BRDF with simple parametric function that is fast to compute.
 - Phong [75]
 - Blinn-Phong [77]
 - Cook-Torrance [81]
 - He et al. [91]
 - Ward [92]
 - Lafortune et al. [97]
 - Ashikhmin et al. [00]
 - etc.

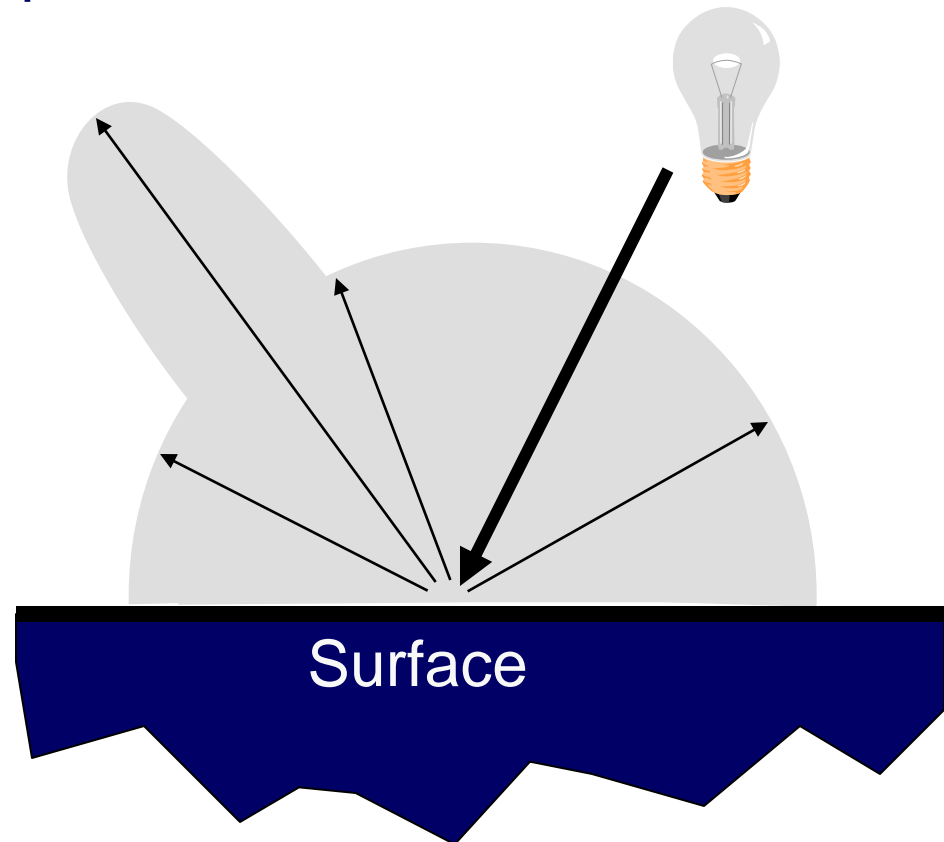


OpenGL Reflectance Model



- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”

Based on model
proposed by Phong

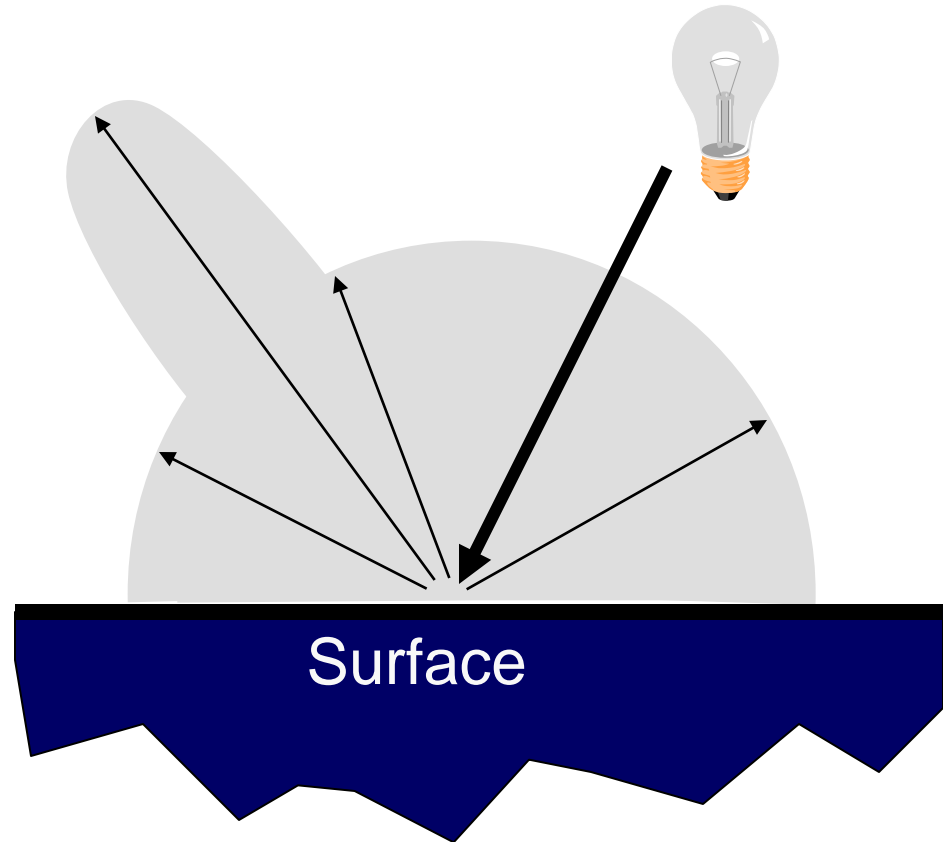


OpenGL Reflectance Model



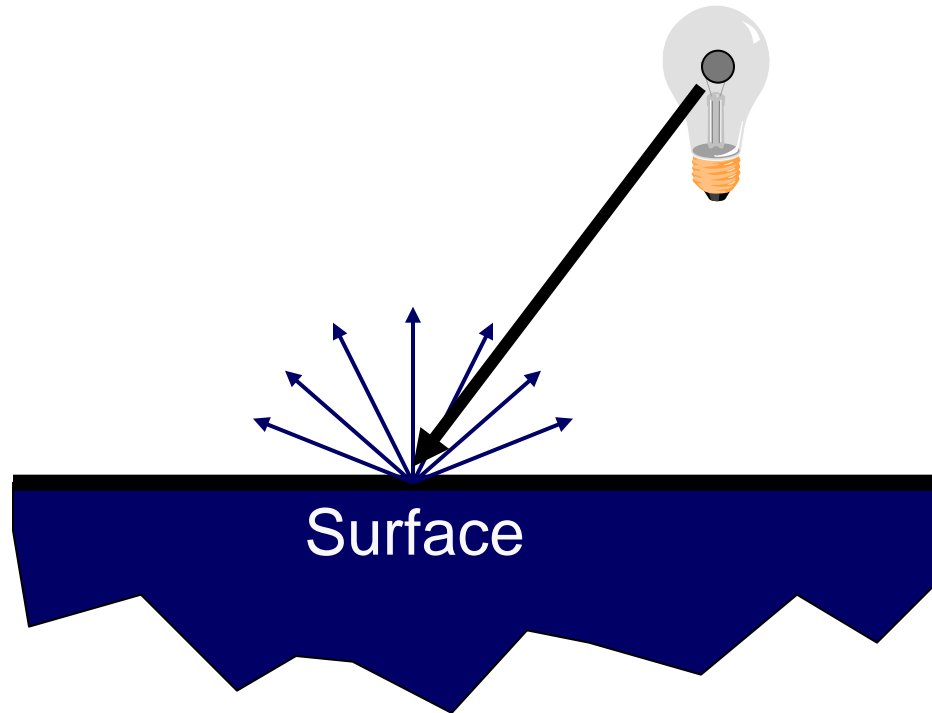
- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”

