



# Subdivision Surfaces

COS 426

# 3D Object Representations



- Raw data
  - Voxels
  - Point cloud
  - Range image
  - Polygons
- Surfaces
  - Mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Octree
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# 3D Object Representations

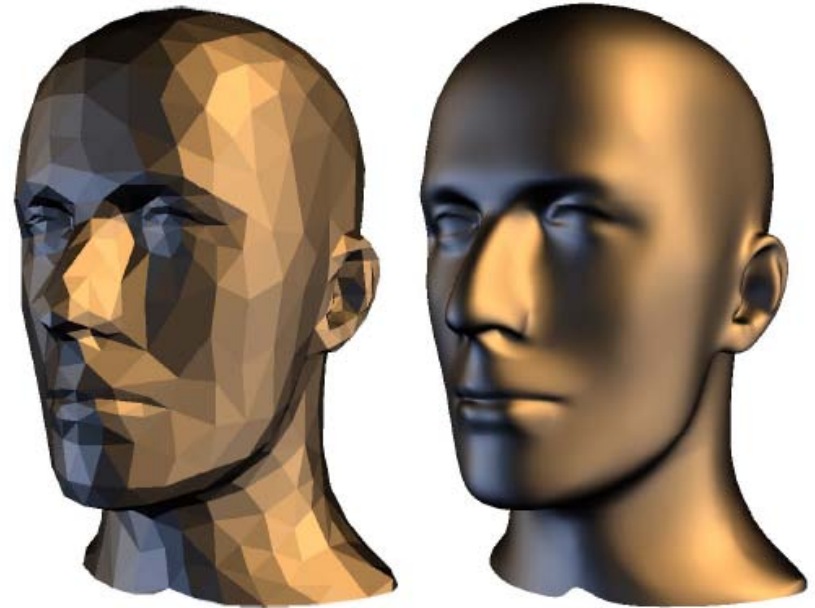


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# Subdivision Surfaces



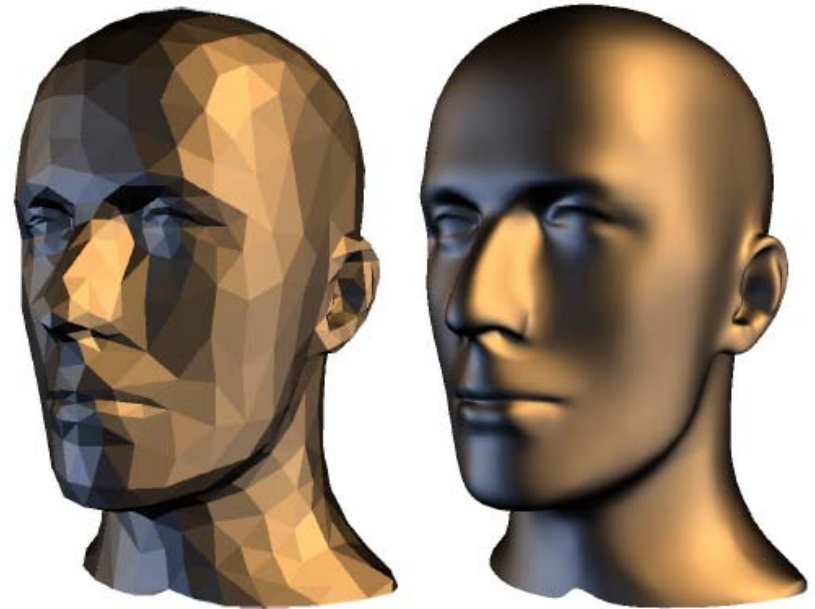
- What makes a good surface representation?
  - o Accurate
  - o Concise
  - o Intuitive specification
  - o Local support
  - o Affine invariant
  - o Arbitrary topology
  - o Guaranteed continuity
  - o Natural parameterization
  - o Efficient display
  - o Efficient intersections



# Subdivision Surfaces



- What makes a good surface representation?
  - o Accurate
  - o Concise
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  - **Guaranteed continuity**
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# Continuity

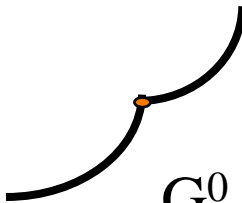


- A curve / surface with  $G^k$  continuity can be differentiated  $k$  times

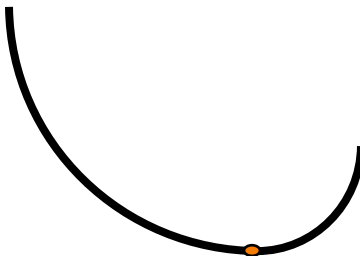
No continuity: “ $G^{-1}$ ”




$G^0$



$G^1$



$G^2$

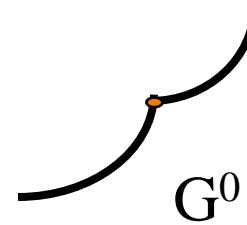


# Continuity

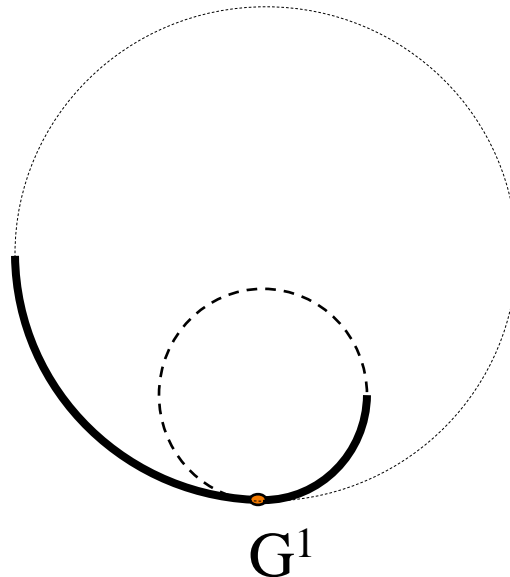


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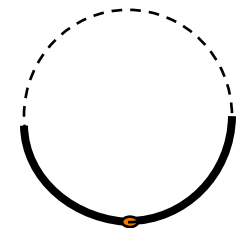
No continuity: “ $G^{-1}$ ”



$G^0$



$G^1$



$G^2$



# Parametric Continuity

- Curve  $x = f(t)$ ,  $y = g(t)$  is  $C^k$  continuous iff

$$\frac{d^k f}{dt^k} \text{ and } \frac{d^k g}{dt^k} \text{ are continuous}$$

- For surfaces:  $x = f(s,t)$ ,  $y = g(s,t)$ ,  $z = h(s,t)$

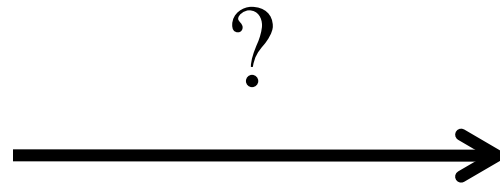
$$C^k \text{ requires continuous } \frac{\partial^k f}{\partial s^k}, \frac{\partial^k f}{\partial t^k}, \frac{\partial^k g}{\partial s^k}, \frac{\partial^k g}{\partial t^k}, \frac{\partial^k h}{\partial s^k}, \frac{\partial^k h}{\partial t^k}$$

- Often easier to check / prove things about



# Subdivision

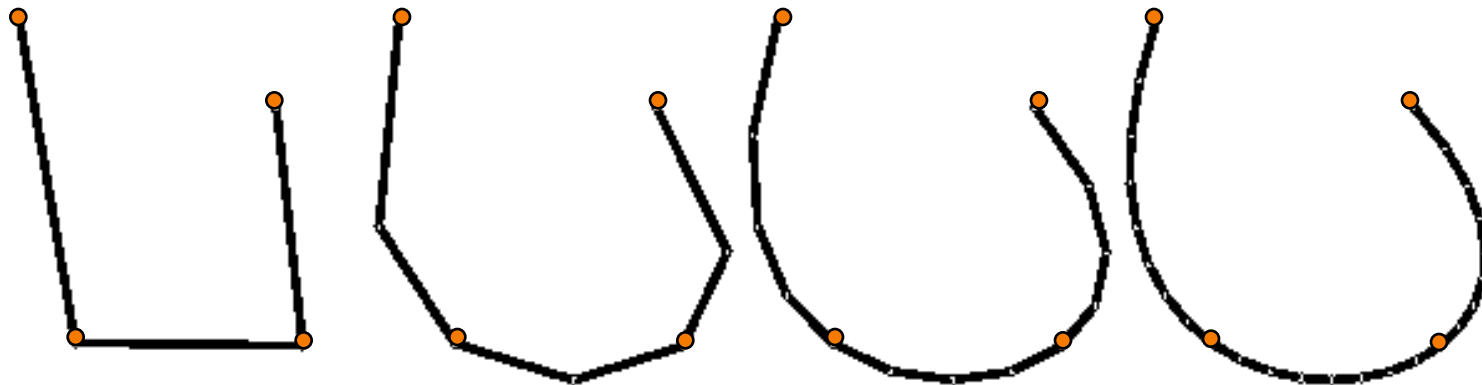
- How do you make a curve with guaranteed continuity?



# Subdivision

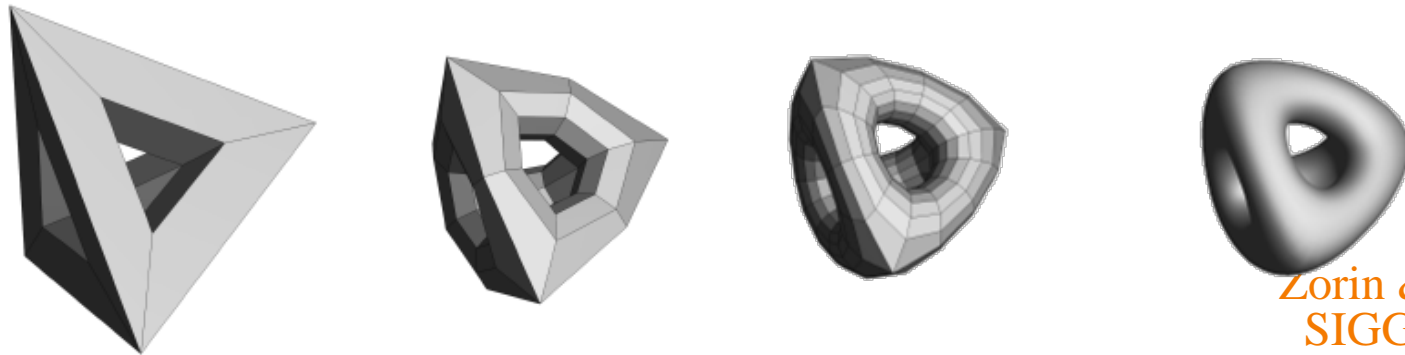


- How do you make a curve with guaranteed continuity? ...



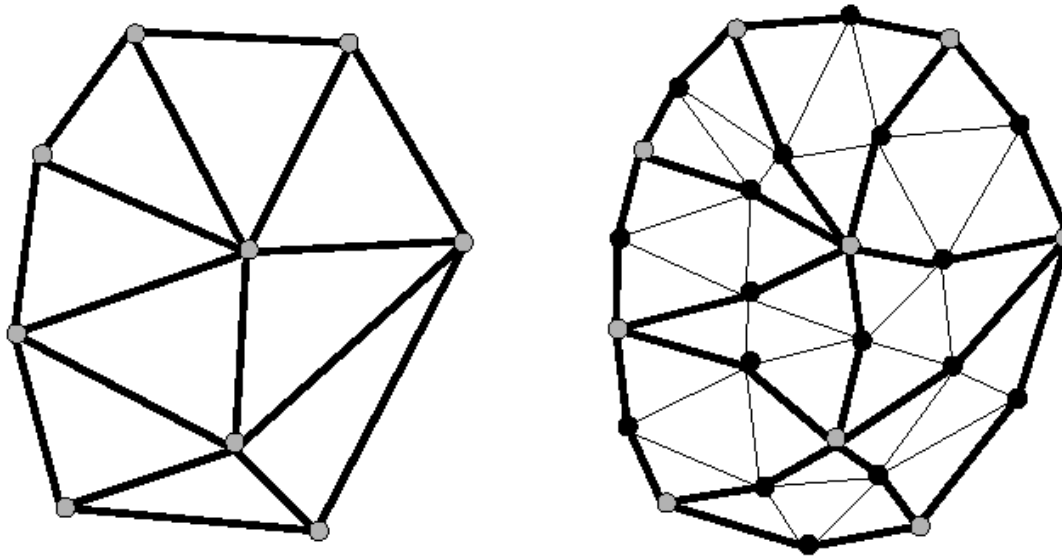
# Subdivision

- How do you make a surface with guaranteed continuity?



# Subdivision Surfaces

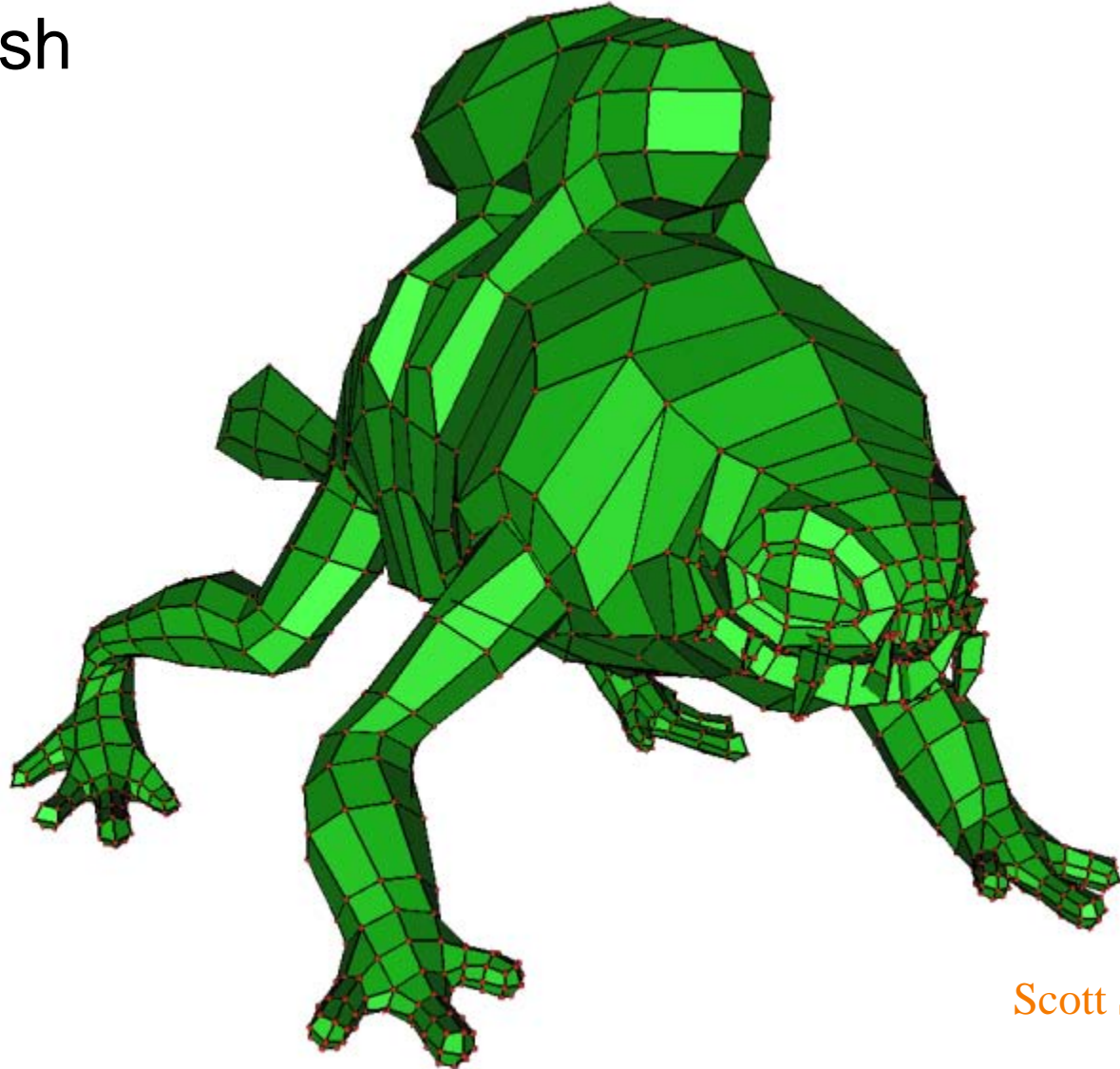
- Repeated application of
  - Topology refinement (splitting faces)
  - Geometry refinement (weighted averaging)



# Subdivision Surfaces – Examples



- Base mesh

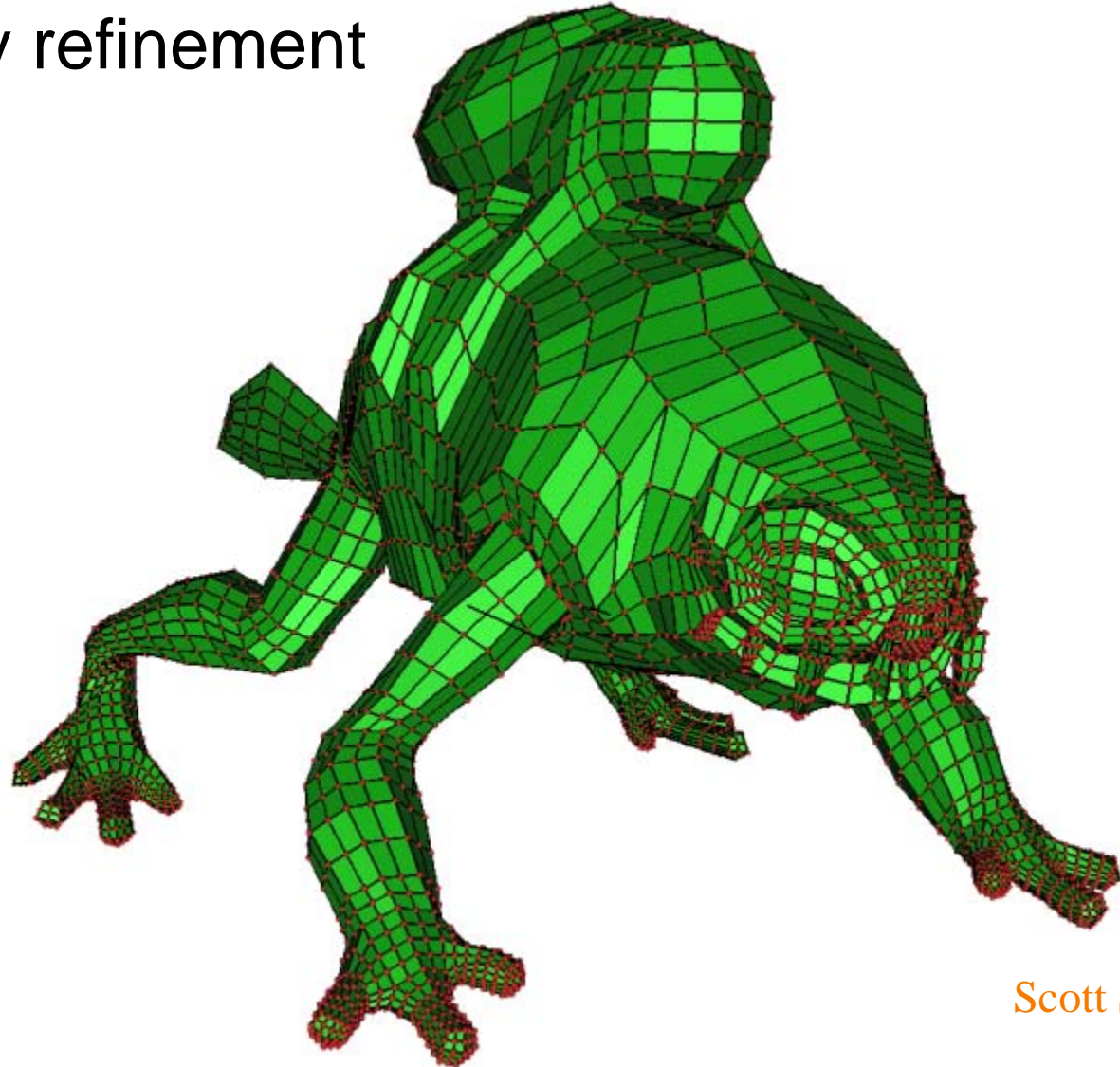


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# Subdivision Surfaces – Examples