Based on model
proposed by Phong



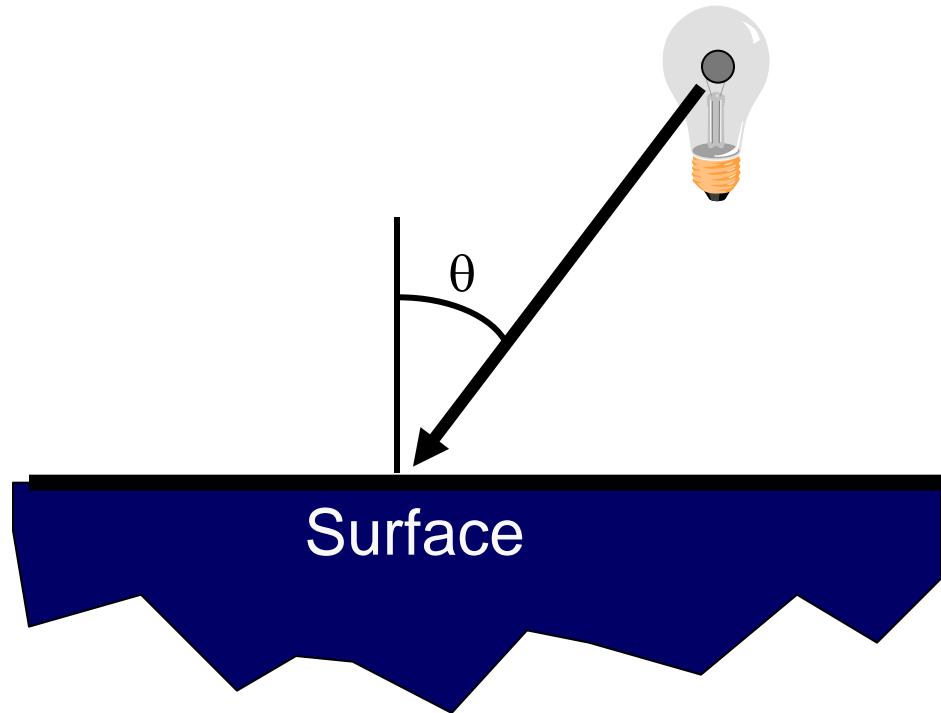
Diffuse Reflection

- Assume surface reflects equally in all directions
 - Examples: chalk, clay



Diffuse Reflection

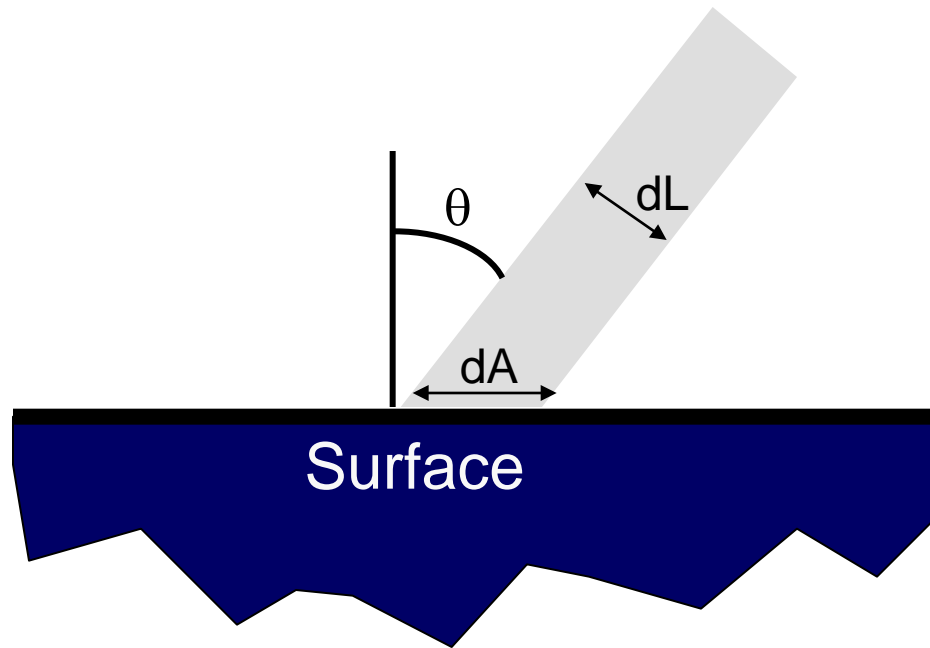
- How much light is reflected?
 - Depends on angle of incident light



Diffuse Reflection

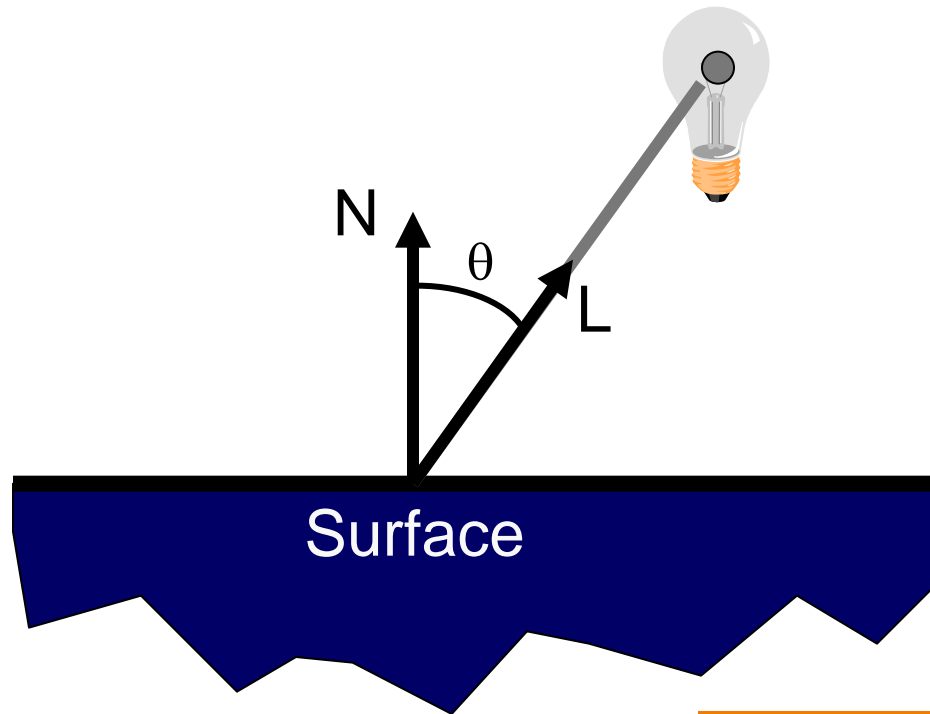
- How much light is reflected?
 - Depends on angle of incident light

$$dL = dA \cos \Theta$$



Diffuse Reflection

- Lambertian model
 - cosine law (dot product)

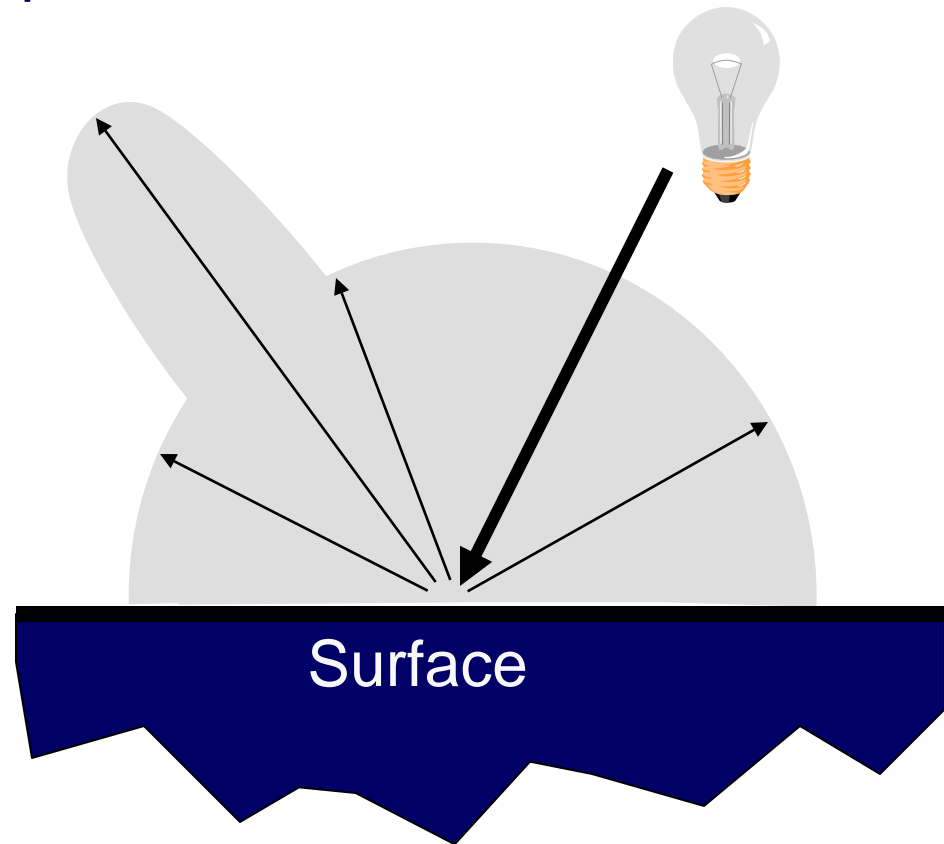


$$I_D = K_D (N \cdot L) I_L$$

OpenGL Reflectance Model



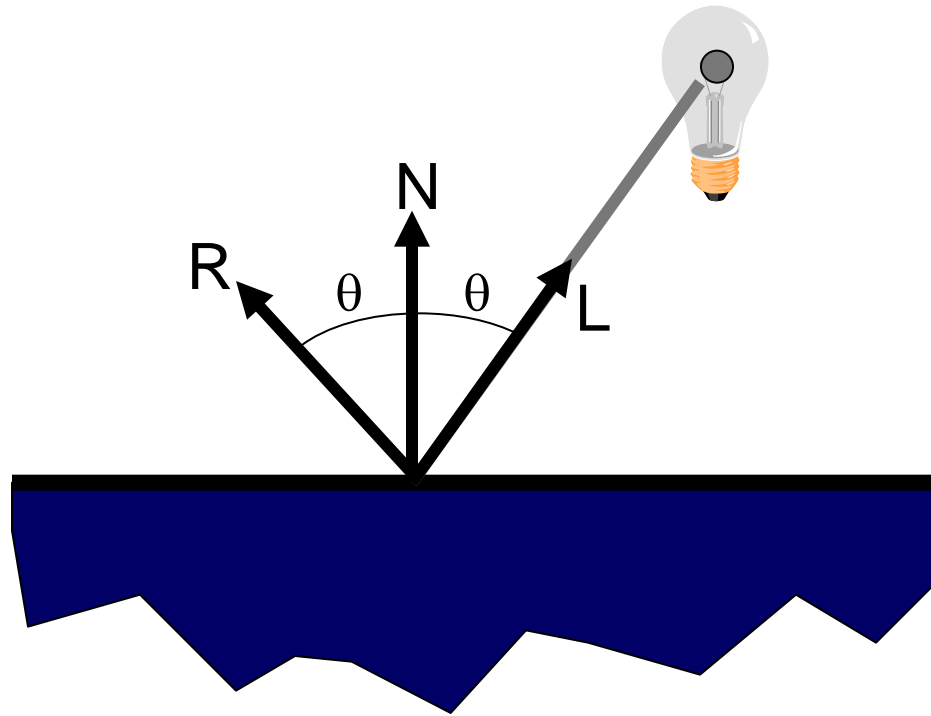
- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”



Specular Reflection



- Reflection is strongest near mirror angle
 - Examples: mirrors, metals

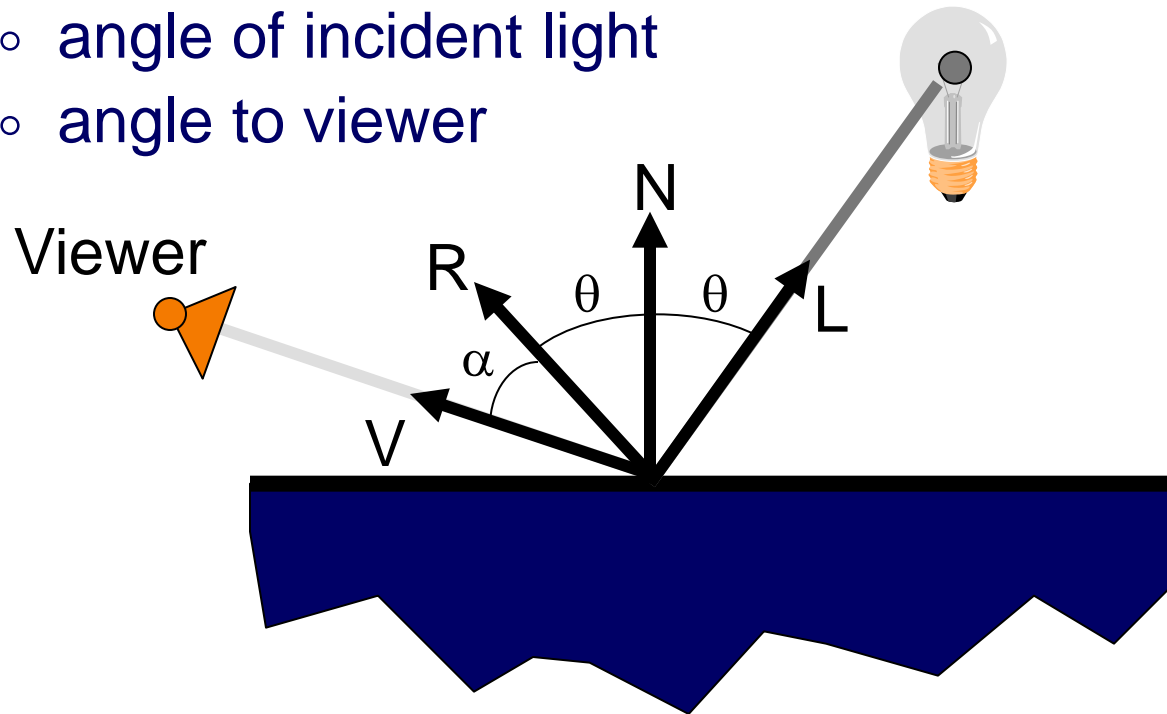


Specular Reflection

How much light is seen?

Depends on:

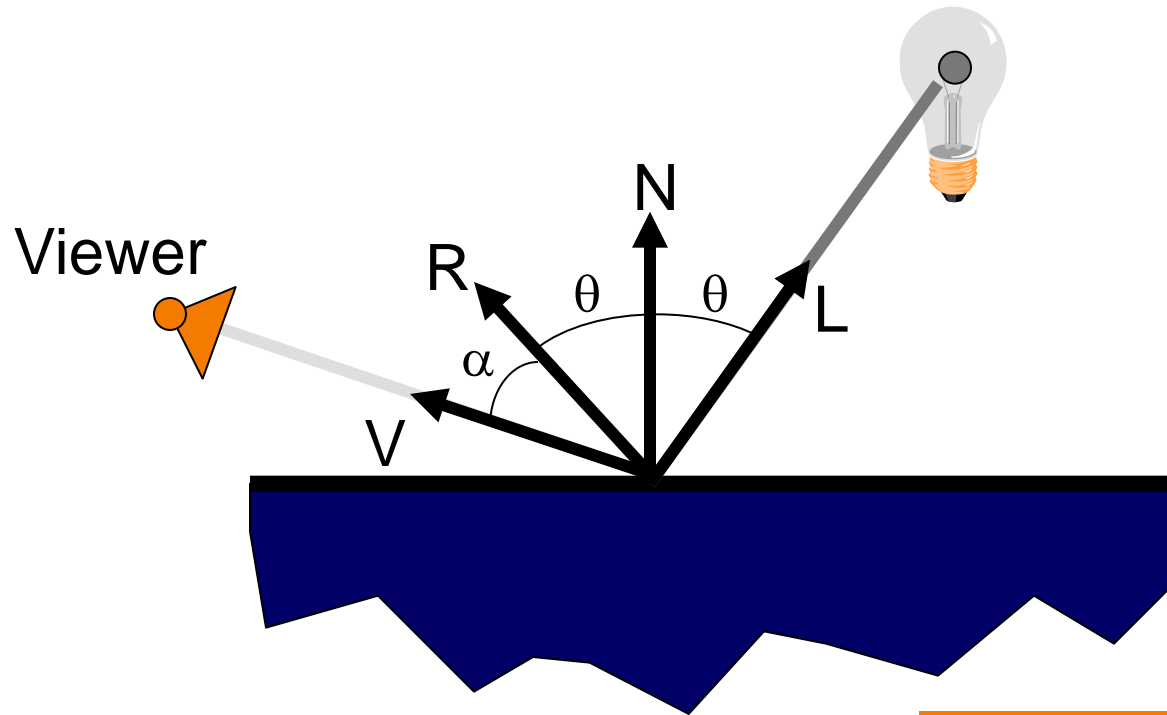
- angle of incident light
- angle to viewer



Specular Reflection

- Phong Model
 - $\cos(\alpha)^n$

This is a physically-motivated hack!

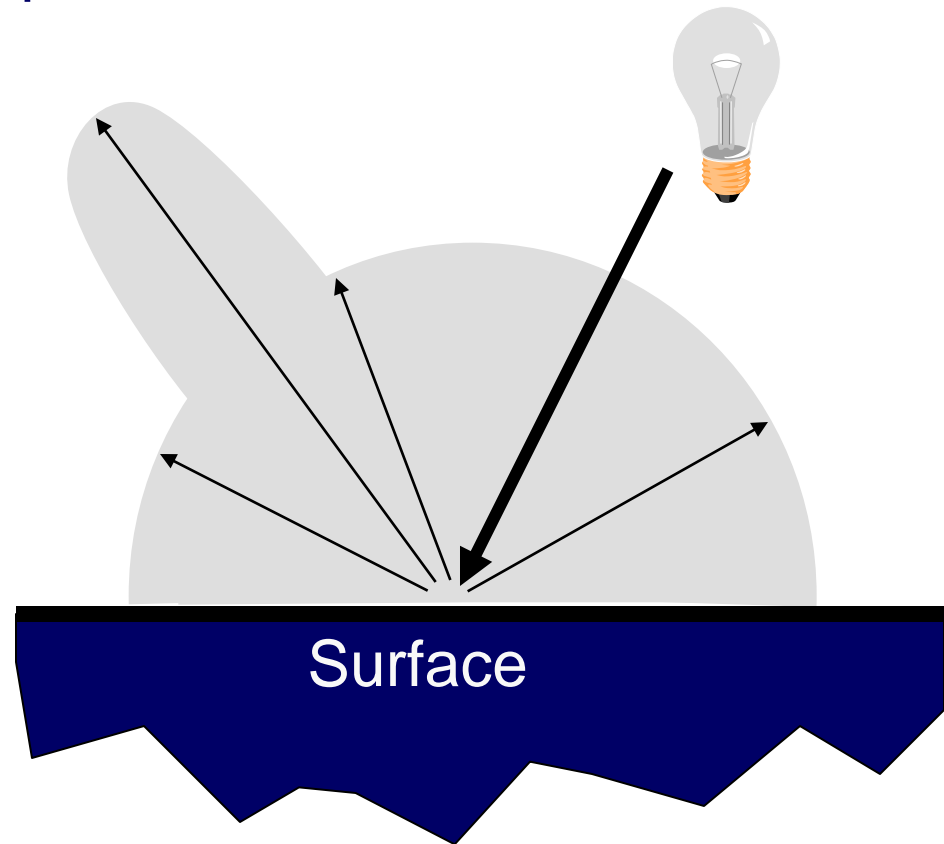


$$I_S = K_S (V \cdot R)^n I_L$$

OpenGL Reflectance Model



- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - **emission** +
 - “ambient”





Emission

- Represents light emanating directly from polygon
 - Note: does not automatically act as light source!
Does not affect other surfaces in scene!