- Topology refinement

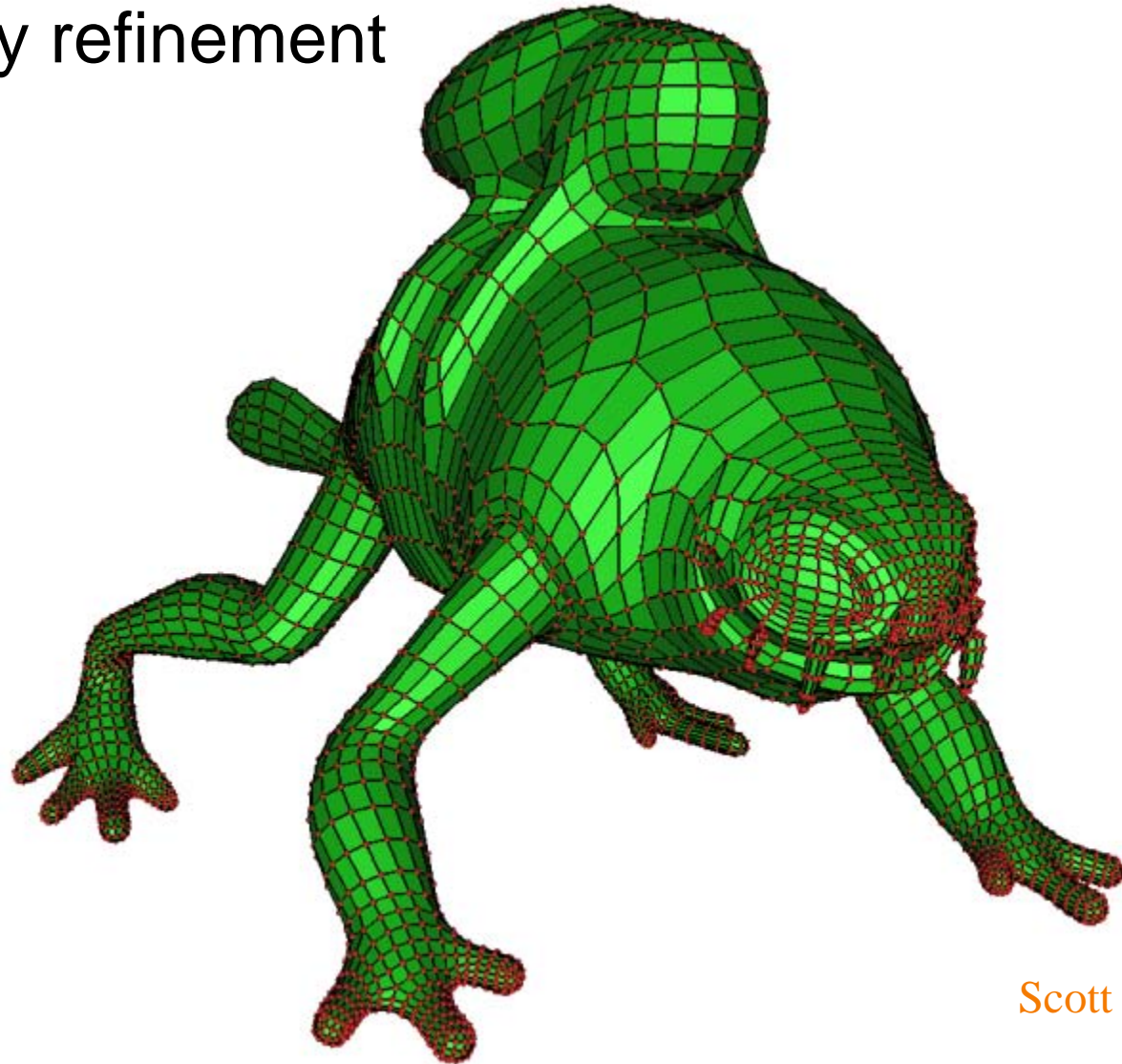


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# Subdivision Surfaces – Examples



- Geometry refinement



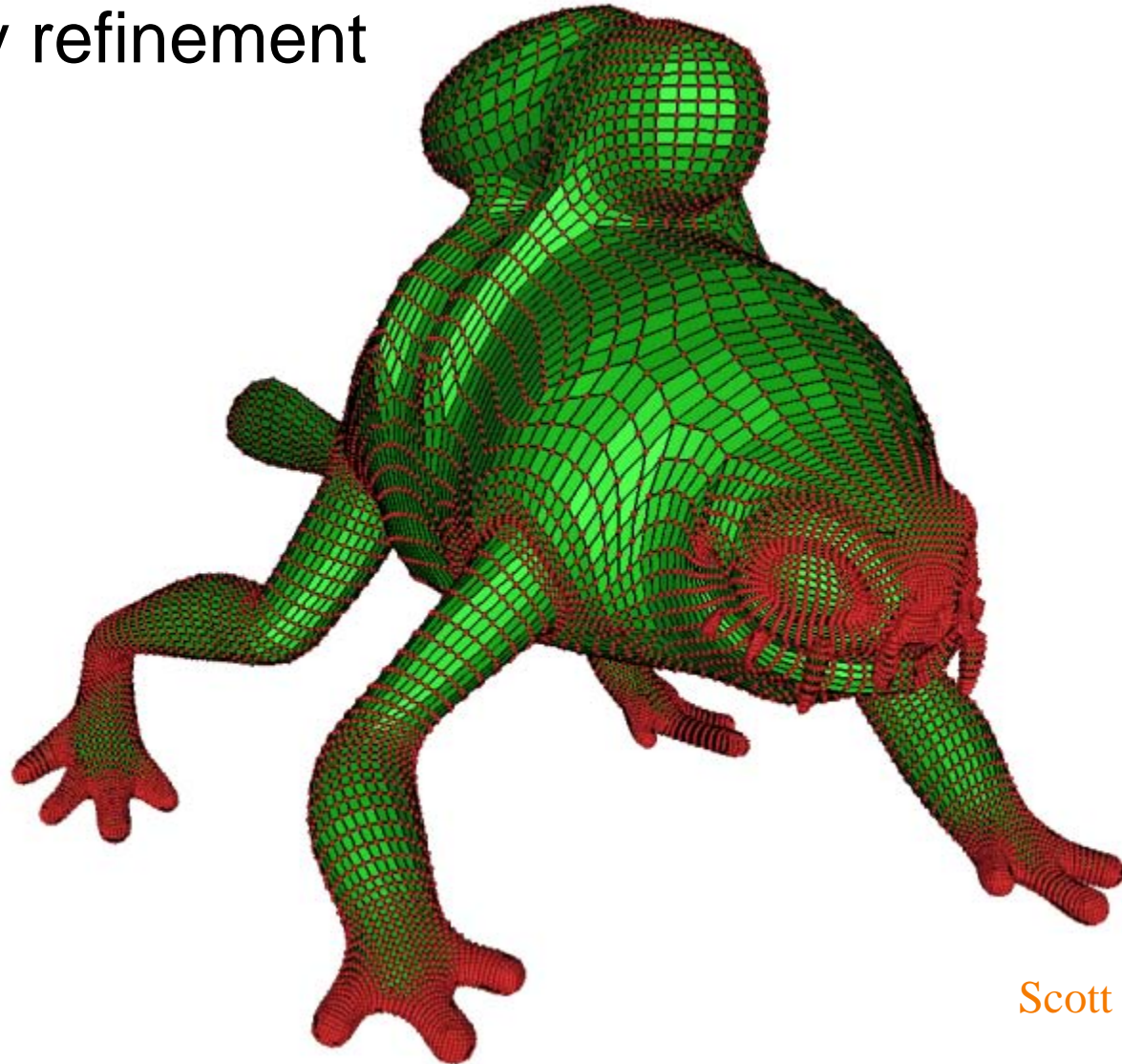
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# Subdivision Surfaces – Examples



- Topology refinement



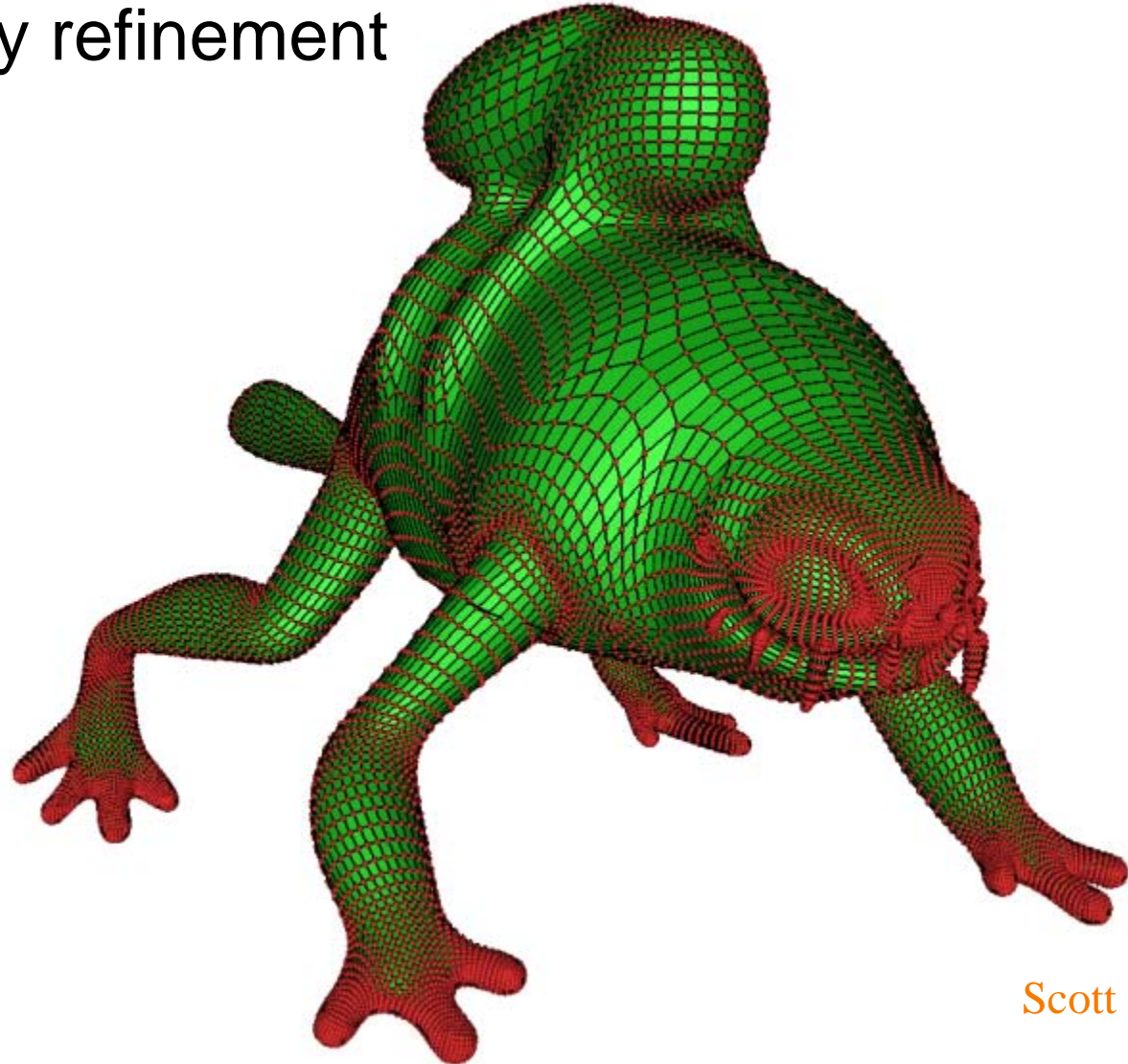
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# Subdivision Surfaces – Examples



- Geometry refinement

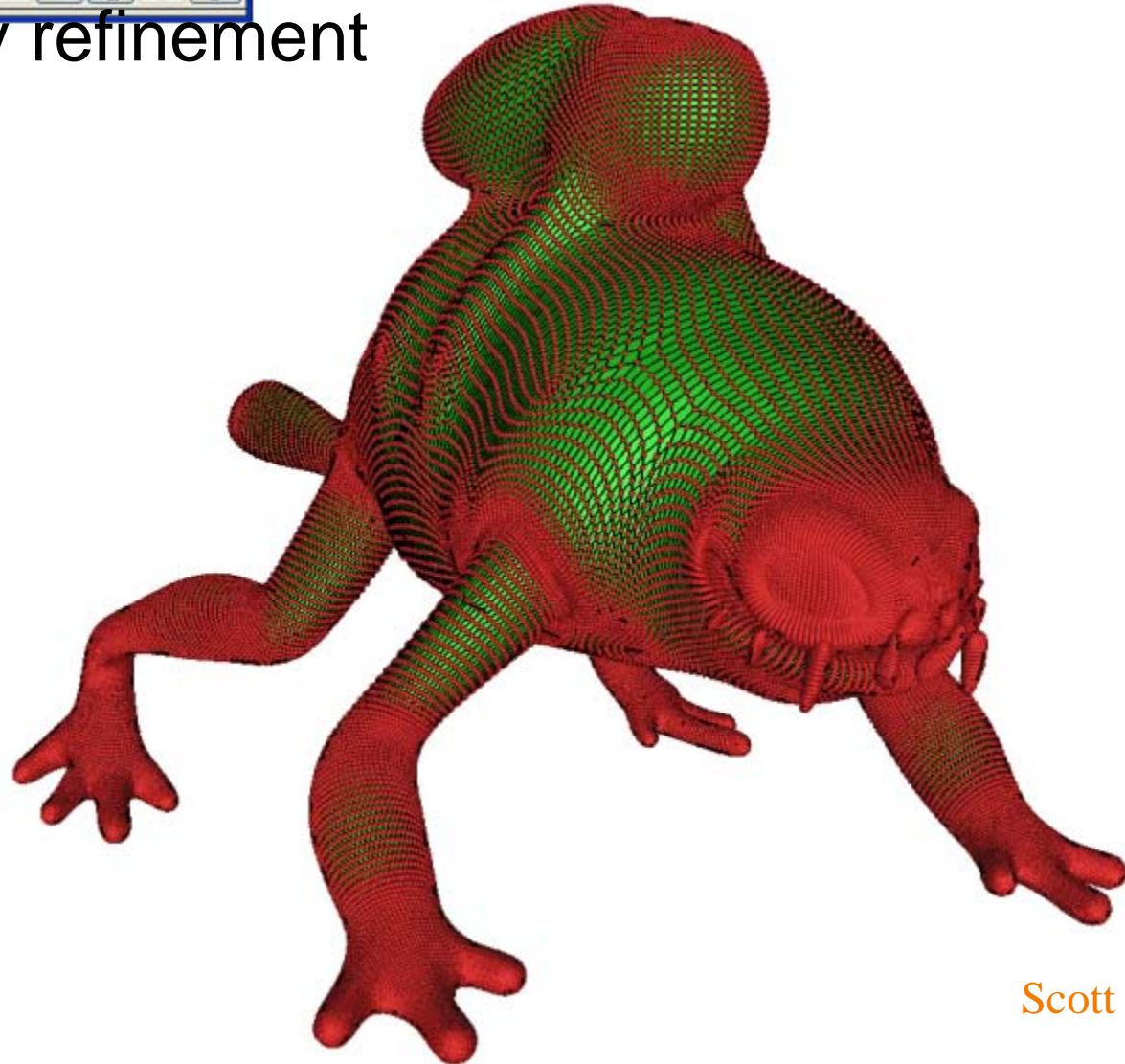


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# Subdivision Surfaces – Examples



- Topology refinement

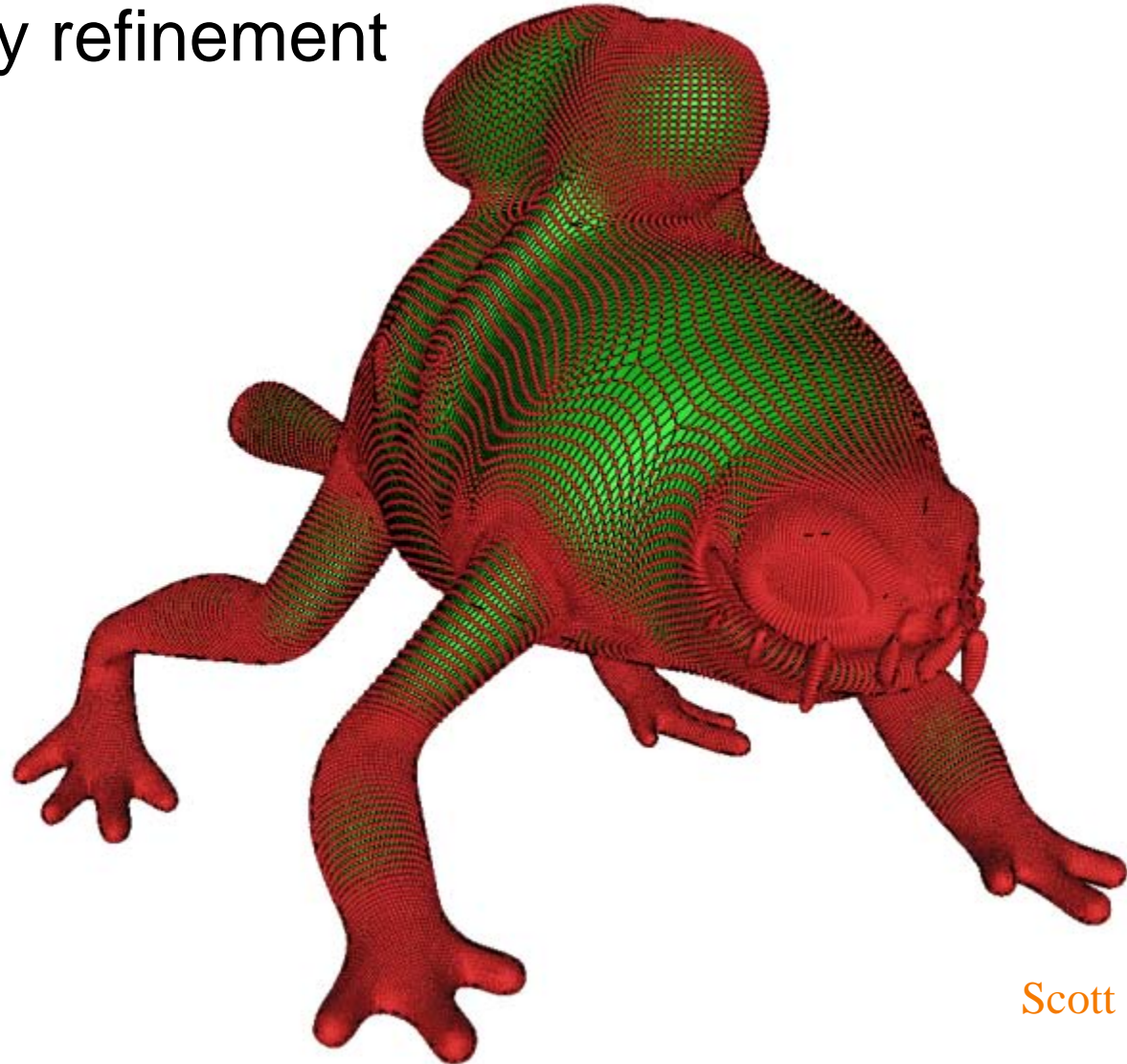


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# Subdivision Surfaces – Examples