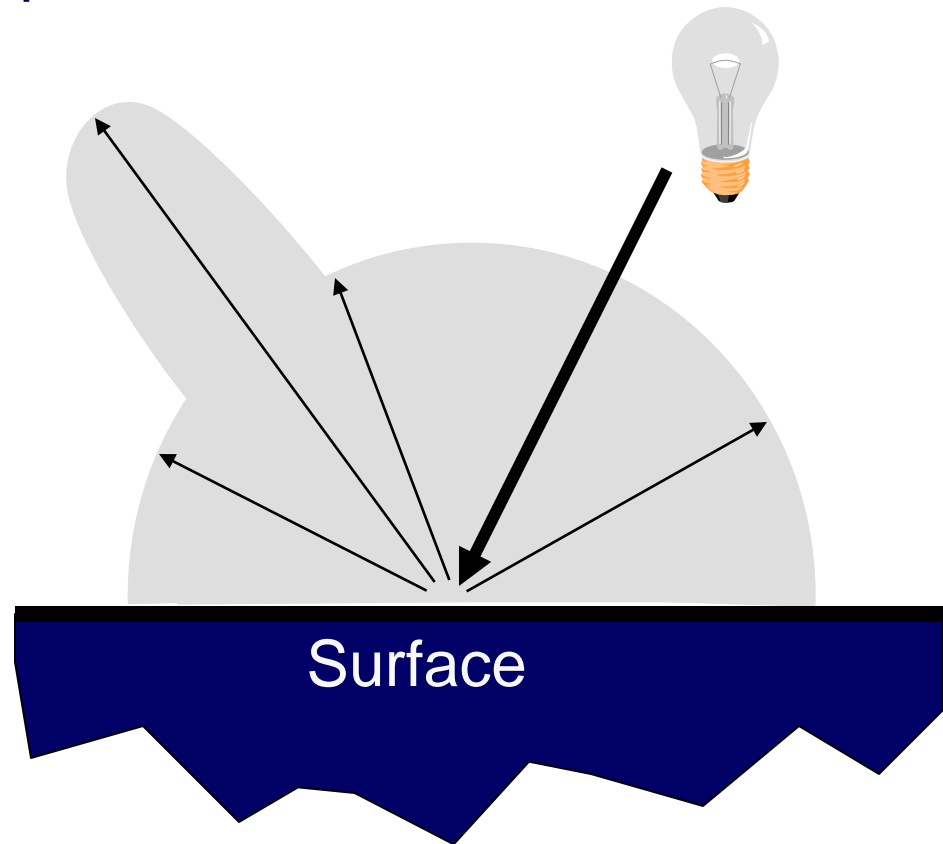


Emission $\neq 0$

OpenGL Reflectance Model



- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”



Ambient Term



- Represents reflection of all indirect illumination

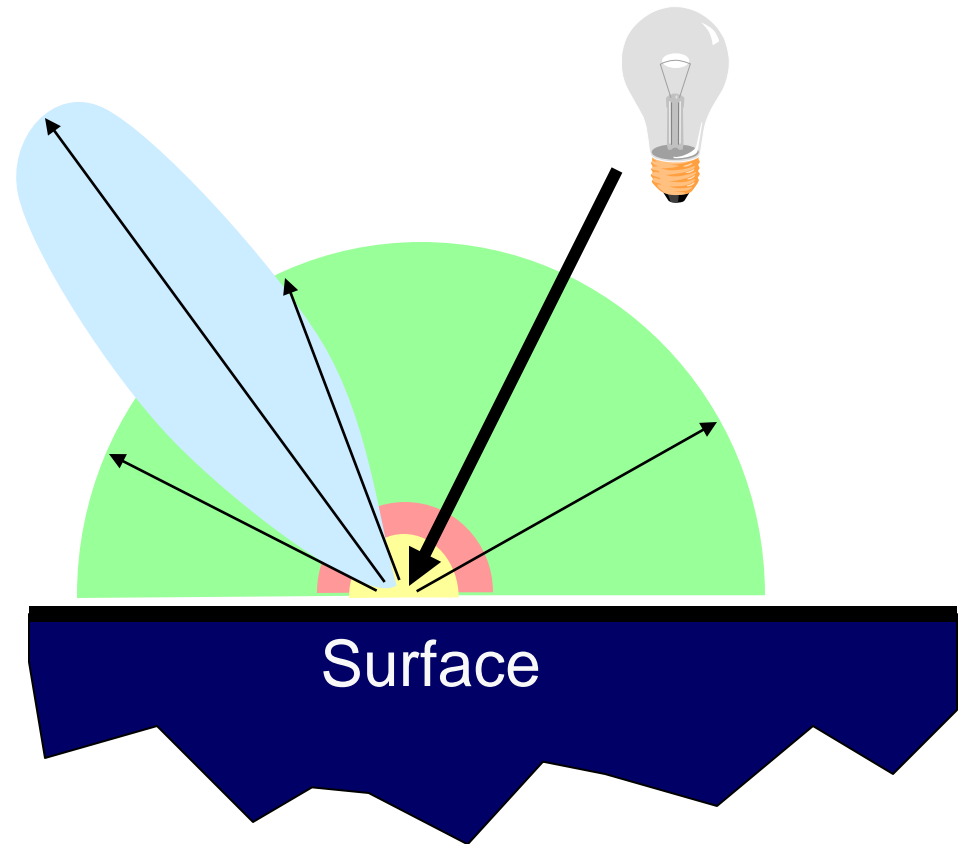


This is a total hack (avoids complexity of global illumination)!

OpenGL Reflectance Model



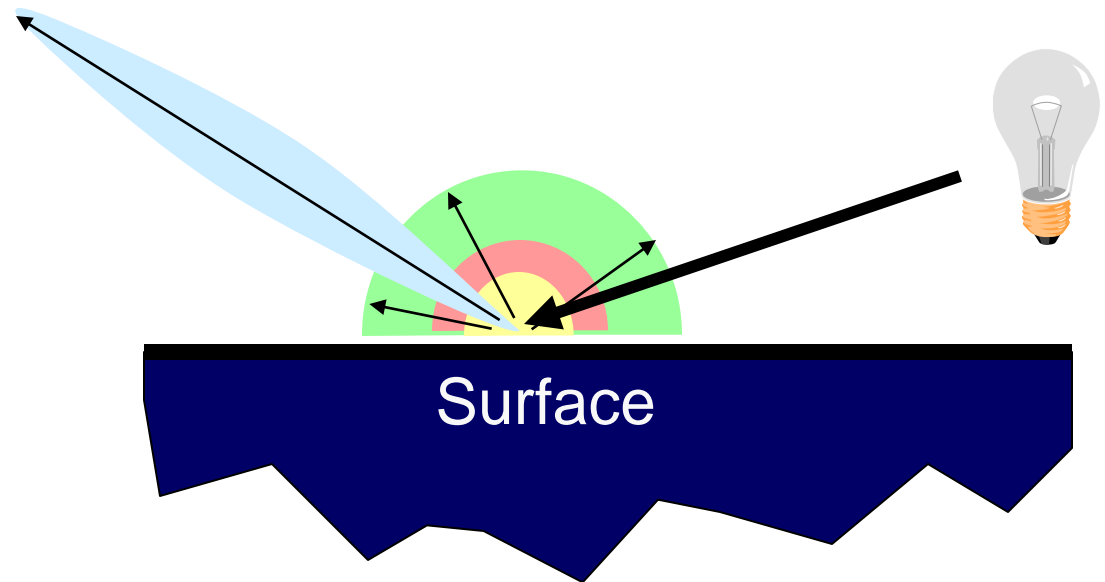
- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”



OpenGL Reflectance Model



- Simple analytic model:
 - diffuse reflection +
 - specular reflection +
 - emission +
 - “ambient”



OpenGL Reflectance Model



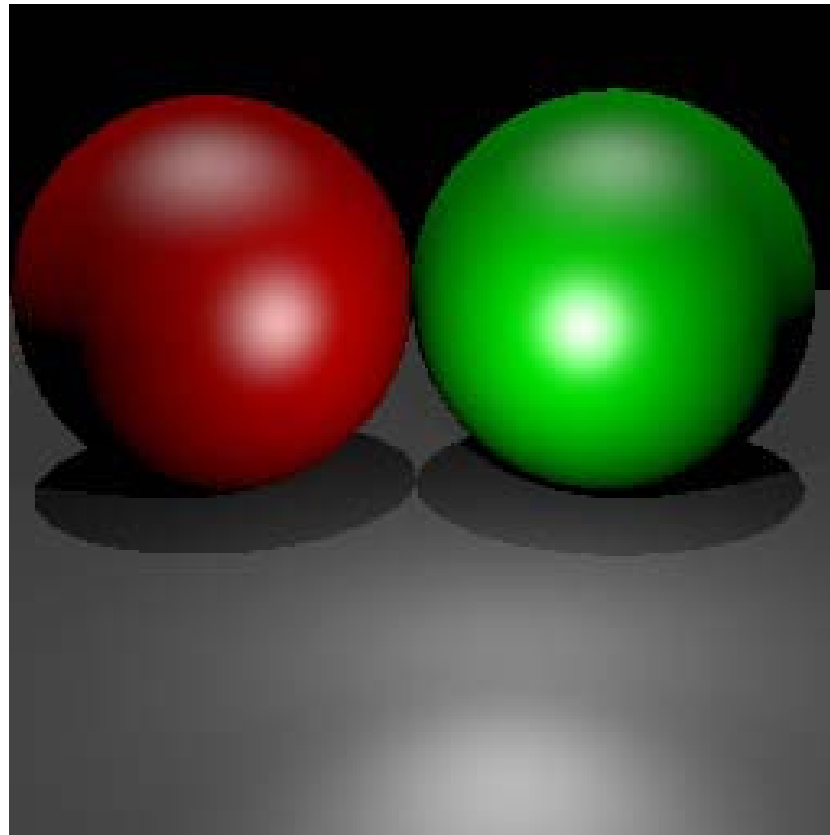
- Sum diffuse, specular, emission, and ambient

Phong	ρ_{ambient}	ρ_{diffuse}	ρ_{specular}	ρ_{total}
$\phi_i = 60^\circ$				
$\phi_i = 25^\circ$				
$\phi_i = 0^\circ$				