- Geometry refinement

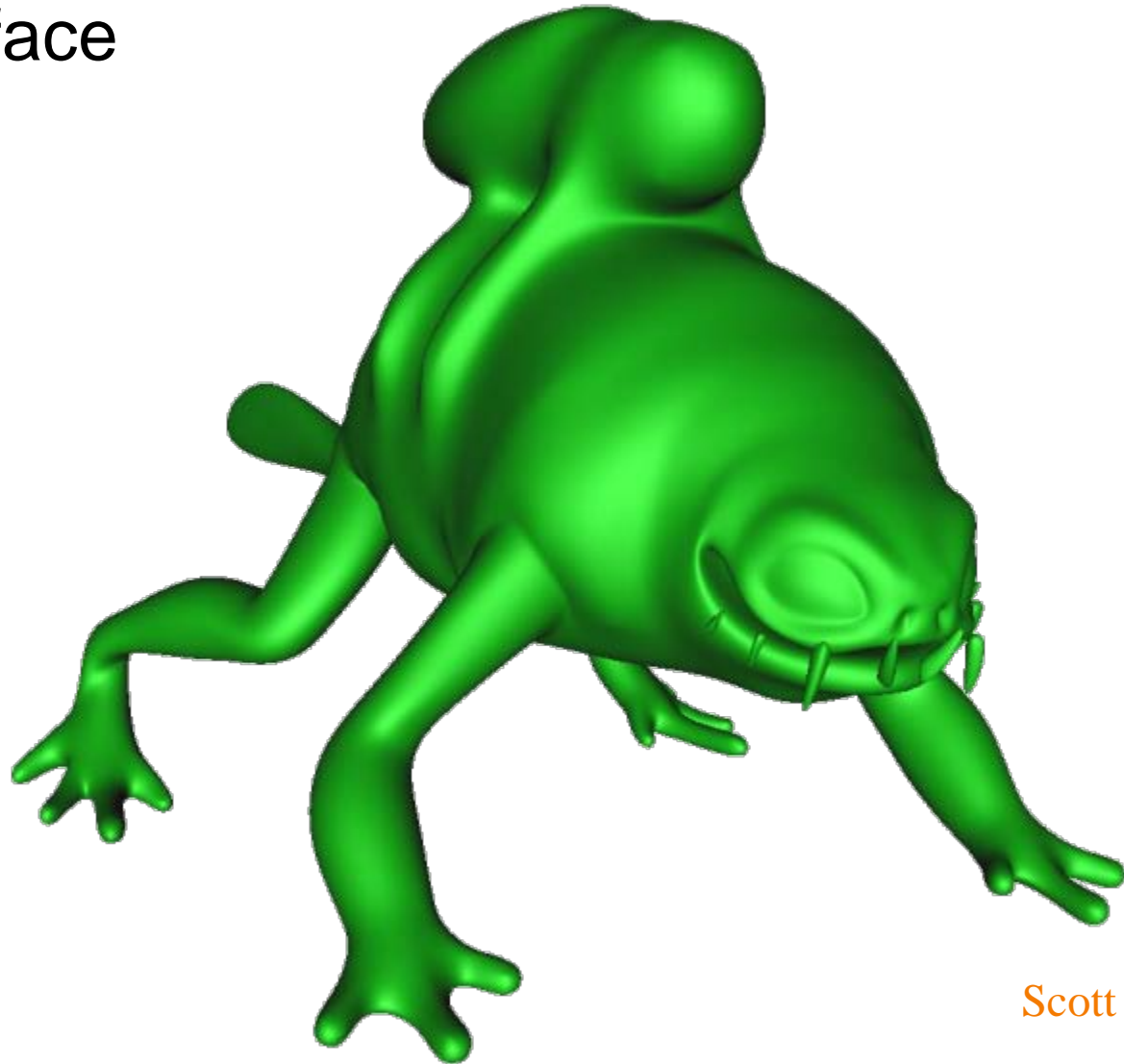


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# Subdivision Surfaces – Examples



- Limit surface



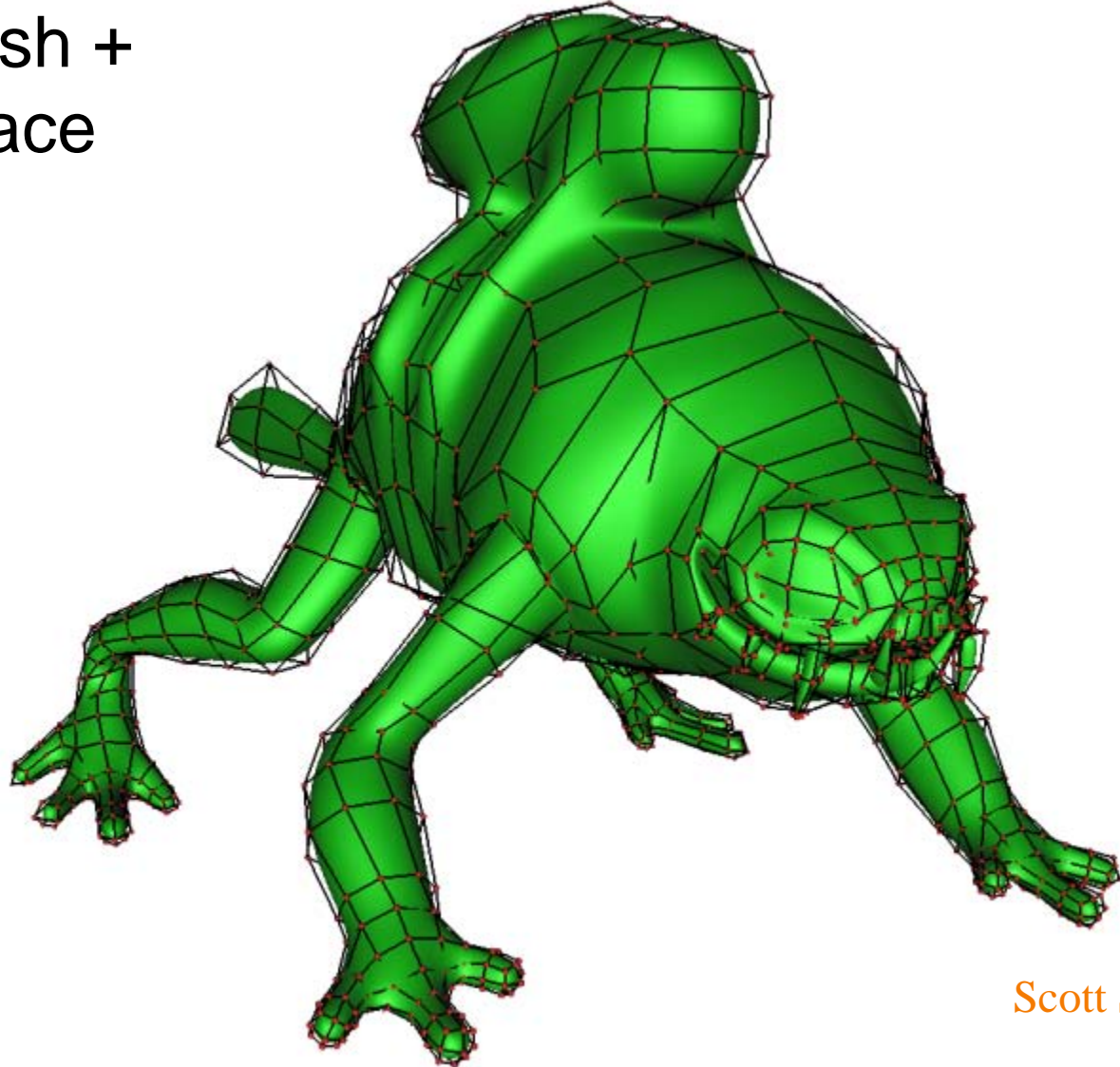
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# Subdivision Surfaces – Examples



- Base mesh + limit surface

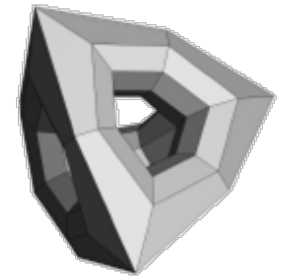
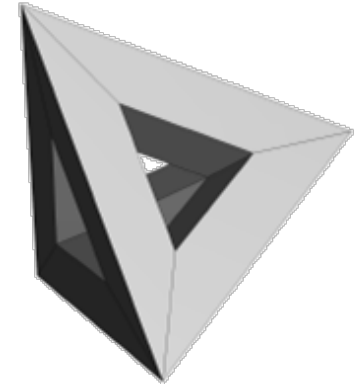


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# Design of Subdivision Rules



- What types of input?
  - Quad meshes, triangle meshes, etc.
- How to refine topology?
  - Simple implementations
- How to refine geometry?
  - Smoothness guarantees in limit surface
    - » Continuity ( $C^0$ ,  $C^1$ ,  $C^2$ , ...?)
  - Provable relationships between limit surface and original control mesh
    - » Interpolation of vertices?



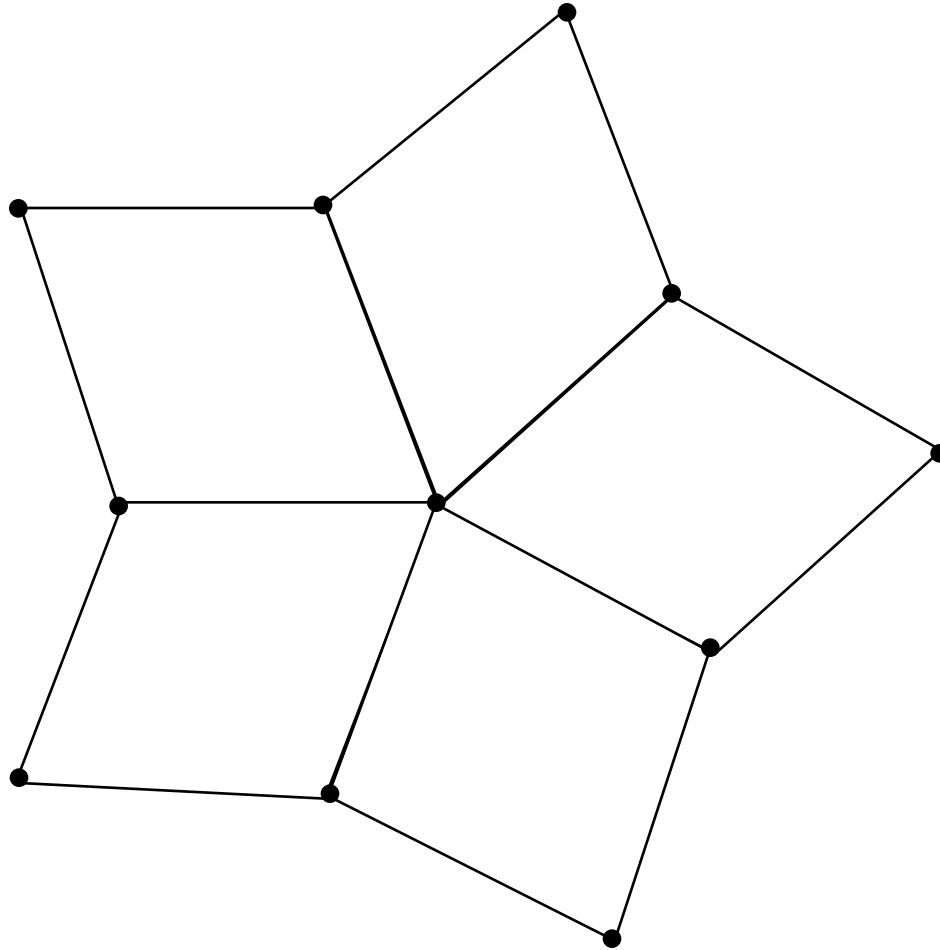


# Linear Subdivision

- Type of input
  - Quad mesh -- four-sided polygons (*quads*)
  - Any number of quads may touch each vertex
- Topology refinement rule
  - Split every quad into four at midpoints
- Geometry refinement rule
  - Average vertex positions

This is a simple example to demonstrate how subdivision schemes work

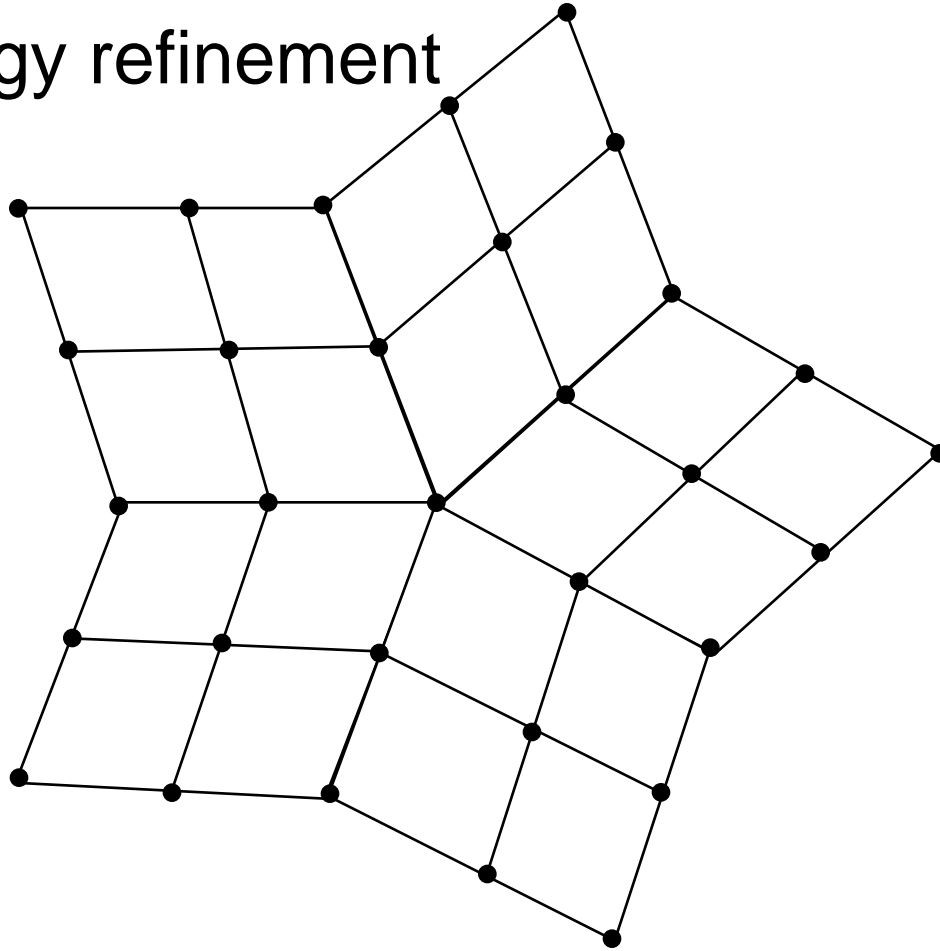
# Linear Subdivision





# Linear Subdivision

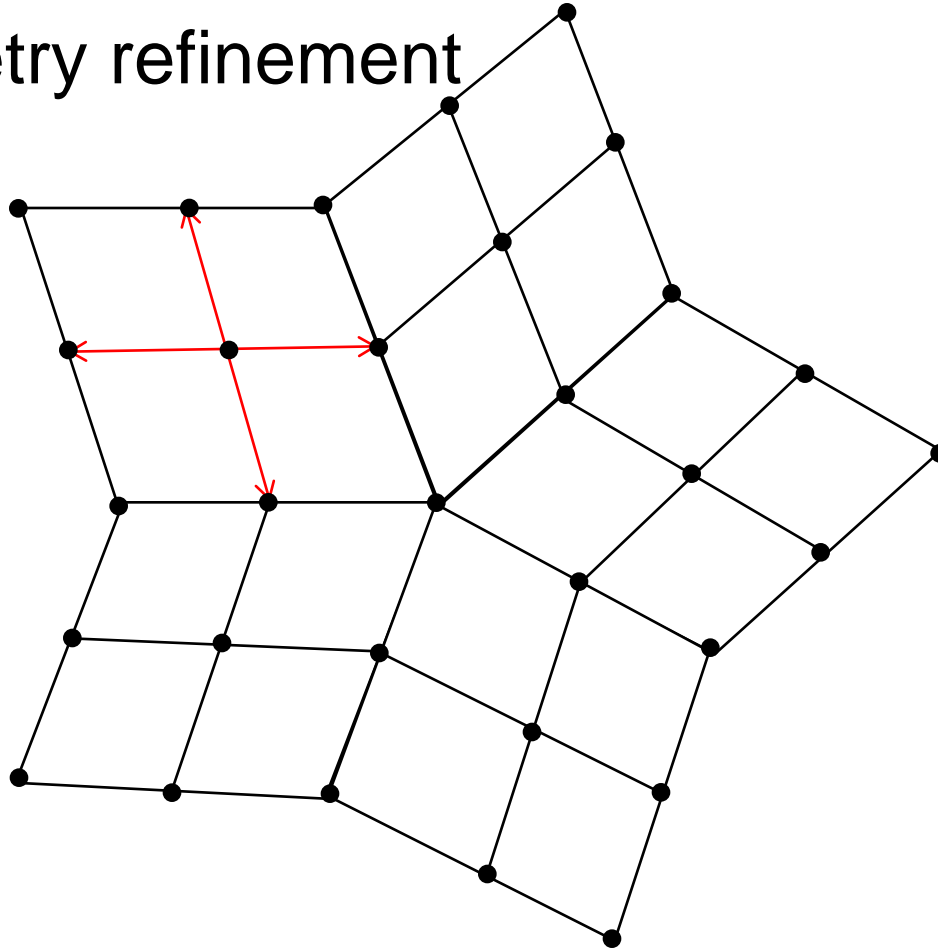
- Topology refinement



# Linear Subdivision



- Geometry refinement





# Linear Subdivision

LinearSubivision  $(F_0, V_0, k)$

for  $i = 1 \dots k$  levels

$(F_i, V_i) = \text{RefineTopology}(F_{i-1}, V_{i-1})$

$\text{RefineGeometry}(F_i, V_i)$

return  $(F_k, V_k)$



# Linear Subdivision

RefineTopology (  $F, V$  )

$newV = V$

$newF = \{ \}$

for each face  $F_i$

    Insert new vertex  $c$  at centroid of  $F_i$  into  $newV$

    for  $j = 1$  to 4

        Insert/lookup in  $newV$  new vertex  $e_j$  at  
        centroid of each edge (  $F_{i,j}, F_{i,j+1}$  )

    for  $j = 1$  to 4

        Insert new face (  $F_{i,j}, e_j, c, e_{j-1}$  ) into  $newF$

return (  $newF, newV$  )



# Linear Subdivision

RefineGeometry(  $F$ ,  $V$  )