OpenGL Reflectance Model

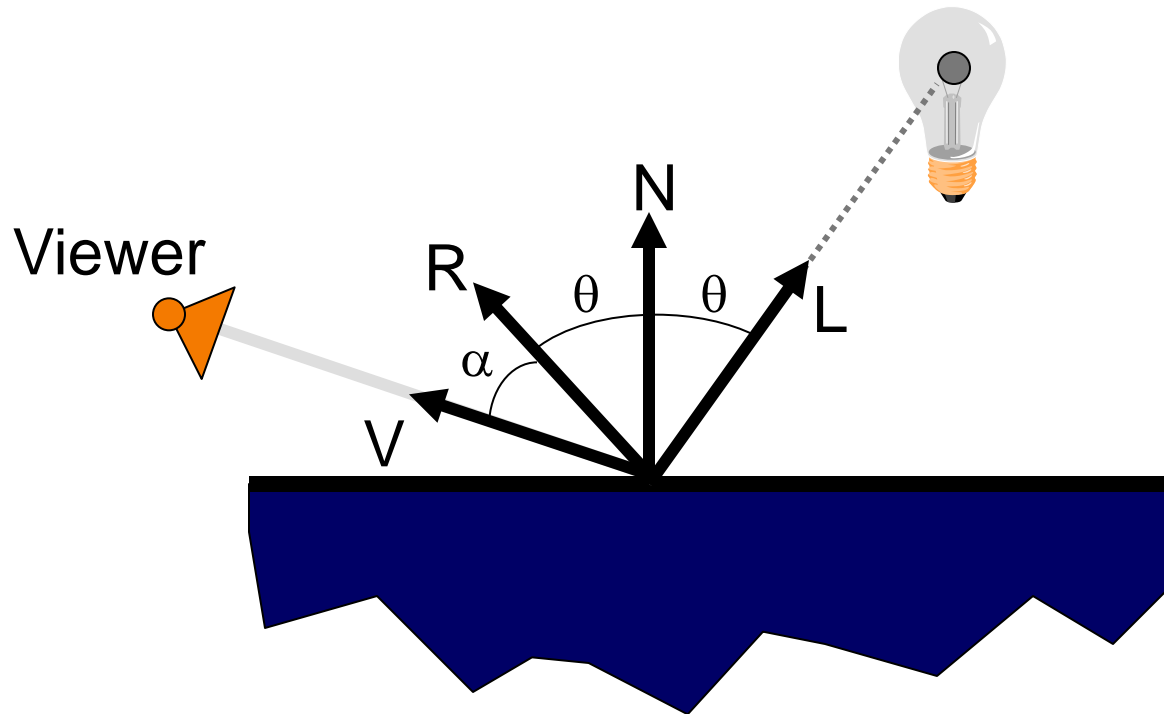


- OK for plastic surfaces, ...



Direct Illumination Calculation

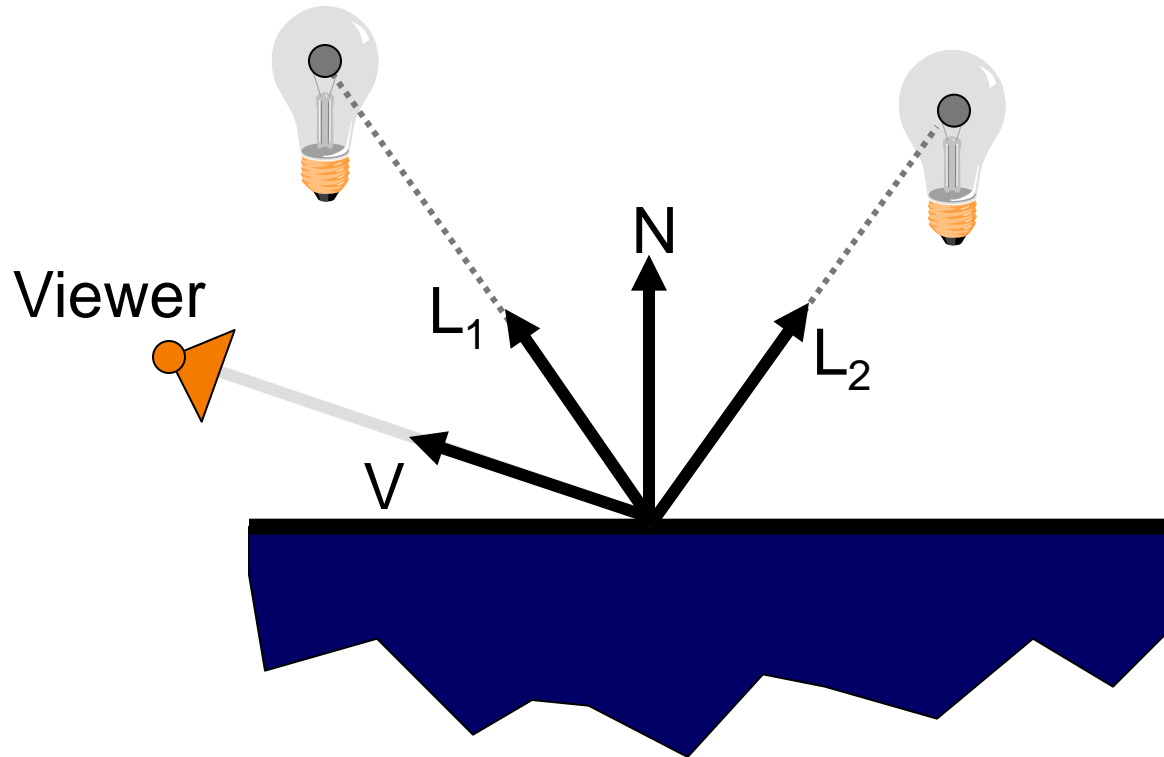
- Single light source:



$$I = I_E + K_A I_{AL} + K_D (N \cdot L) I_L + K_S (V \cdot R)^n I_L$$

Direct Illumination Calculation

- Multiple light sources:

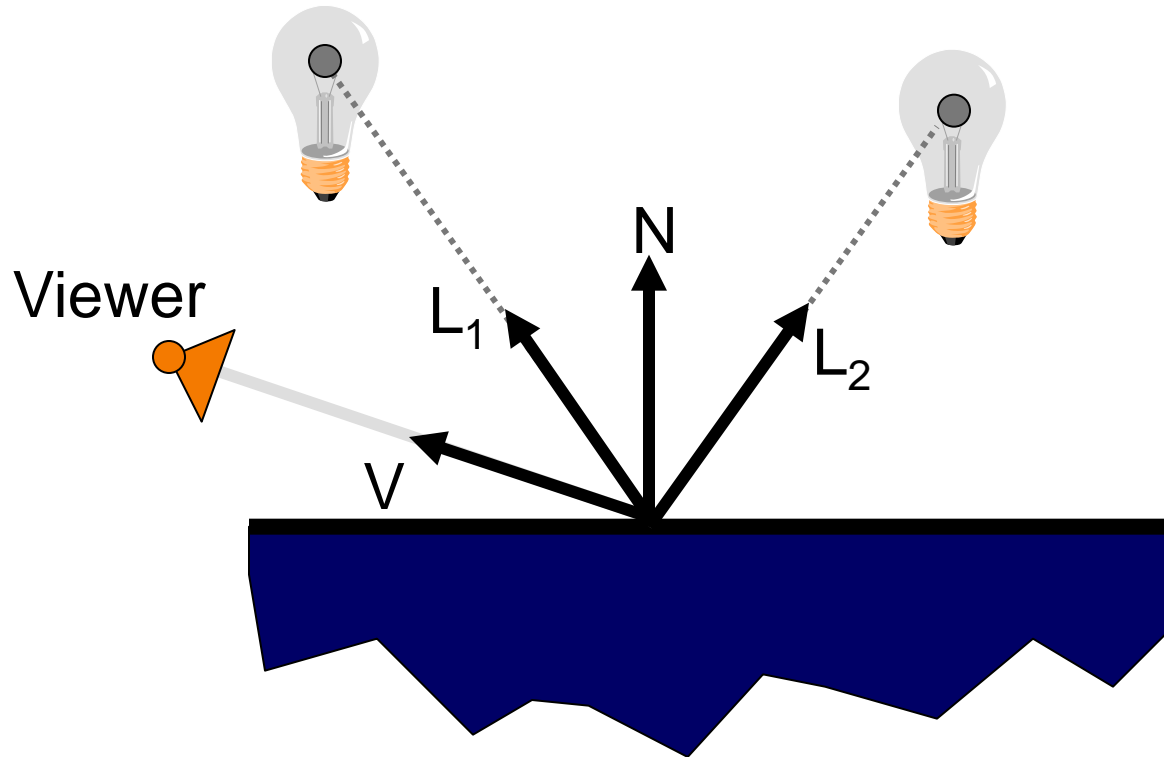


$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)$$



Direct Illumination Calculation

- Multiple light sources:



Note:
this is
shorthand
for
(I_r, I_g, I_b)

$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)$$

Overview



- Direct Illumination
 - Emission at light sources
 - Scattering at surfaces
- Global illumination
 - Shadows
 - Transmissions
 - Inter-object reflections