$newV = 0 * V$

$wt$  = array of 0 whose size is number of vertices

$newF = F$

for each face  $F_i$

$cent$  = centroid for  $F_i$

$newV[F_i] += cent$  // syntax: repeat for all vtx indices in  $F_i$

$wt[F_i] += 1$  // syntax: repeat for all vtx indices in  $F_i$

for each vertex  $newV[i]$

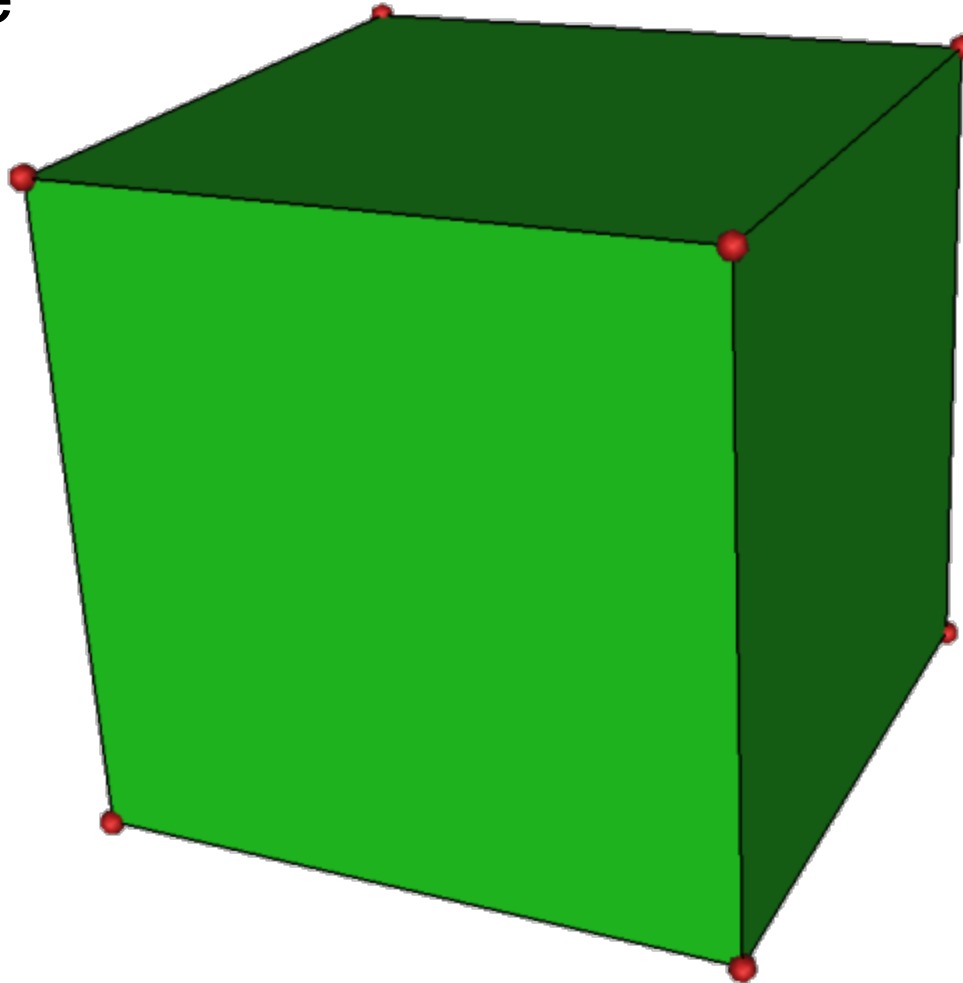
$newV[i] /= wt[i]$

return ( $newF$ ,  $newV$ )

# Linear Subdivision



- Example



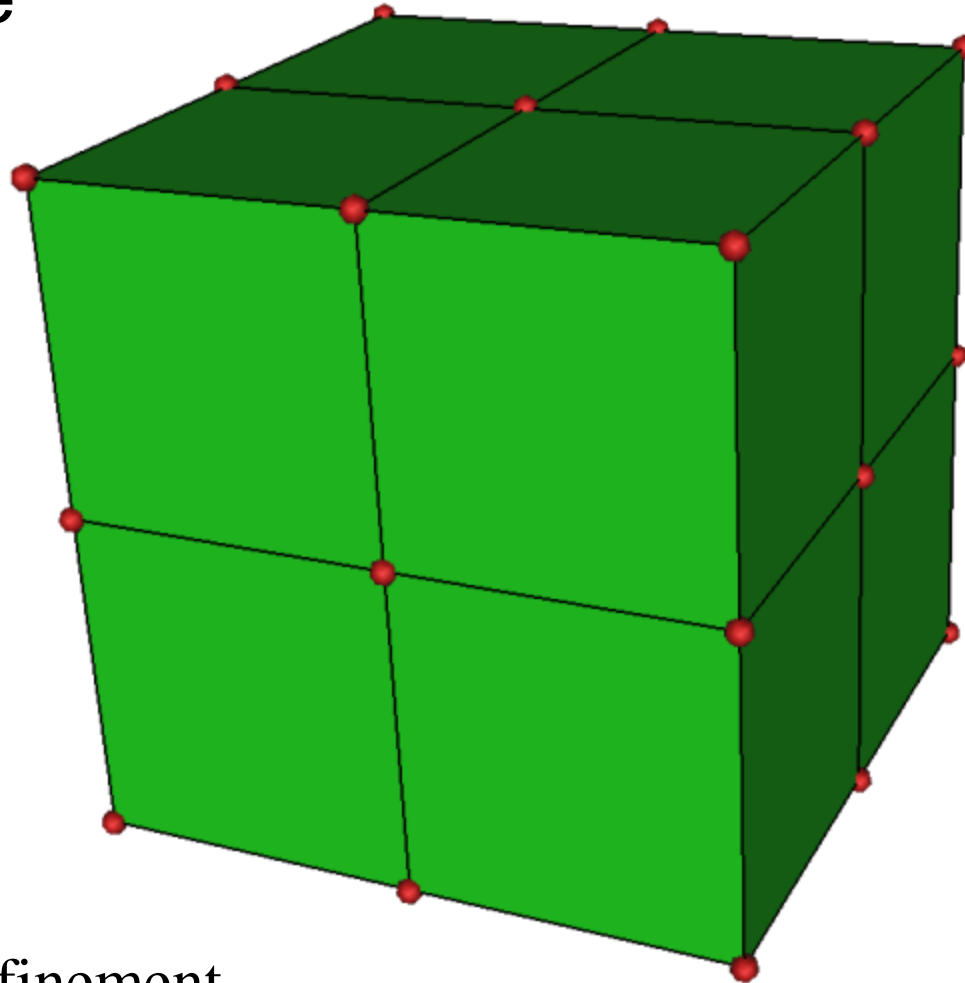
Input mesh

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# Linear Subdivision



- Example



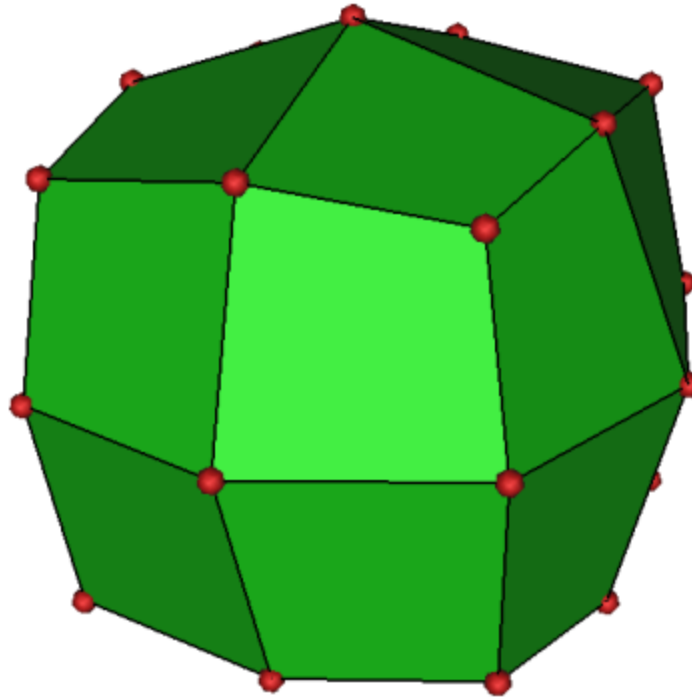
Topology refinement

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# Linear Subdivision



- Example



Geometry refinement

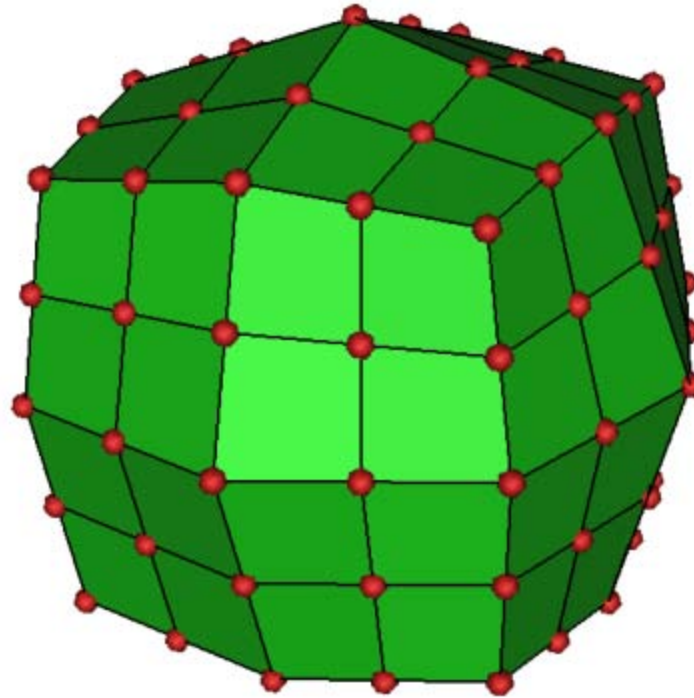
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# Linear Subdivision



- Example



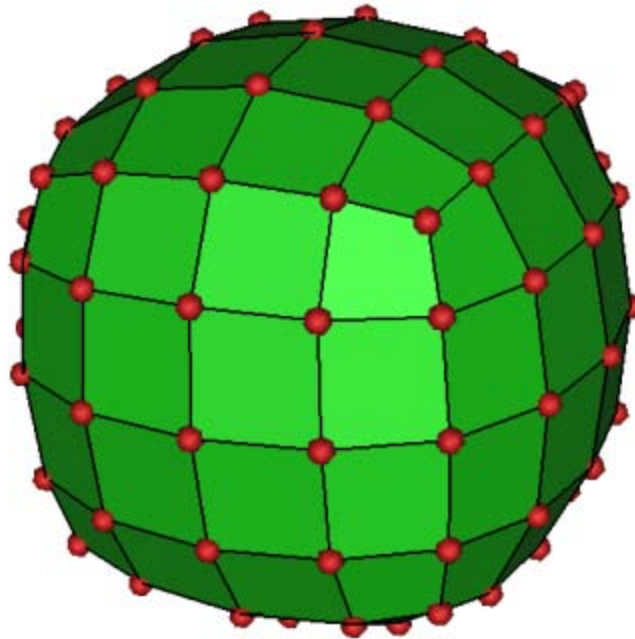
Topology refinement

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# Linear Subdivision



- Example



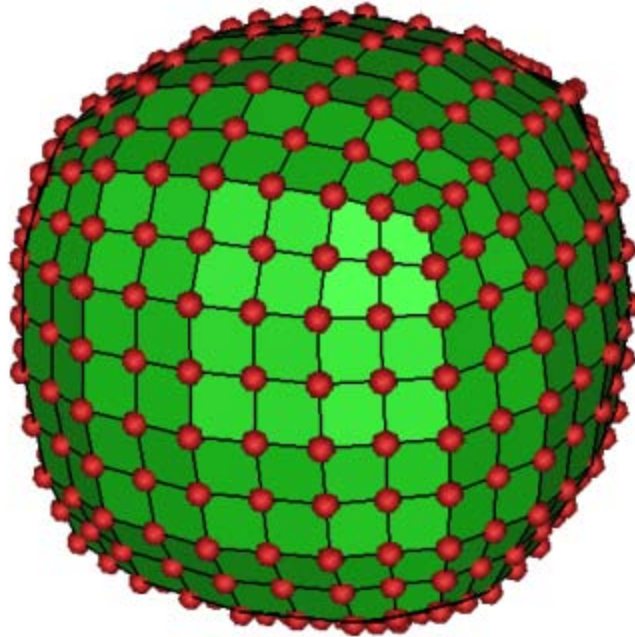
Geometry refinement

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# Linear Subdivision



- Example



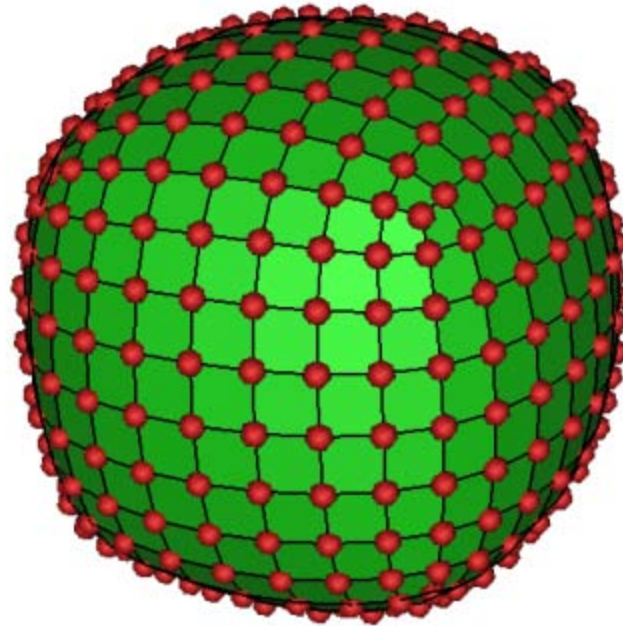
Topology refinement

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# Linear Subdivision



- Example



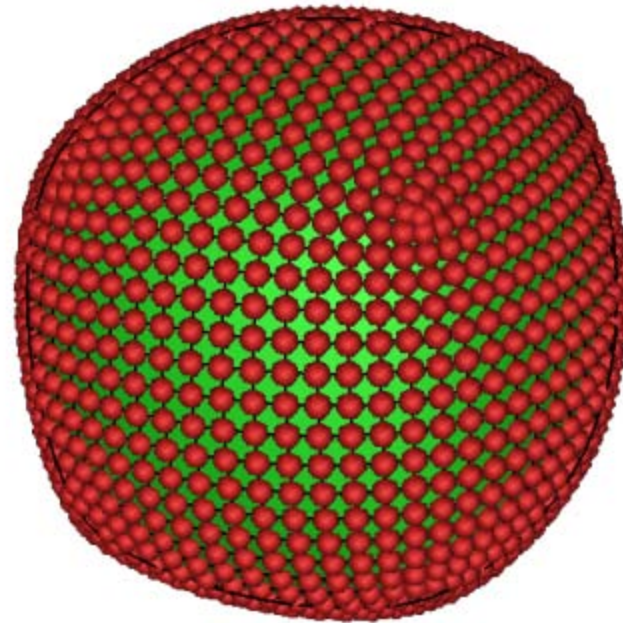
Geometry refinement

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# Linear Subdivision



- Example



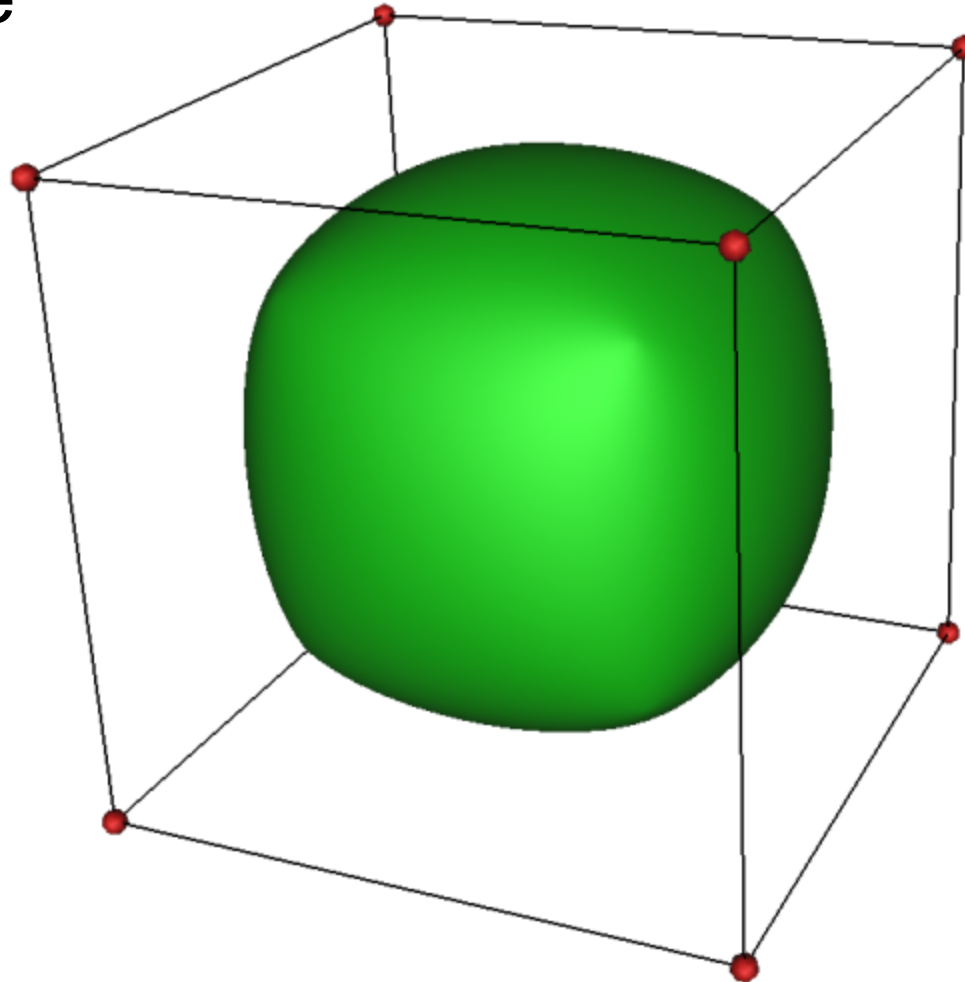
Topology refinement

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# Linear Subdivision



- Example



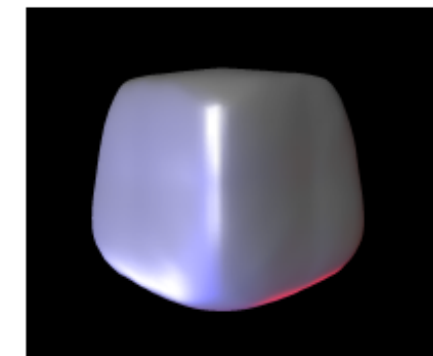
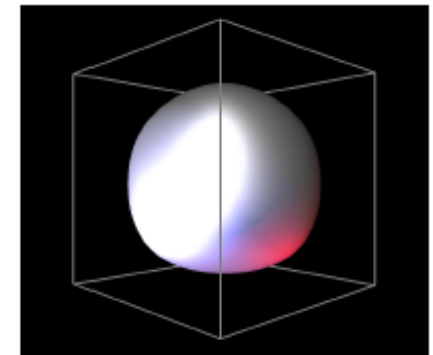
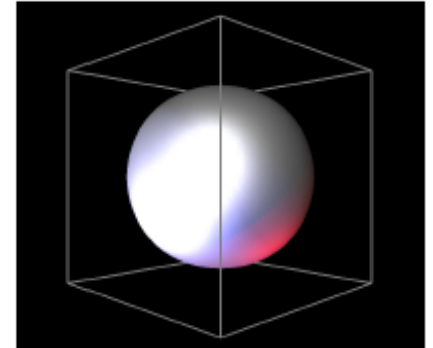
Final result