Global Illumination

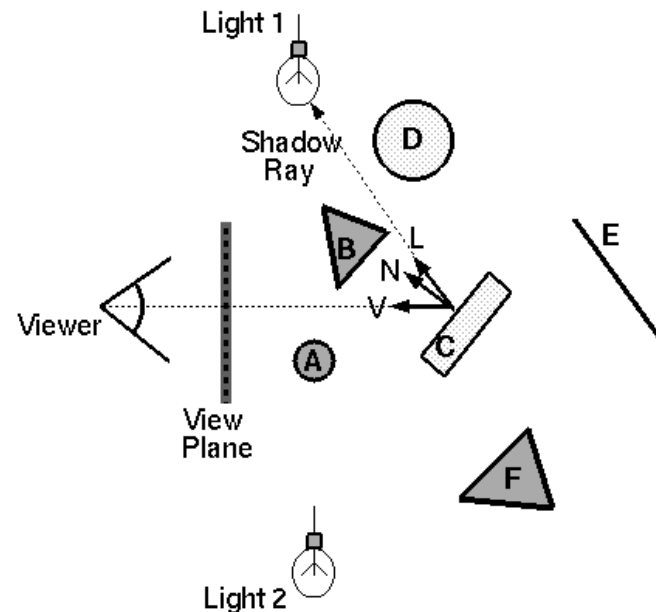
Global Illumination



Greg Ward

Shadows

- Shadow term tells if light sources are blocked
 - Cast ray towards each light source L_i
 - $S_i = 0$ if ray is blocked, $S_i = 1$ otherwise

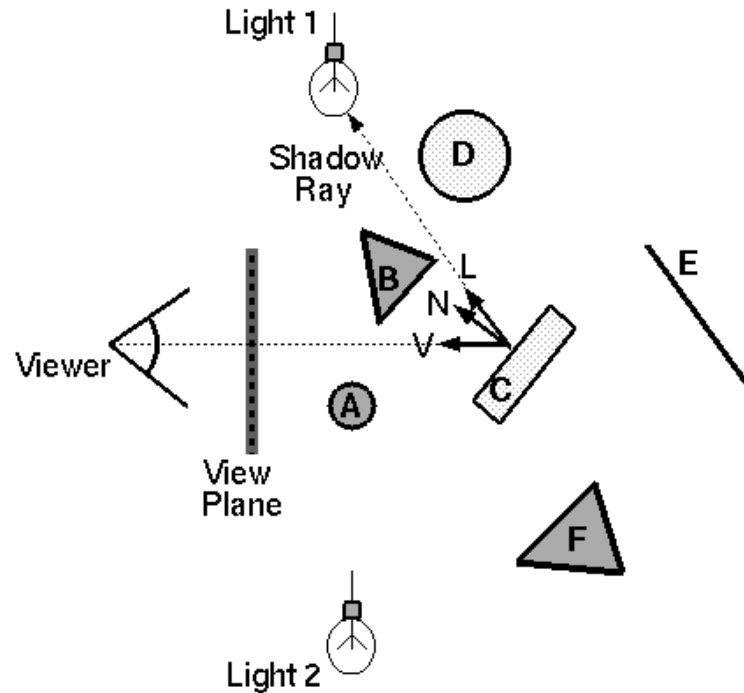


Shadow
Term

$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L$$

Ray Casting (last lecture)

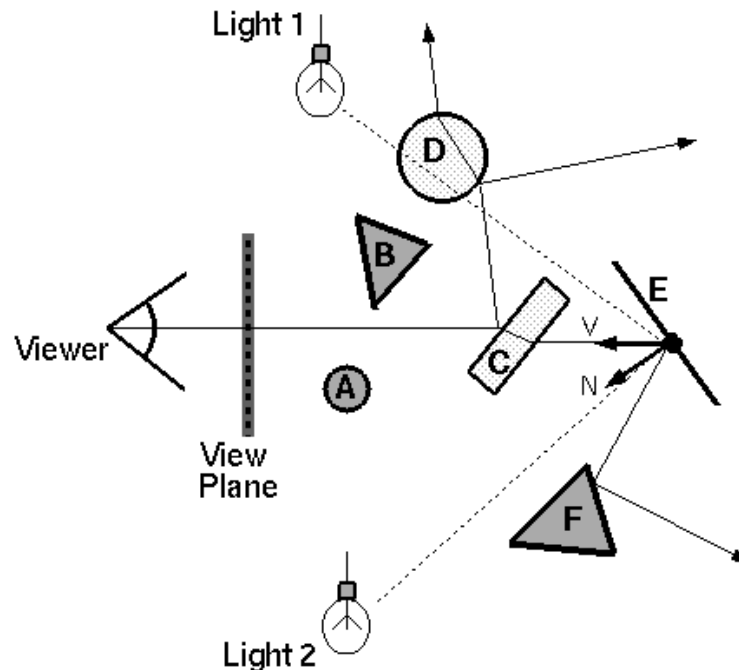
- Trace primary rays from camera
 - Direct illumination from unblocked lights only



$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L$$

Recursive Ray Tracing

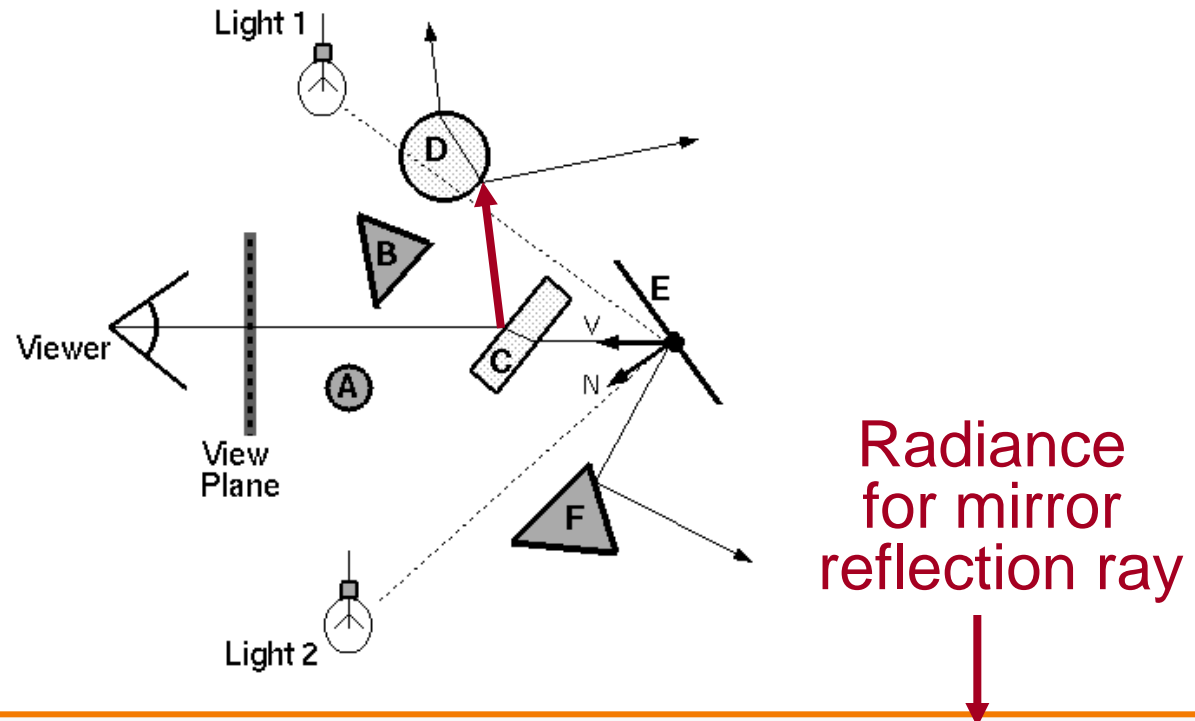
- Also trace secondary rays from hit surfaces
 - Global illumination from mirror reflection and transparency



$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S I_R + K_T I_T$$

Mirror reflections

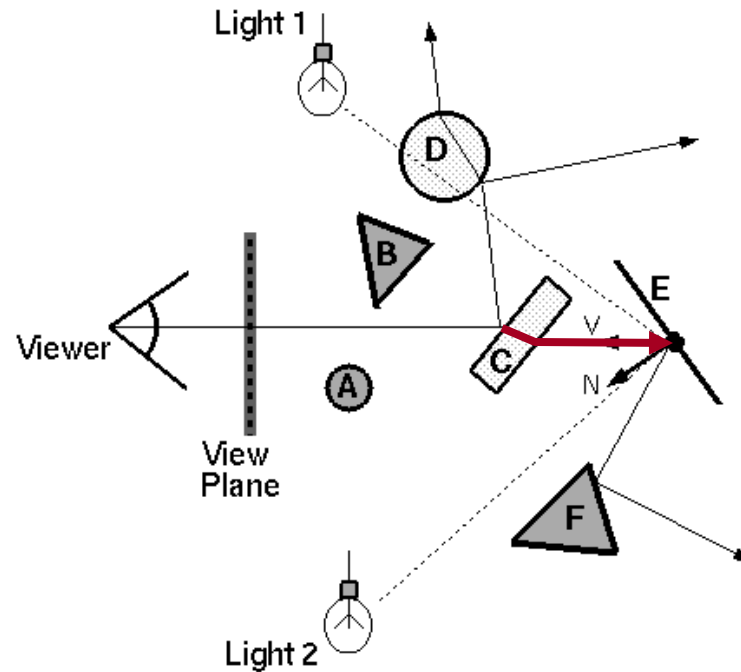
- Trace secondary ray in mirror direction
 - Evaluate radiance along secondary ray and include it into illumination model



$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S \mathbf{I}_R + K_T I_T$$

Transparency

- Trace secondary ray in direction of refraction
 - Evaluate radiance along secondary ray and include it into illumination model