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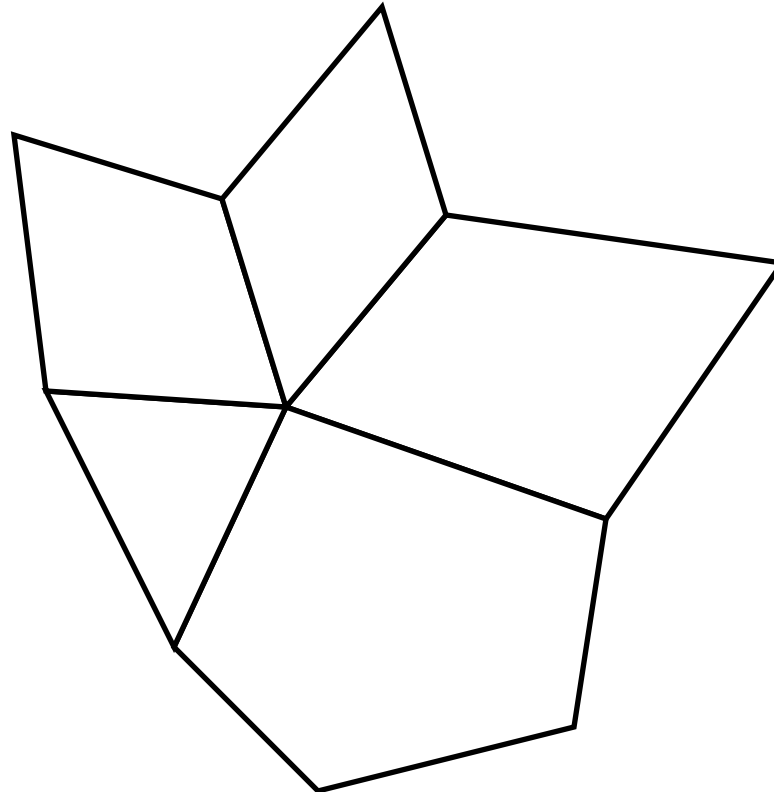
# Subdivision Schemes



- Common subdivision schemes
    - Catmull-Clark
    - Loop
    - Many others
  - Differ in ...
    - Input topology
    - How refine topology
    - How refine geometry
- ... which makes differences in ...
- Provable properties



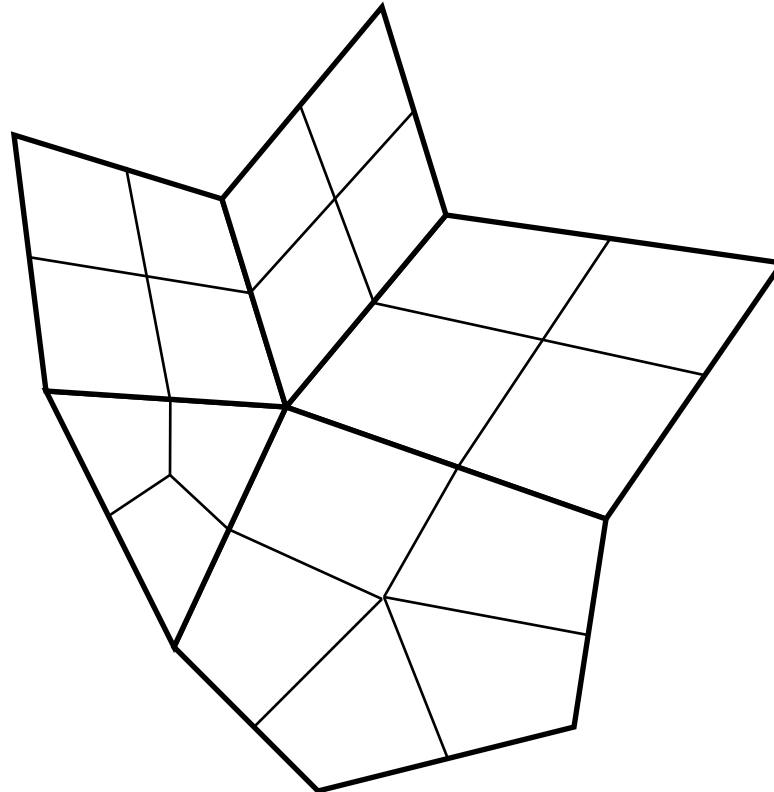
# Catmull-Clark Subdivision



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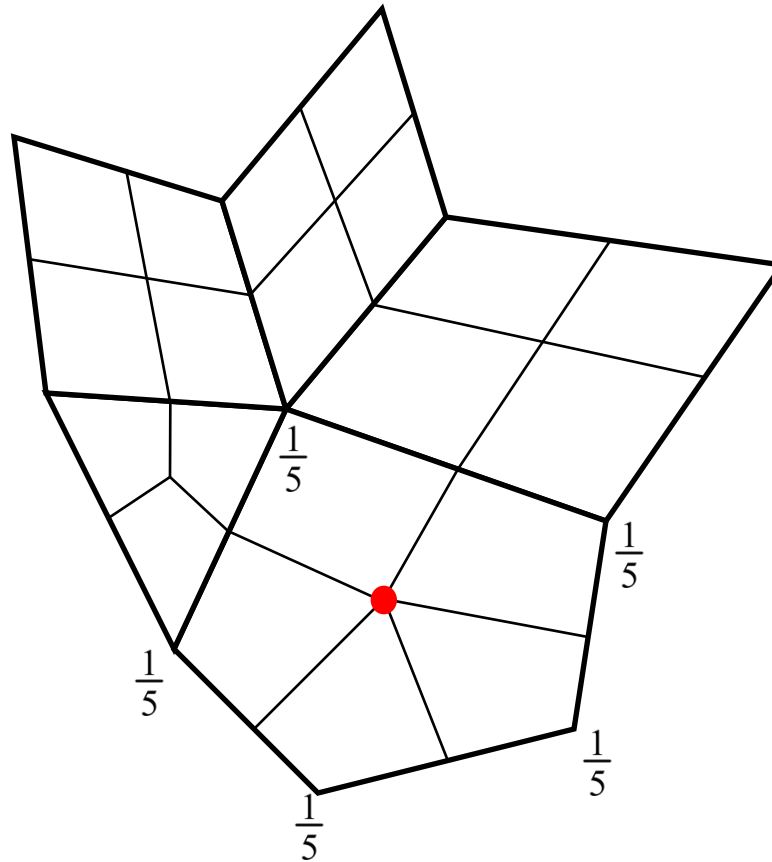


# Catmull-Clark Subdivision

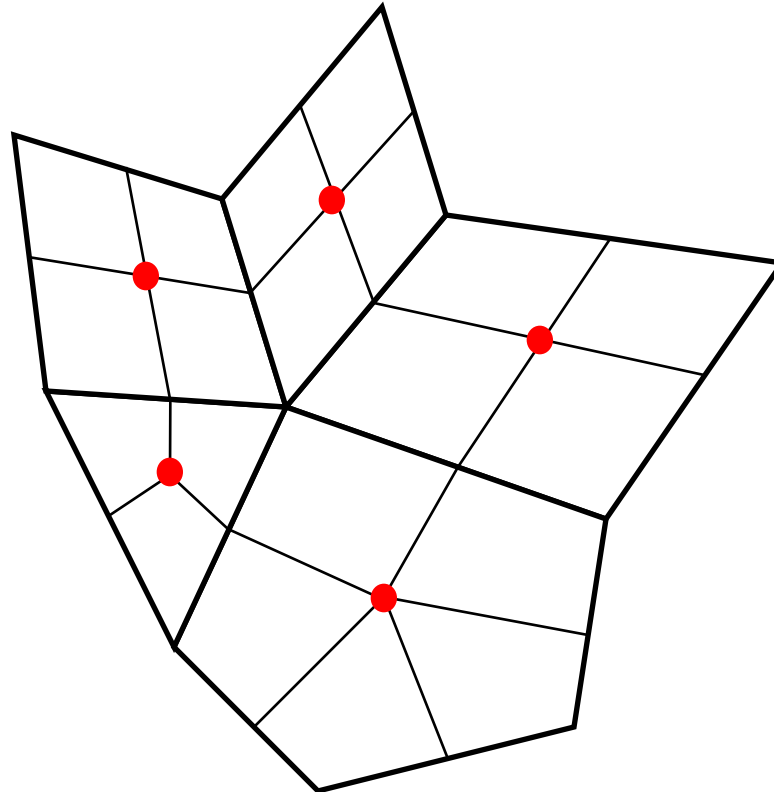


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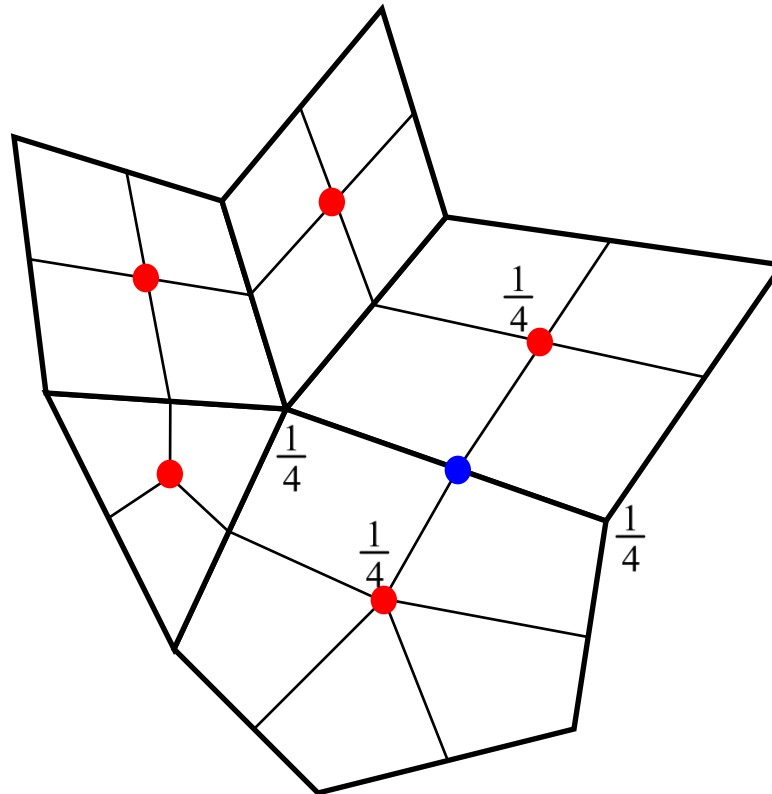
# Catmull-Clark Subdivision



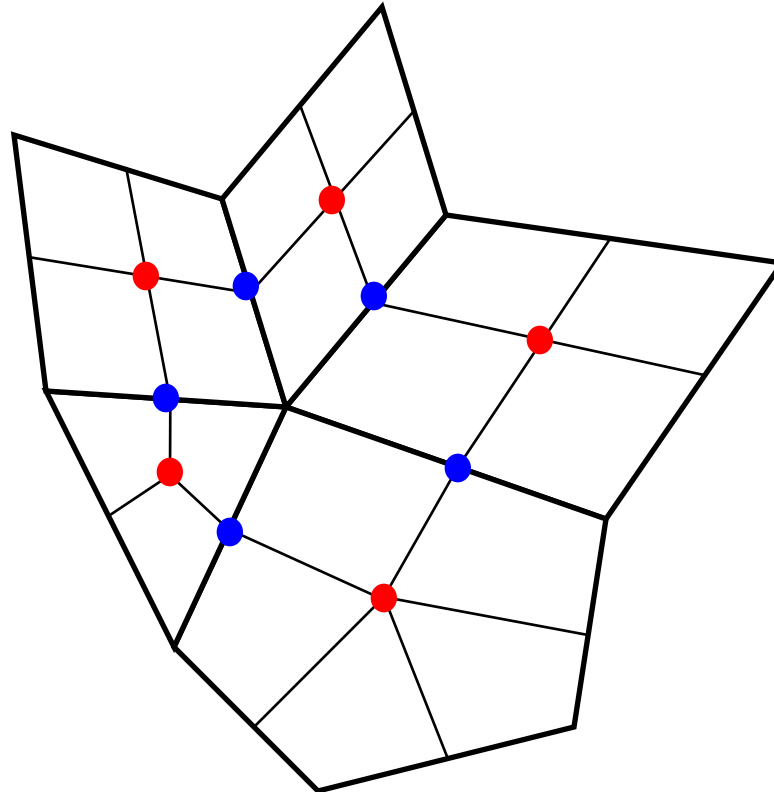
# Catmull-Clark Subdivision



# Catmull-Clark Subdivision



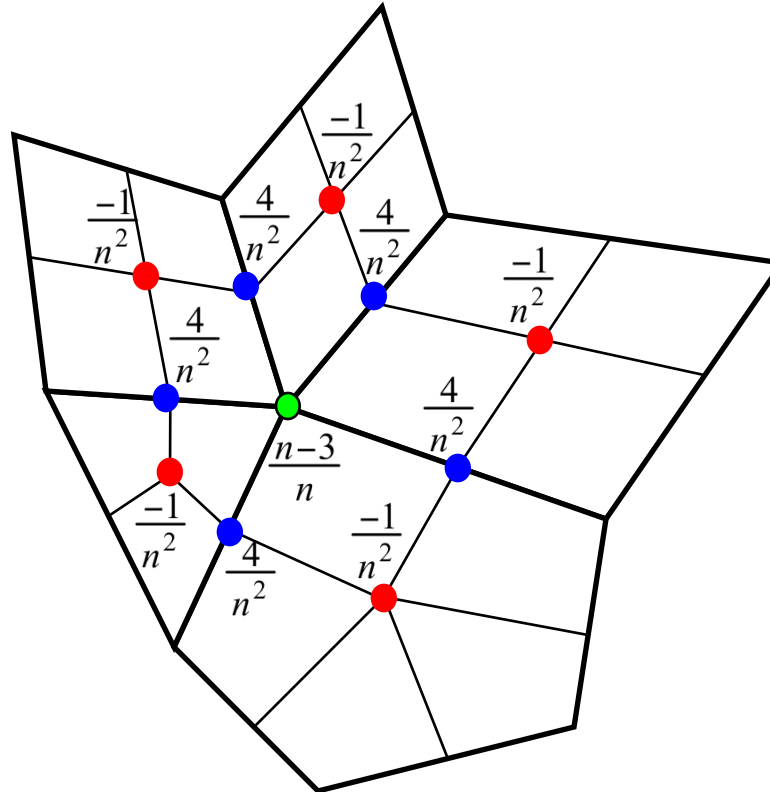
# Catmull-Clark Subdivision



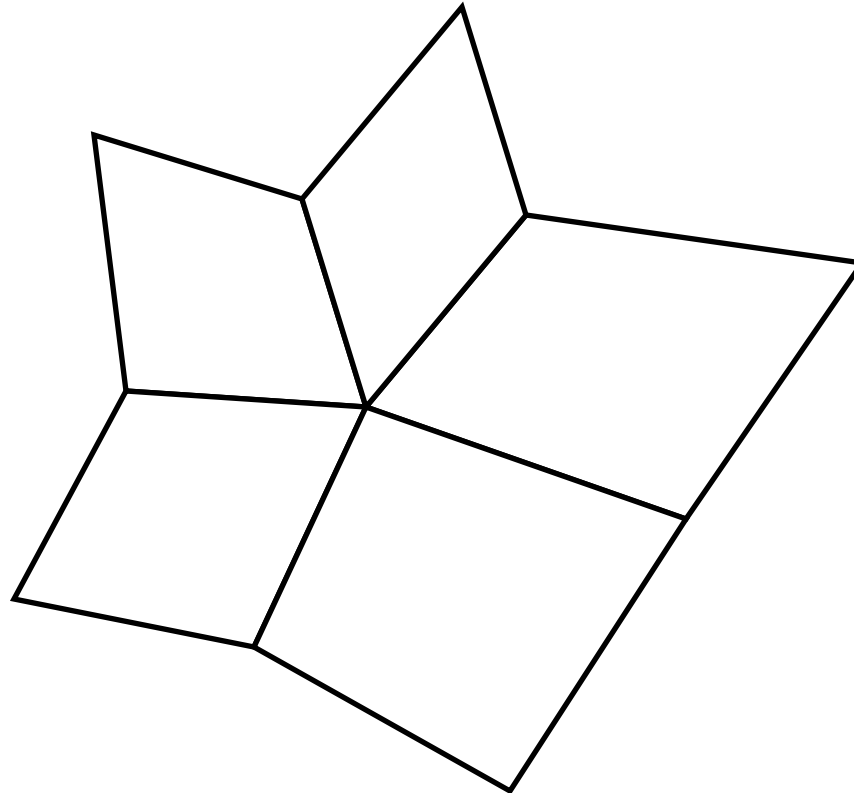
# Catmull-Clark Subdivision



$$\text{New } \bullet = \left( 4 * \text{avg of } \bullet - 1 * \text{avg of } \bullet + (n-3) * \bullet \right) / n$$