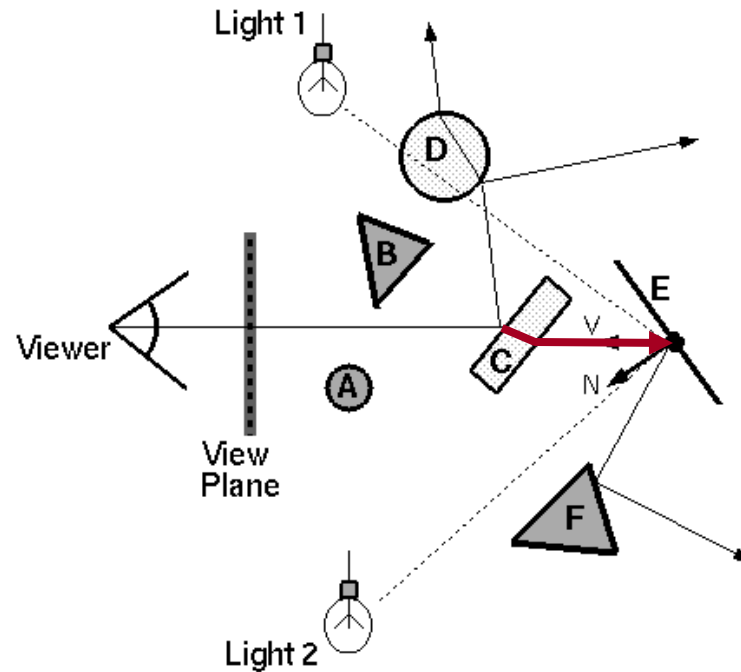


Radiance for refraction ray

$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S I_R + K_T I_T$$

Transparency

- Transparency coefficient is fraction transmitted
 - $K_T = 1$ for translucent object, $K_T = 0$ for opaque
 - $0 < K_T < 1$ for object that is semi-translucent



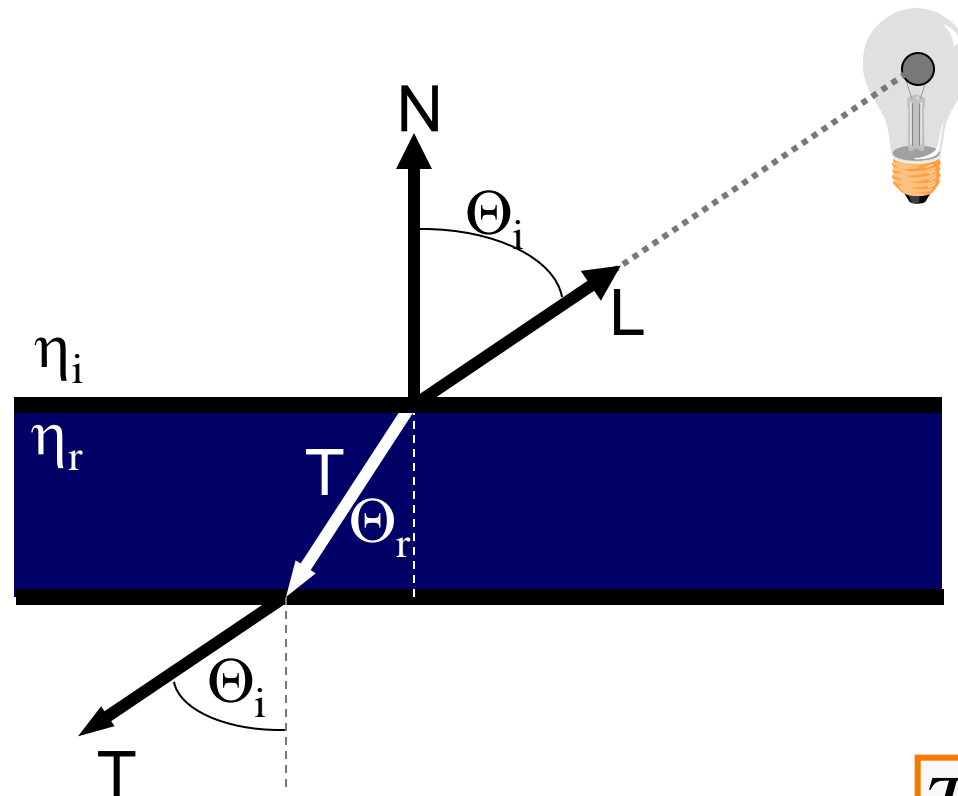
Transparency
Coefficient

$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S I_R + K_T I_T$$



Refractive Transparency

- For thin surfaces, can ignore change in direction
 - Assume light travels straight through surface

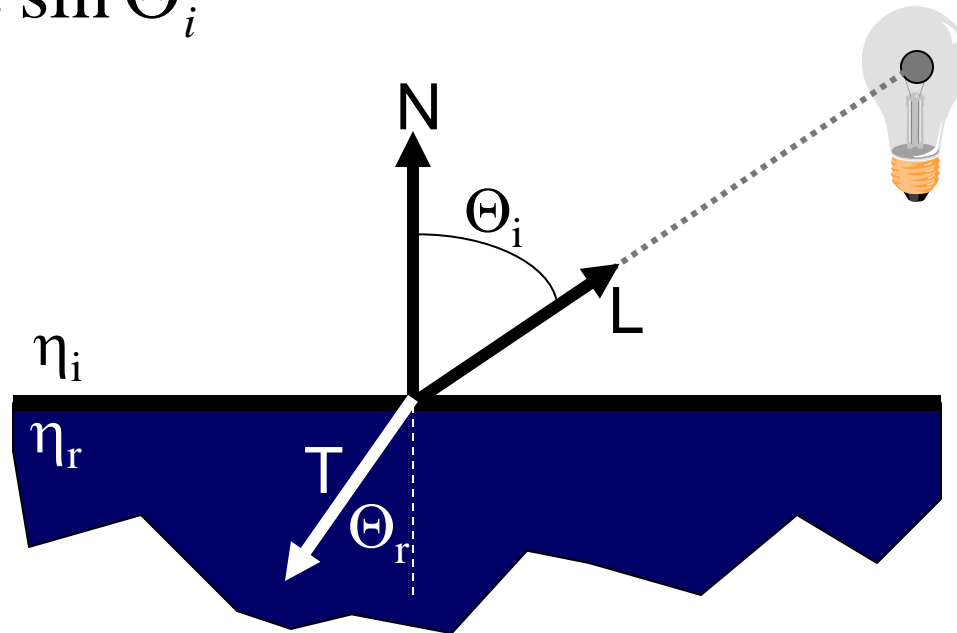


$$T \cong -L$$

Refractive Transparency

For solid objects, apply Snell's law:

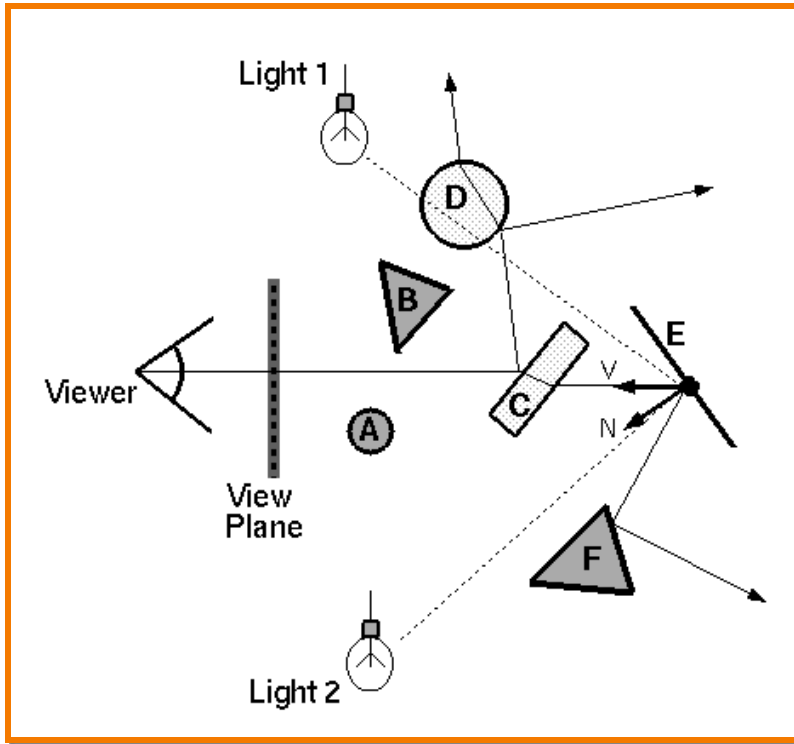
$$\eta_r \sin \Theta_r = \eta_i \sin \Theta_i$$



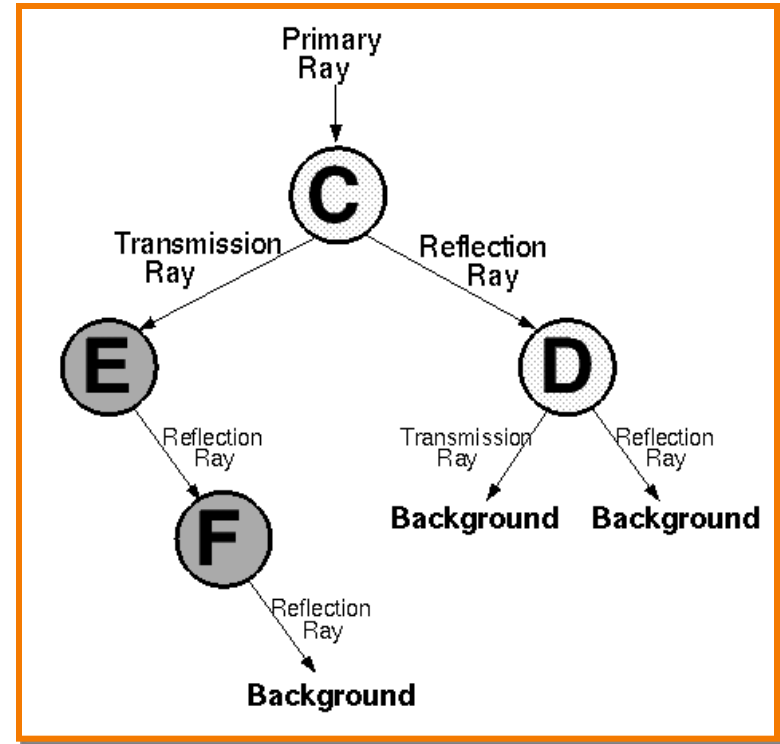
$$T = \left(\frac{\eta_i}{\eta_r} \cos \Theta_i - \cos \Theta_r \right) N - \frac{\eta_i}{\eta_r} L$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene