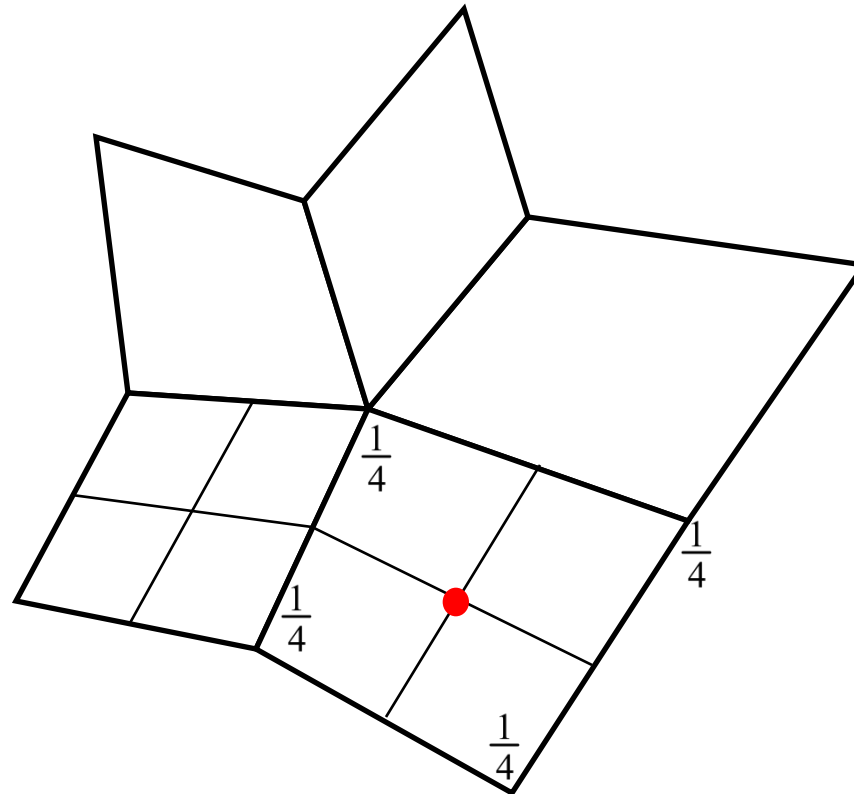


# Catmull-Clark Subdivision



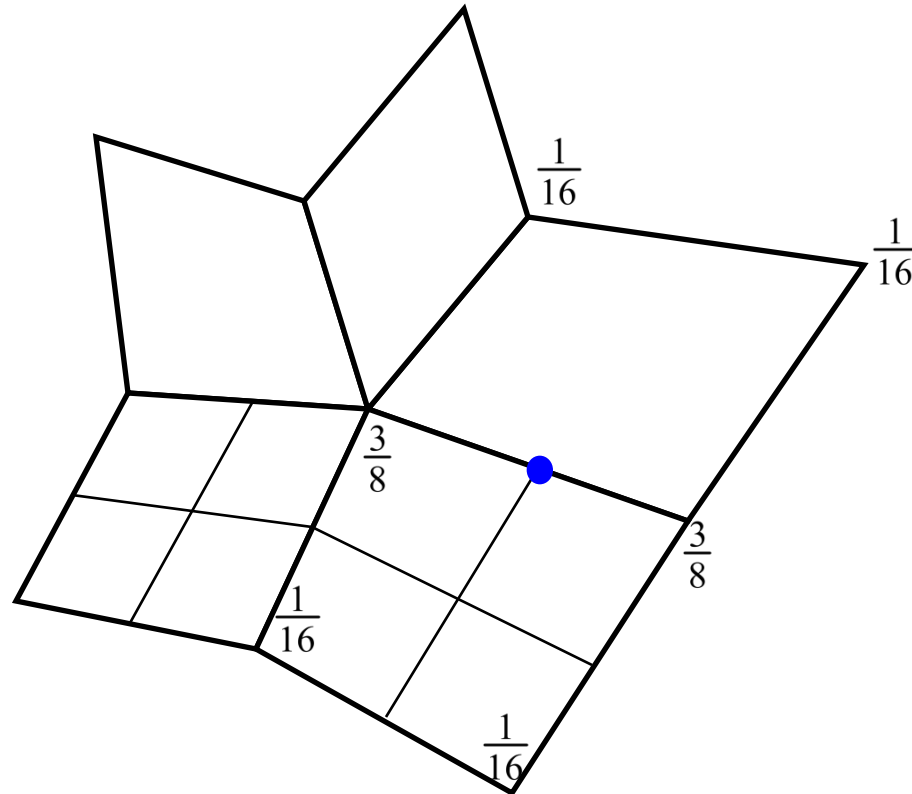
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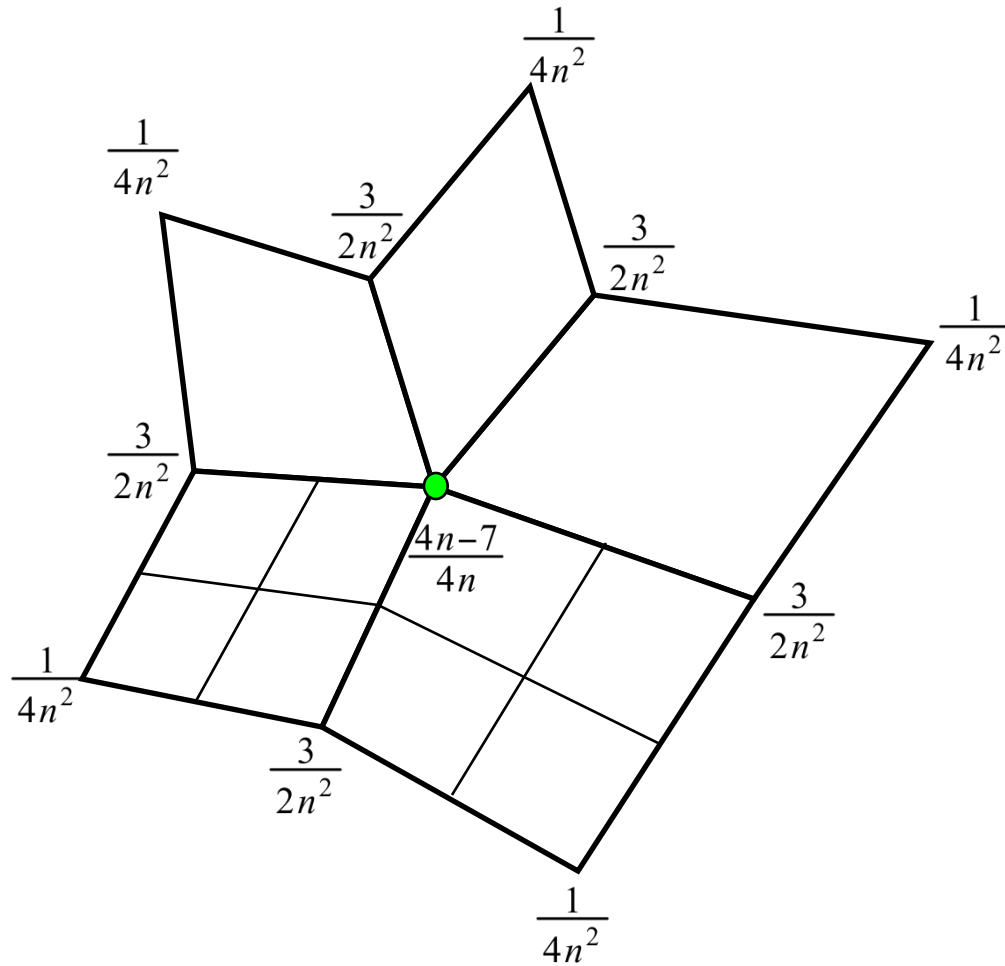




# Catmull-Clark Subdivision



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# Catmull-Clark Subdivision

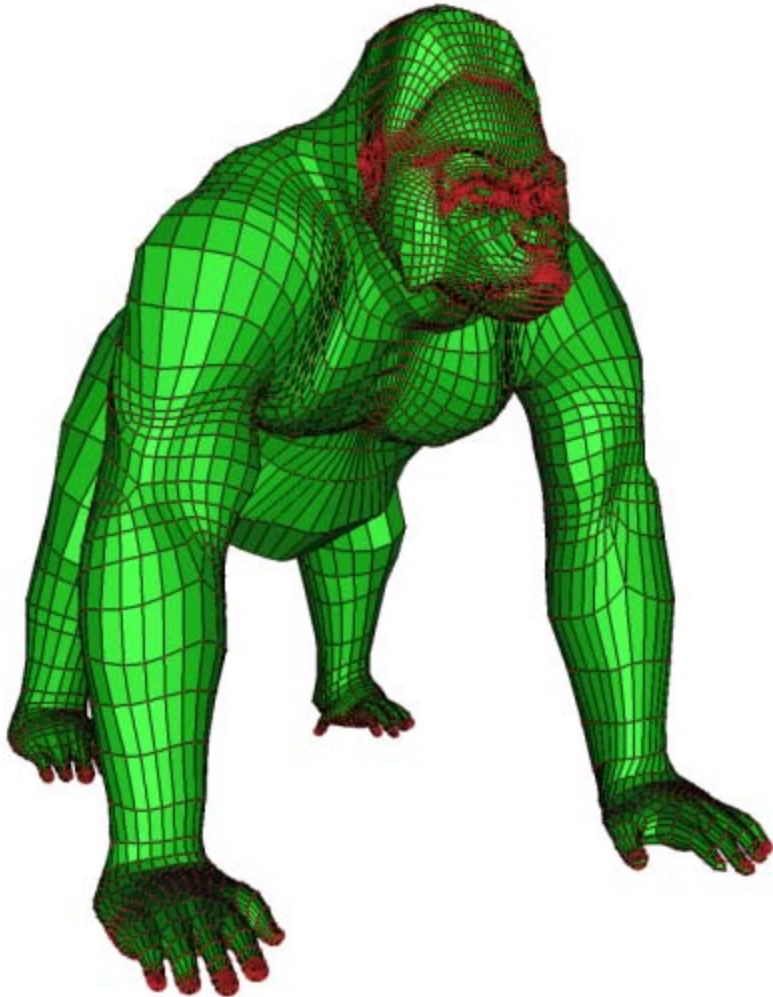


Repeated  
Averaging



Catmull-Clark  
Subdivision

# Catmull-Clark Subdivision



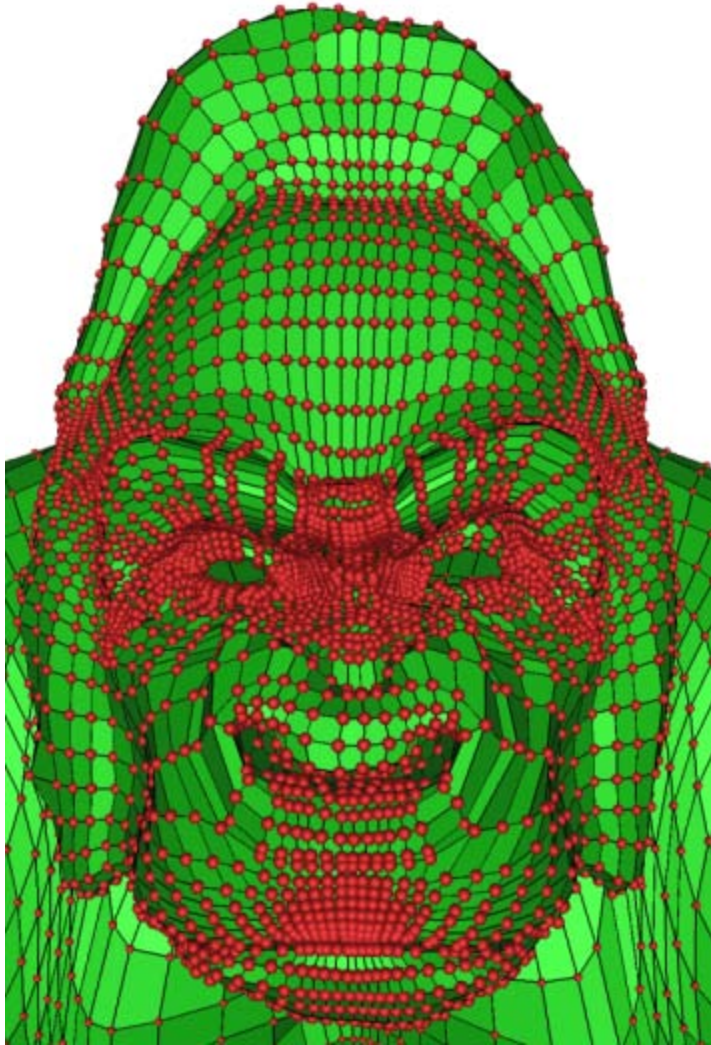
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# Catmull-Clark Subdivision



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# Catmull-Clark Subdivision



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# Catmull-Clark Subdivision



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# Catmull-Clark Subdivision



- One round of subdivision produces all quads
- Smoothness of limit surface
  - $C^2$  almost everywhere
  - $C^1$  at vertices with valence  $\neq 4$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
- Most commonly used subdivision scheme in the movies...

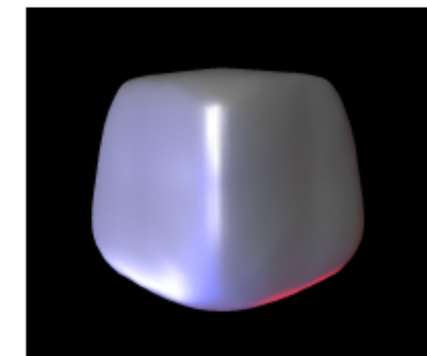
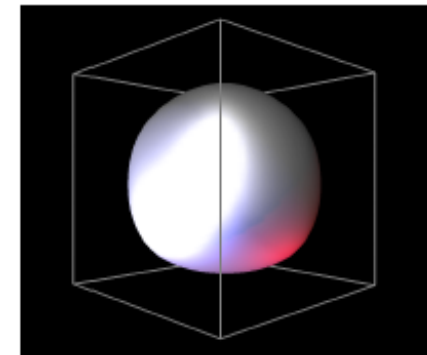
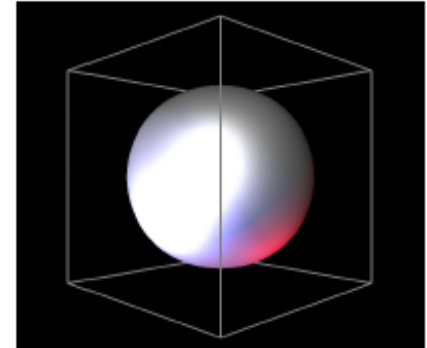


Pixar



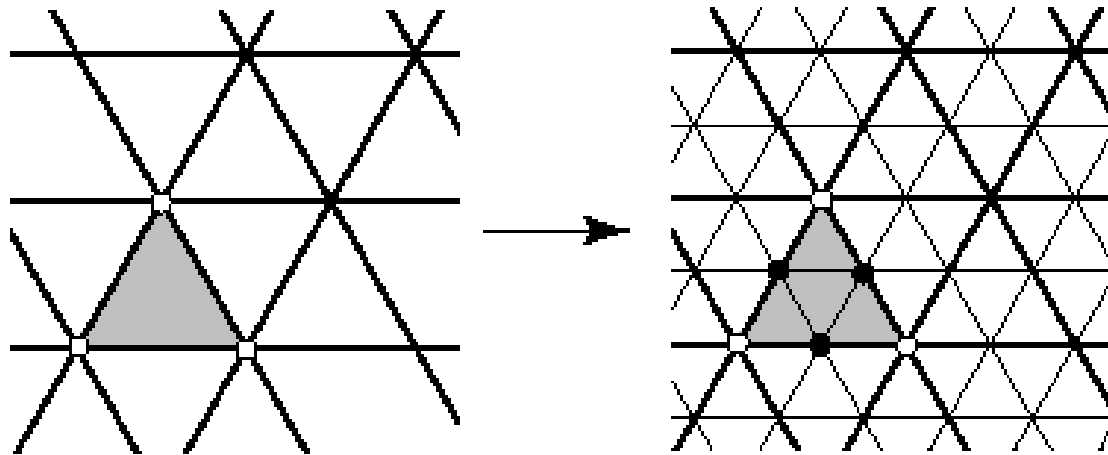
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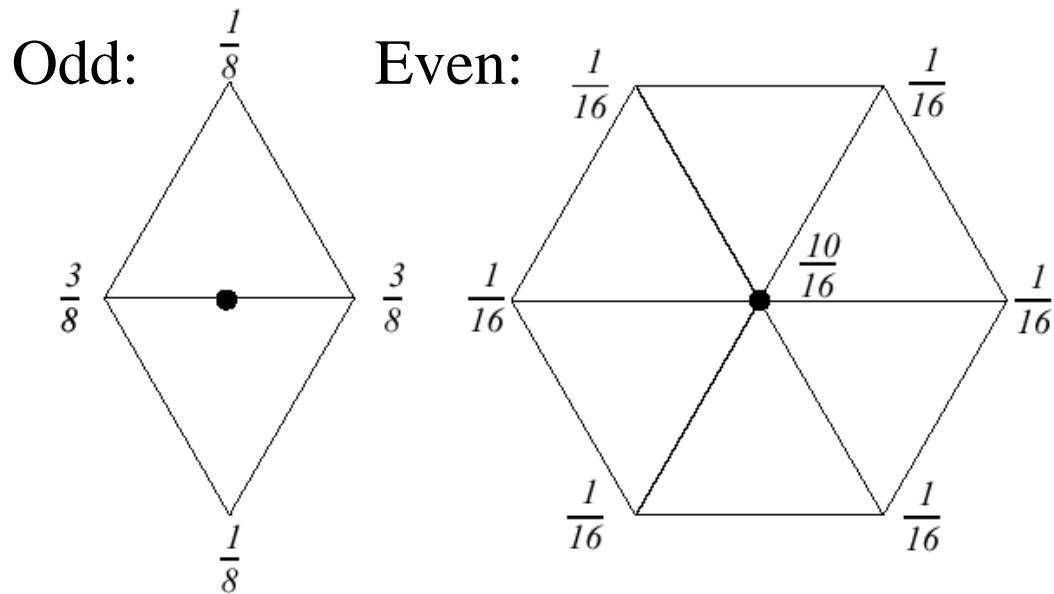
# Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - Linear subdivision
  - Averaging rules for “even / odd” (white / black) vertices



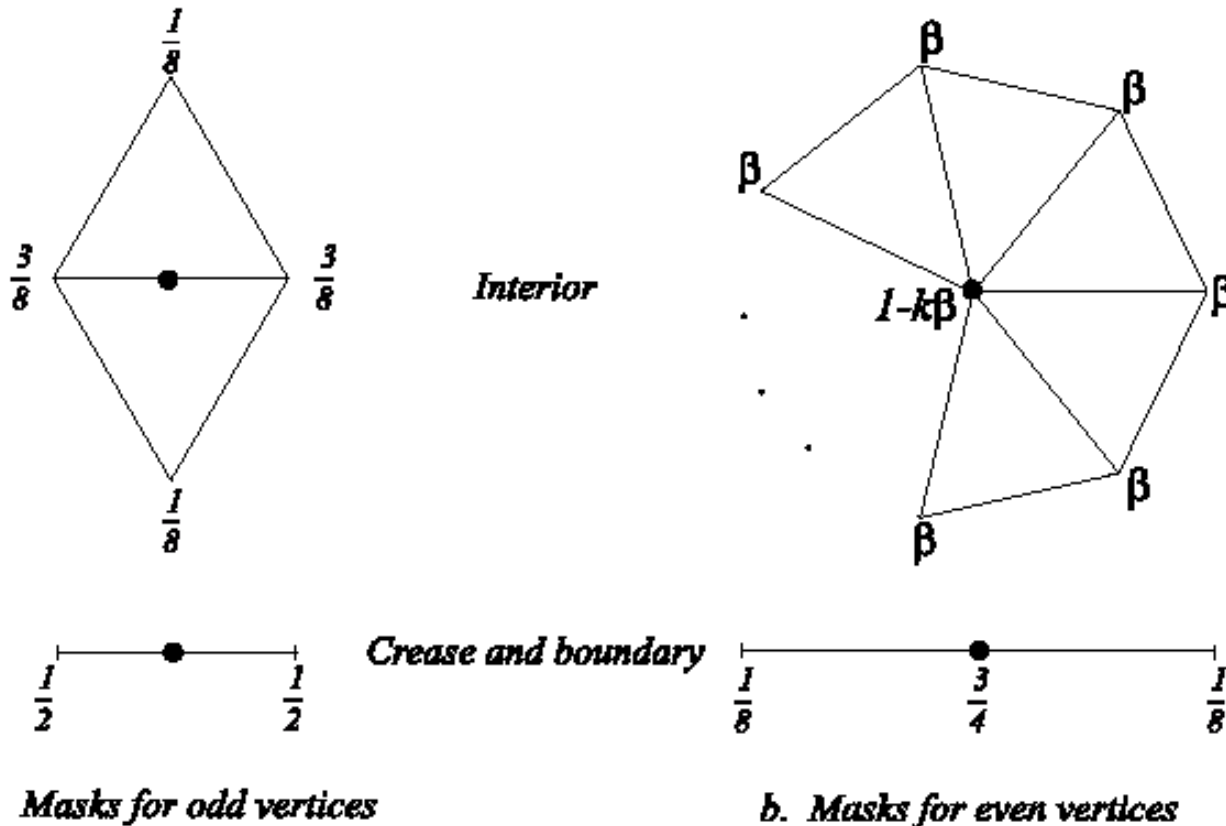
# Loop Subdivision

- Averaging rules
  - Weights for “odd” and “even” vertices



# Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:





# Loop Subdivision

- How to choose  $\beta$ ?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness
  - Involves calculating eigenvalues of matrices

» Original Loop

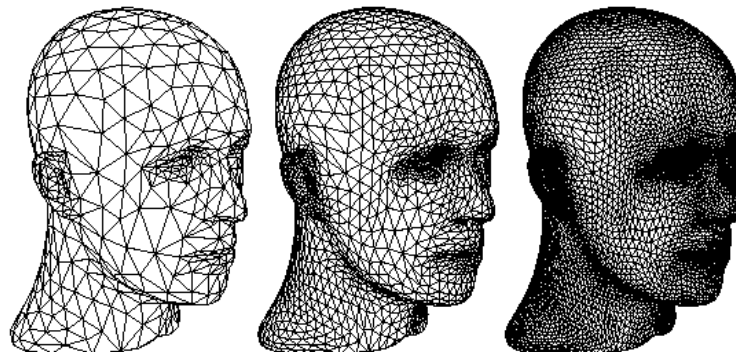
$$\beta = \frac{1}{n} \left( \frac{5}{8} - \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right)$$

» Warren

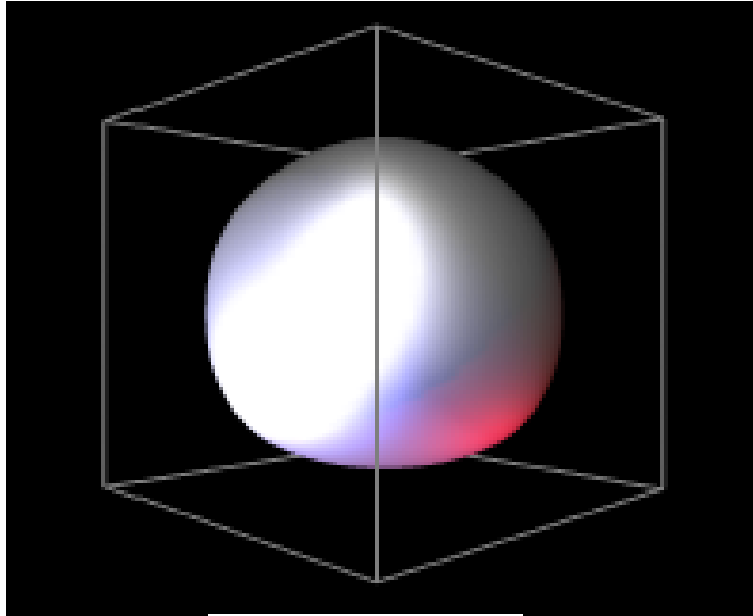
$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

# Loop Subdivision

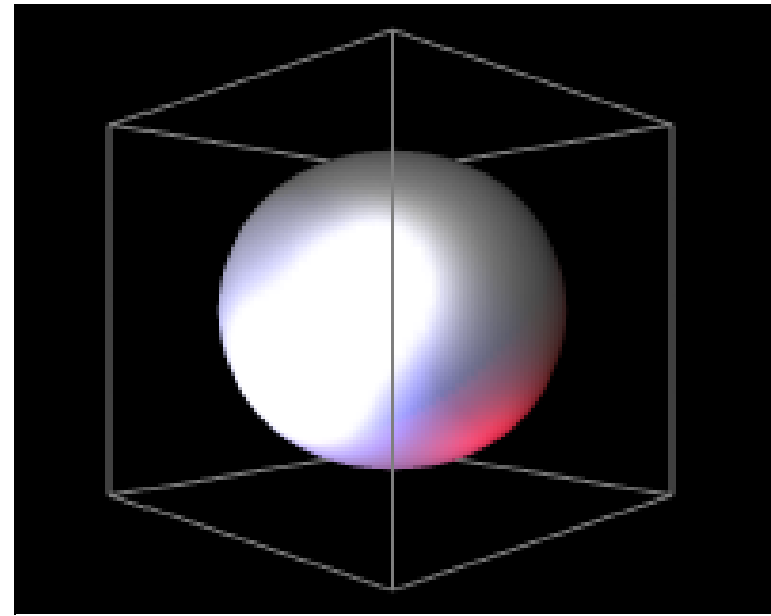
- Operates only on triangle meshes
- Smoothness of limit surface
  - $C^2$  almost everywhere
  - $C^1$  at vertices with valence  $\neq 6$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull



# Subdivision Schemes

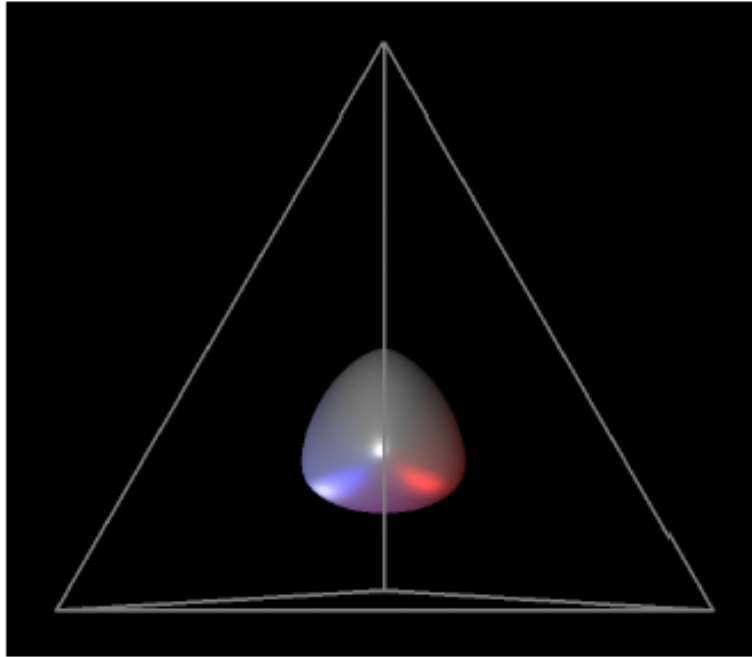


Loop

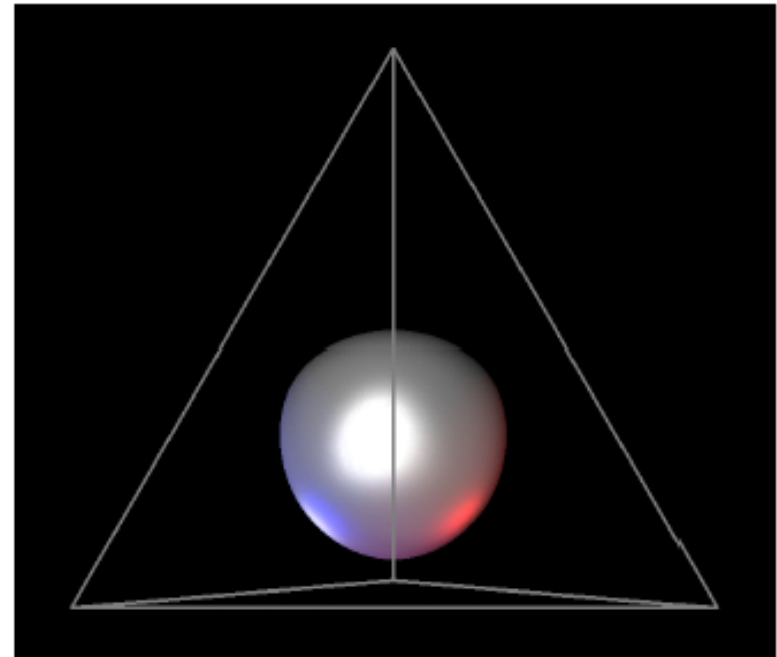


Catmull-Clark

# Subdivision Schemes



Loop



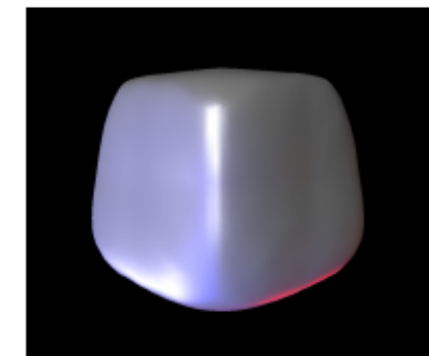
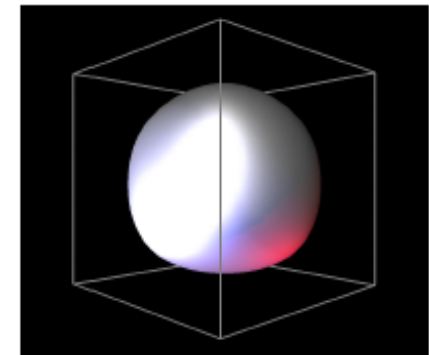
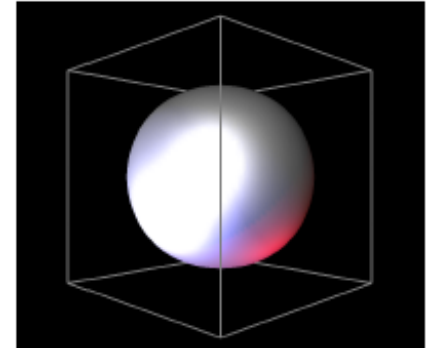
Catmull-Clark



# Subdivision Schemes



- Common subdivision schemes
    - Catmull-Clark
    - Loop
    - Many others
  - Differ in ...
    - Input topology
    - How refine topology
    - How refine geometry
- ... which makes differences in ...
- Provable properties





# Subdivision Schemes

- Other subdivision schemes

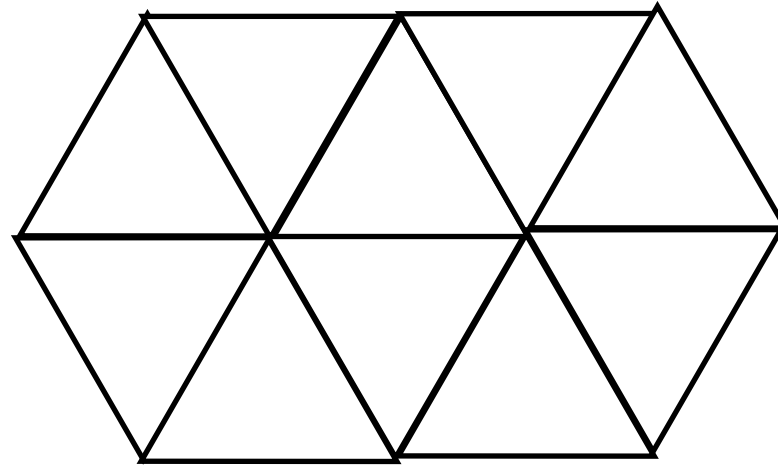
Face split		
	<i>Triangular meshes</i>	<i>Quad. meshes</i>
<i>Approximating</i>	Loop ( $C^2$ )	Catmull-Clark ( $C^2$ )
<i>Interpolating</i>	Mod. Butterfly ( $C^1$ )	Kobbelt ( $C^1$ )

Vertex split
Doo-Sabin, Midedge ( $C^1$ )
Biquartic ( $C^2$ )

# Other Subdivision Schemes



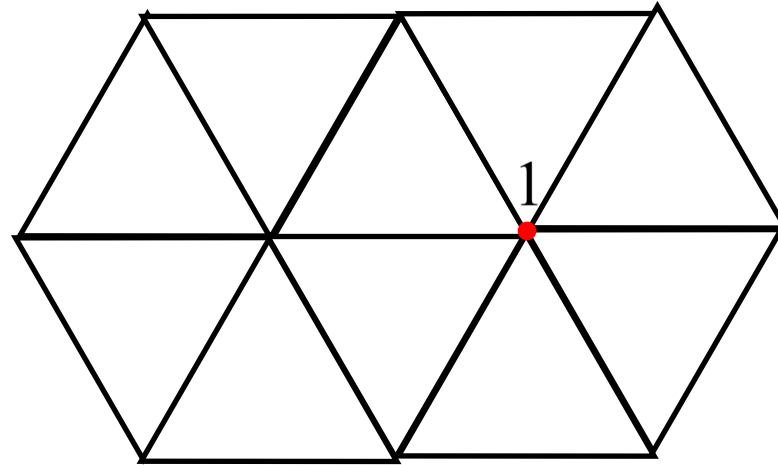
- Butterfly subdivision



# Other Subdivision Schemes



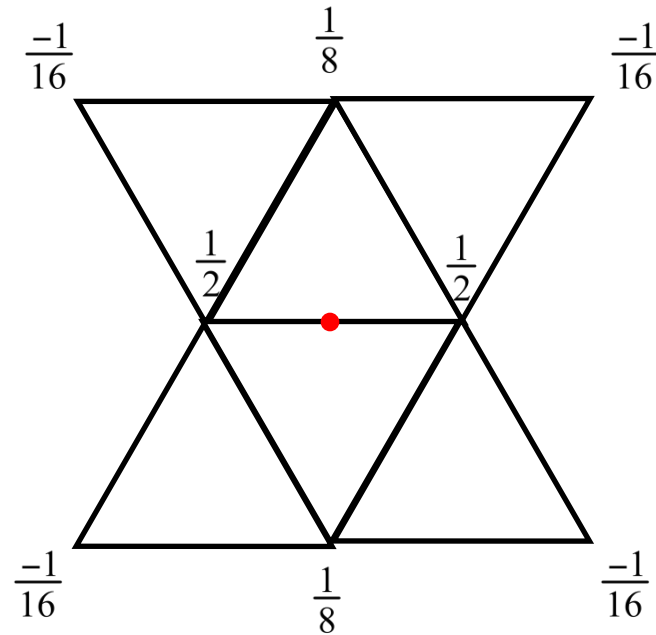
- Butterfly subdivision



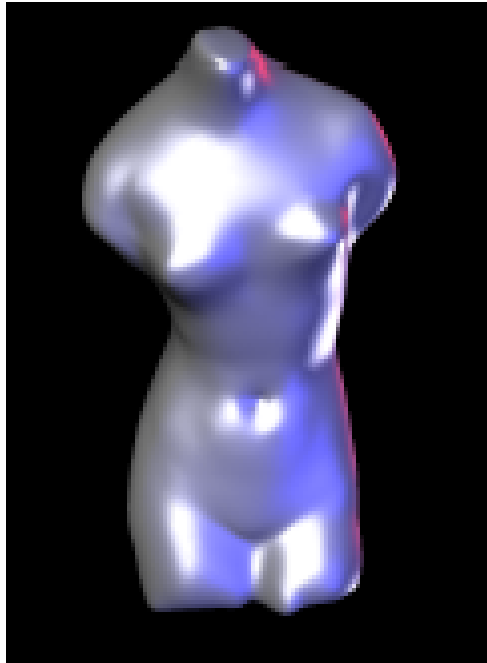
# Other Subdivision Schemes



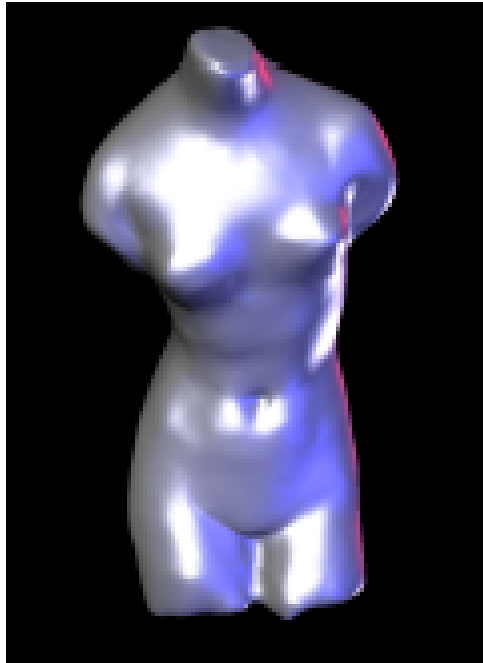
- Butterfly subdivision



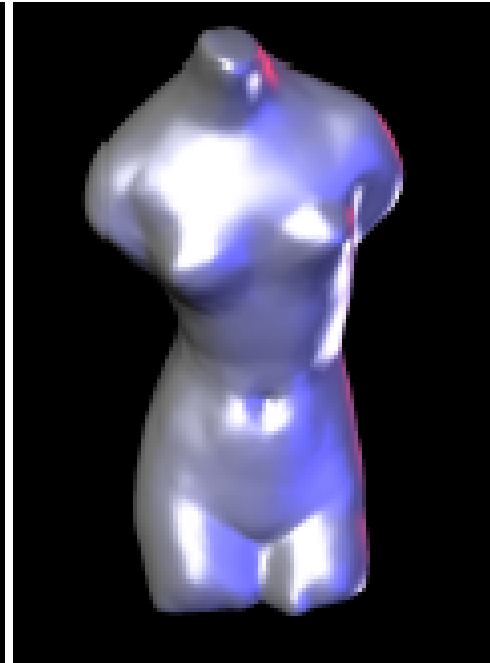
# Other Subdivision Schemes



Loop



Butterfly

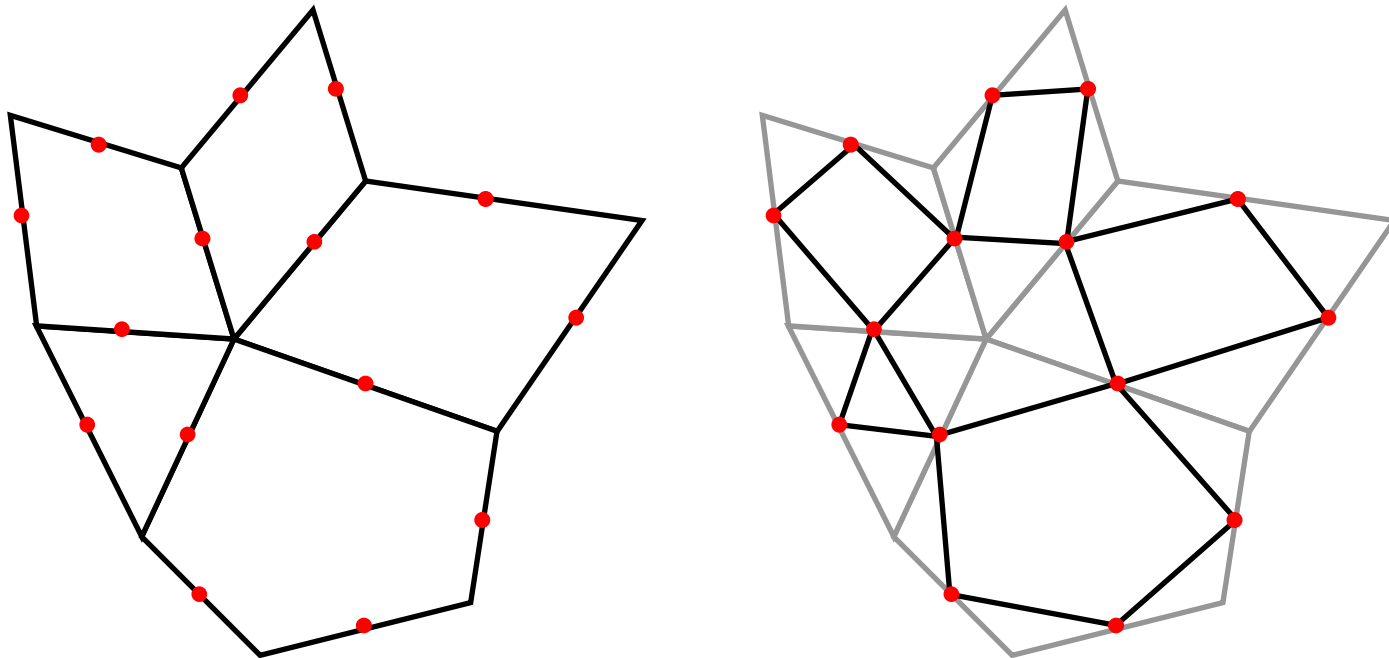


Catmull-Clark

# Other Subdivision Schemes



- Vertex-split subdivision  
(Doo-Sabin, Midedge, Biquartic)