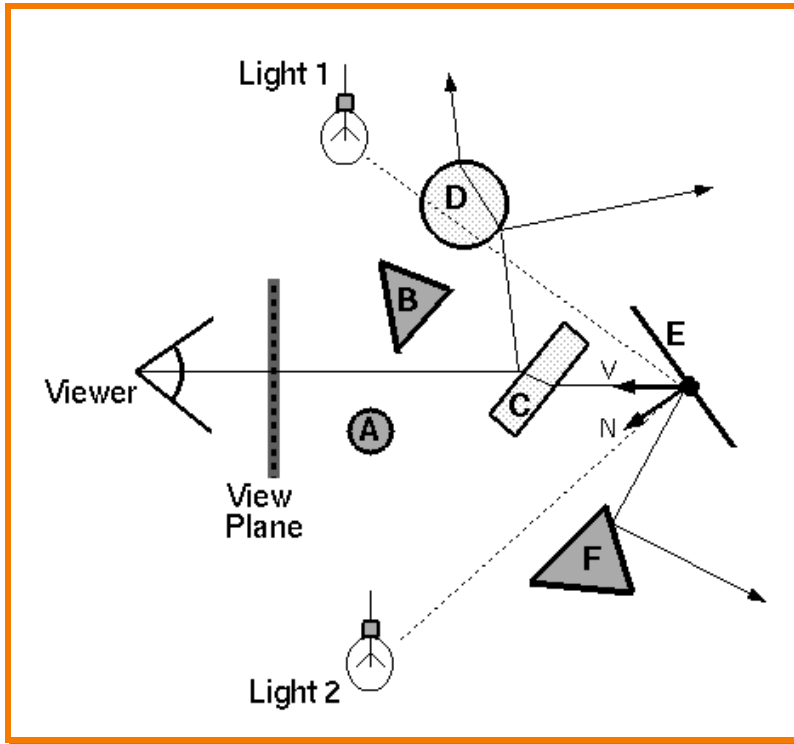


Ray tree

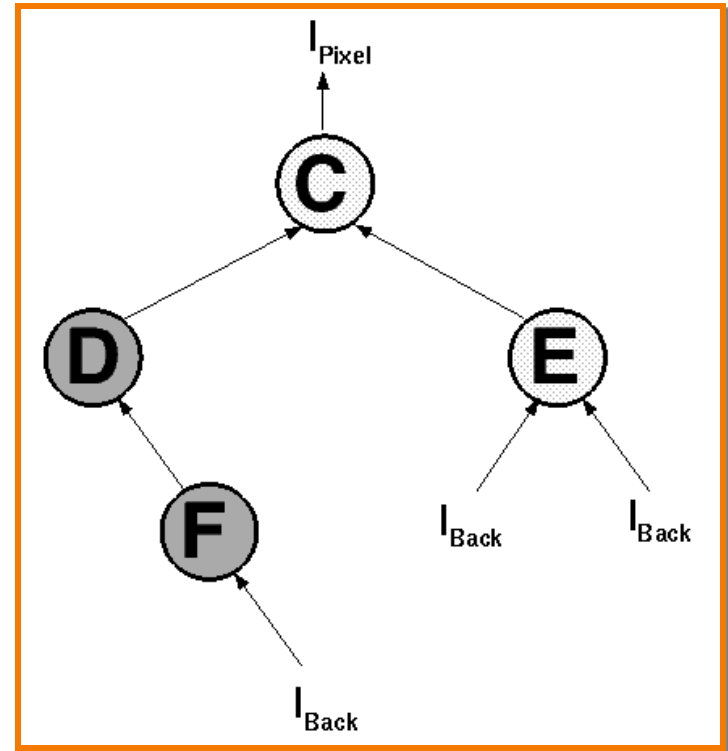
$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene



Ray tree

$$I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) S_L I_L + K_S I_R + K_T I_T$$

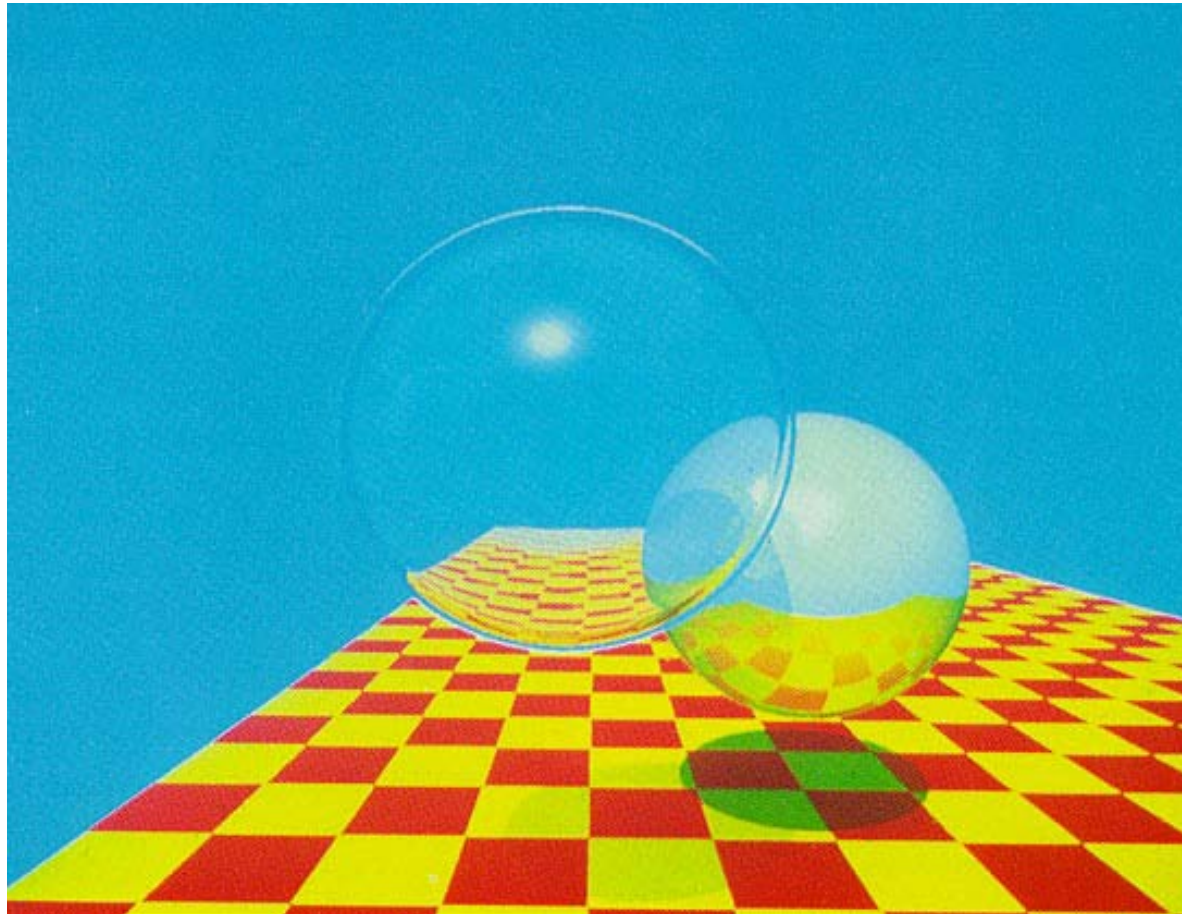
Recursive Ray Tracing



- ComputeRadiance is called recursively

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray, R3Intersection& hit)
{
    R3Ray specular_ray = SpecularRay(ray, hit);
    R3Ray refractive_ray = RefractiveRay(ray, hit);
    R3Rgb radiance = Phong(scene, ray, hit) +
                    Ks * ComputeRadiance(scene, specular_ray) +
                    Kt * ComputeRadiance(scene, refractive_ray);
    return radiance;
}
```

Example



Turner Whitted, 1980

Summary



- Ray casting (direct Illumination)
 - Usually use simple analytic approximations for light source emission and surface reflectance
- Recursive ray tracing (global illumination)
 - Incorporate shadows, mirror reflections, and pure refractions

All of this is an approximation
so that it is practical to compute

More on global illumination next time!



Illumination Terminology

- Radiant power [flux] (Φ)
 - Rate at which light energy is transmitted (in Watts).
- Radiant Intensity (I)
 - Power radiated onto a unit solid angle in direction (in Watts/sr)
 - » e.g.: energy distribution of a light source (inverse square law)
- Radiance (L)
 - Radiant intensity per unit projected surface area (in Watts/m²sr)
 - » e.g.: light carried by a single ray (no inverse square law)
- Irradiance (E)
 - Incident flux density on a locally planar area (in Watts/m²)
 - » e.g.: light hitting a surface at a point
- Radiosity (B)
 - Exitant flux density from a locally planar area (in Watts/m²)