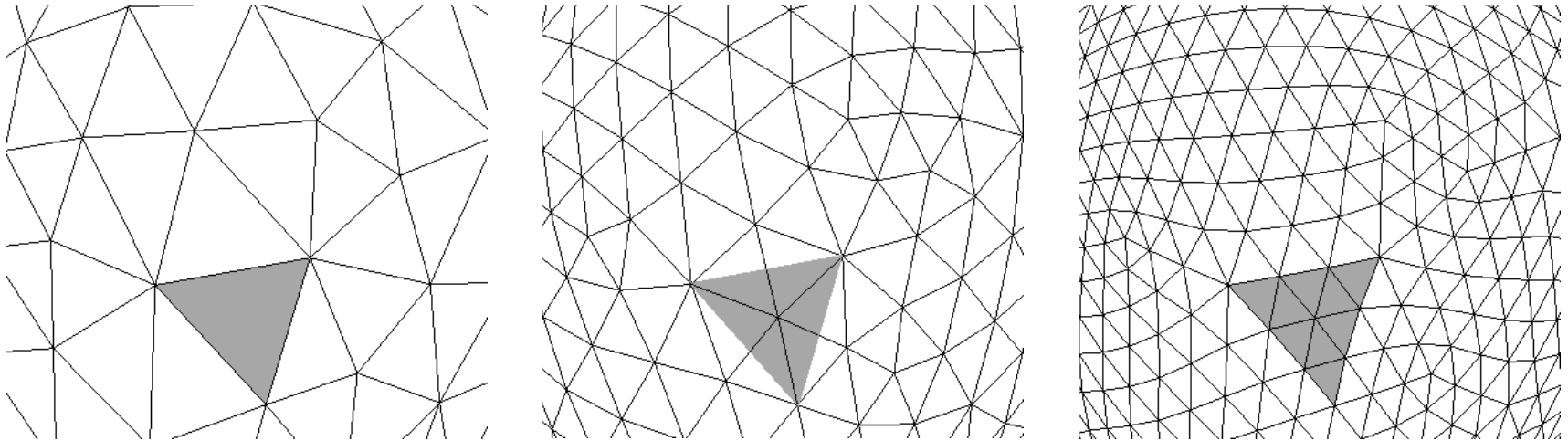


One step of Midedge subdivision

# Other Subdivision Schemes



- Sqrt(3) subdivision



Rotating grid of sqrt(3) subdivision





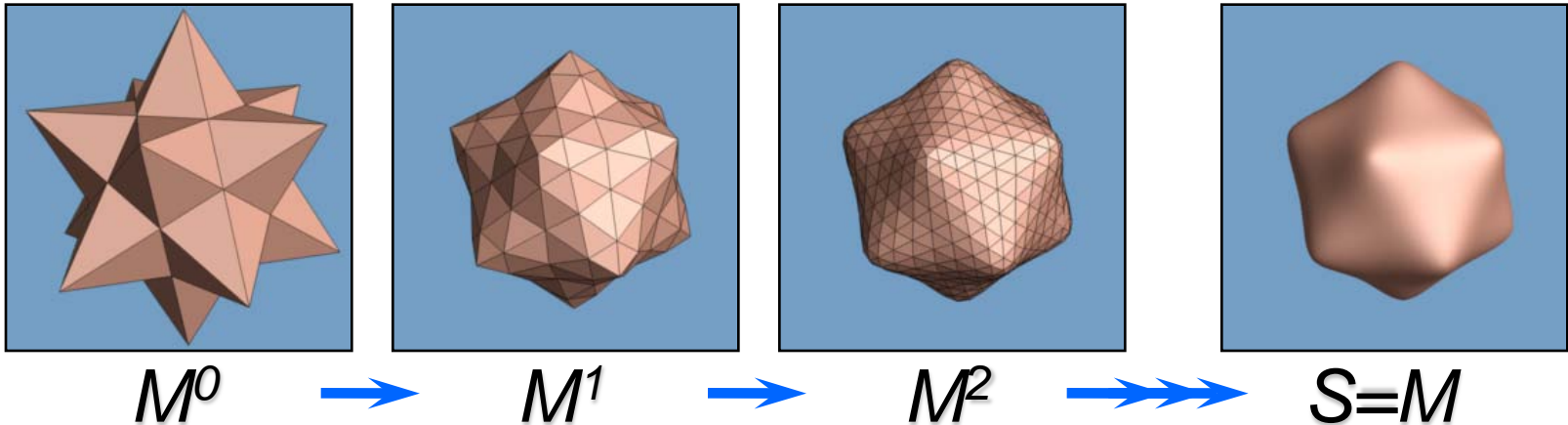
# Extensions

- Common schemes assume *stationary* rules
- Locally altering subdivision rules allows for additional effects
- Examples for non-stationary rules:
  - Edge preservation
  - Adaptive subdivision

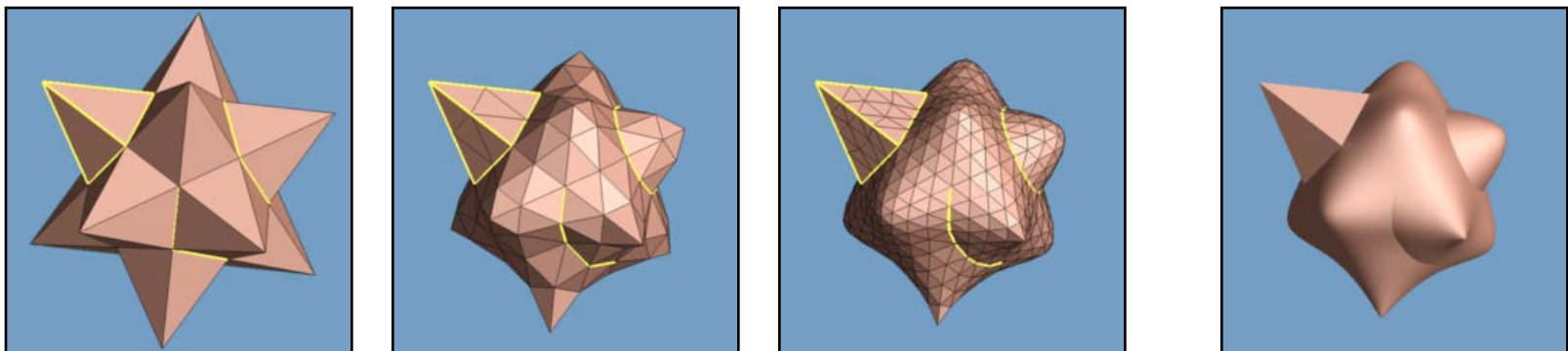
# Edge Preservation

- Treat selected control mesh edges as boundary:

[Loop '87]



[Hoppe et al. '94]



tagged mesh

# Adaptive Subdivision



- Goal:
  - Best possible approximation of smooth limit surface
  - With limited triangle budget
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature



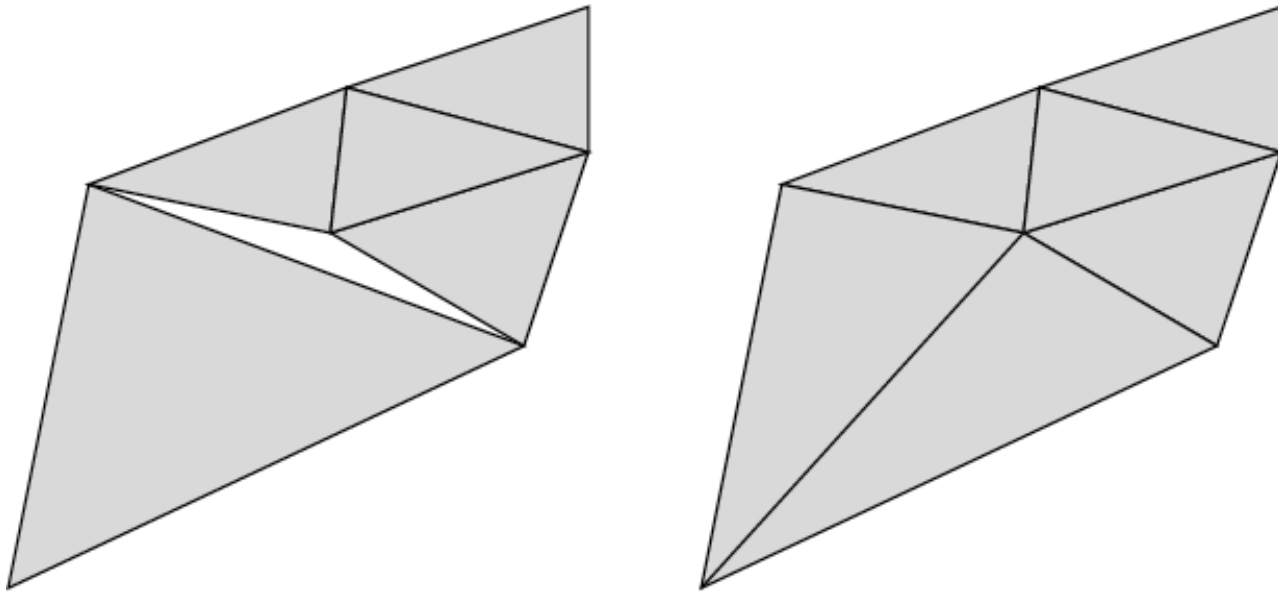
# Adaptive Subdivision

- Goal:
  - Best possible approximation of smooth limit surface
  - With limited triangle budget
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature
- Solution:
  - Stop subdivision at different levels across the surface
  - Stop-criterion depending on quality measure
  - Project each vertex onto limit surface

# Adaptive Subdivision



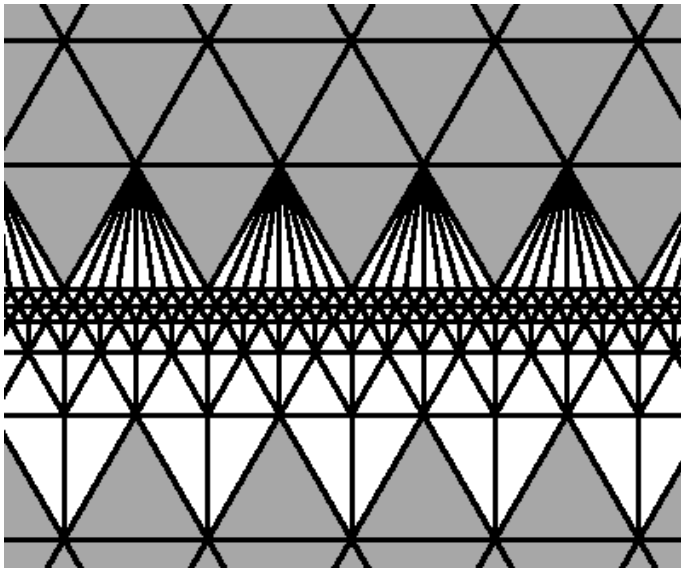
- Problem:
  - Different levels of subdivision may lead to gaps in the surface



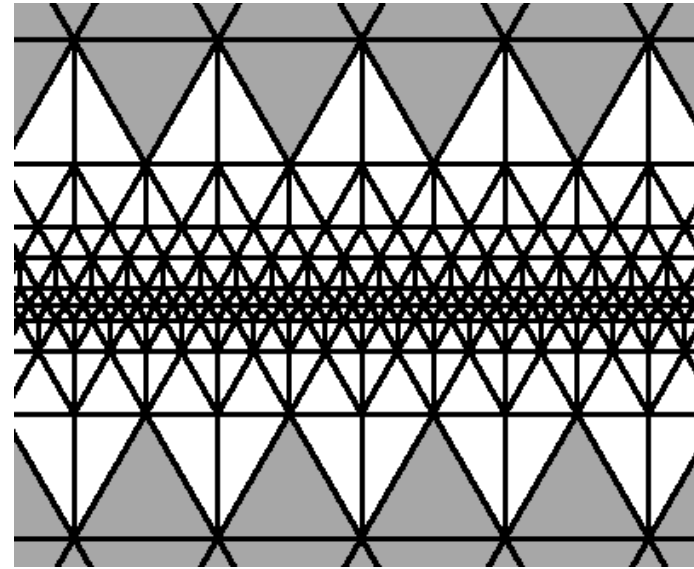
# Adaptive Subdivision



- Solution:
  - Replacing incompatible coarse triangles by *triangle fan*
  - Balanced subdivision: neighboring subdivision levels must not differ by more than one



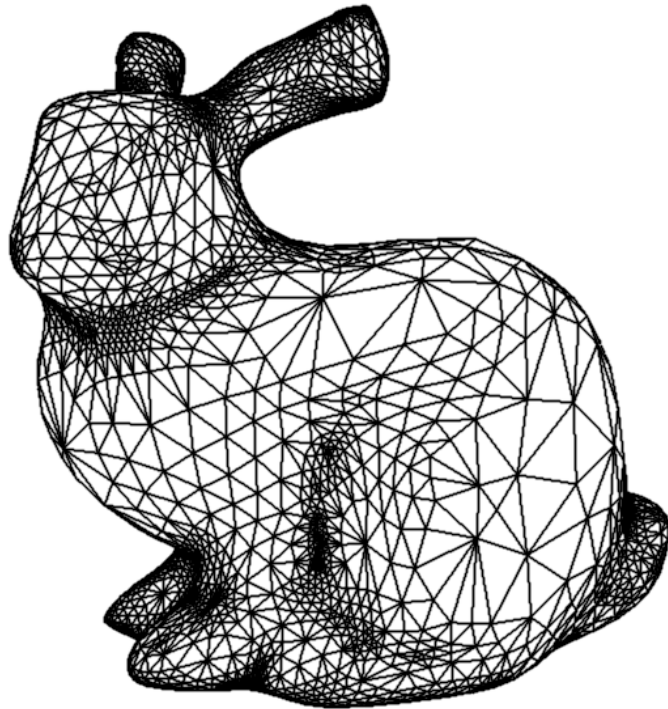
Unbalanced



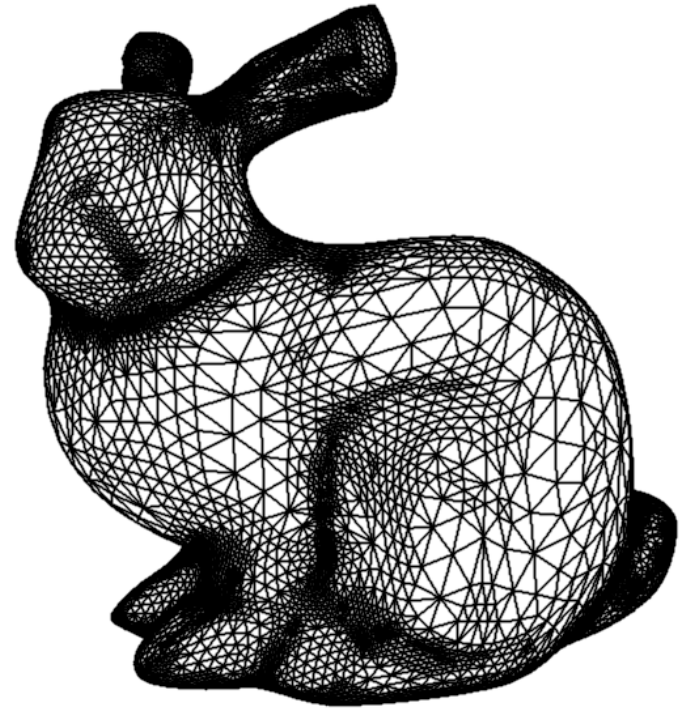
Balanced

[Kobbelt 2000]

# Adaptive Subdivision



10072 Triangles



228654 Triangles

[Kobbelt 2000]

# Subdivision Surfaces



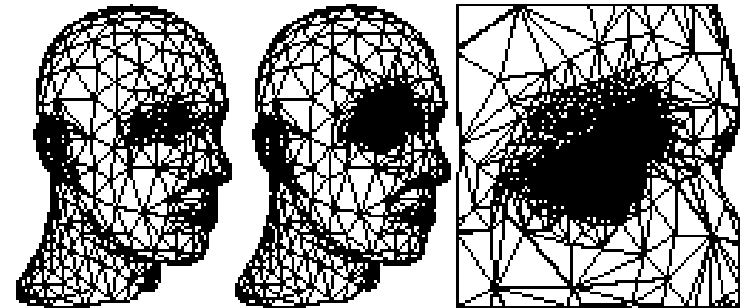
- Properties:
  - o Accurate
  - o Concise
  - o Intuitive specification
  - o Local support
  - o Affine invariant
  - o Arbitrary topology
  - o Guaranteed continuity
  - o Natural parameterization
  - o Efficient display
  - o Efficient intersections





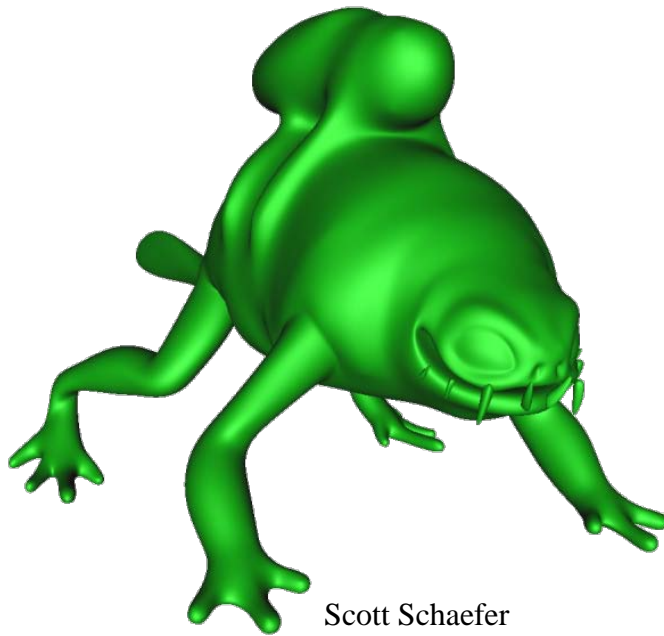
# Subdivision Surfaces

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Local support
  - Guaranteed continuity
  - Multiresolution
- Difficulties:
  - Intuitive specification
  - Parameterization
  - Intersections

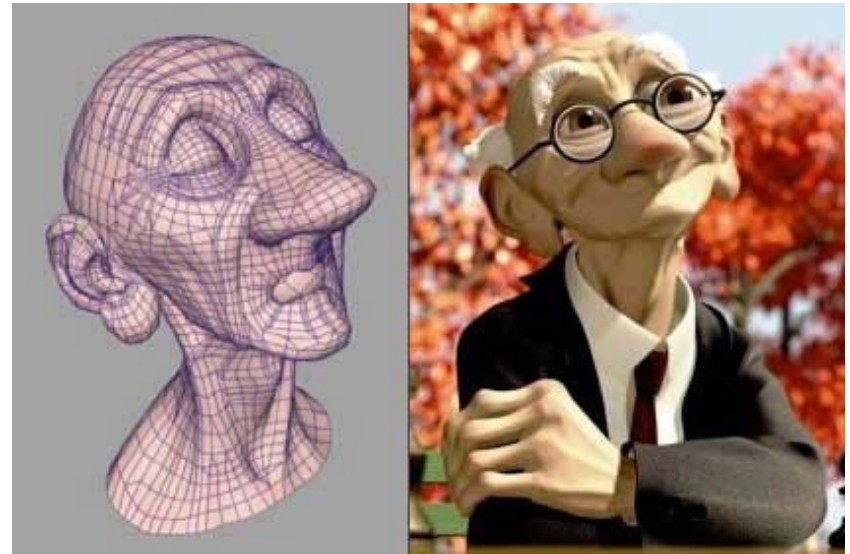


# Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software



Scott Schaefer



Geri's Game © Pixar Animation Studios

# Summary



Feature	Polygonal Mesh	Subdivision Surface
Accurate	No	Yes
Concise	No	Yes
Intuitive specification	No	No
Local support	Yes	Yes
Affine invariant	Yes	Yes
Arbitrary topology	Yes	Yes
Guaranteed continuity	No	Yes
Natural parameterization	No	No
Efficient display	Yes	Yes
Efficient intersections	No	No