Bulow & Klemperer 96 (#7)

Auctions vs. Negotiations — we'll restrict ourselves to I.P.V. model, risk neutral

Simple example — Seller wants to sell 1 object, value = 0. v's uniform on $[0,1]$, (always $\bar{v} = 1$)

Case I. sells to one buyer. optimal auction is achieved if v_* is chosen so that $M(v_*) = 0$.

marginal revenue

$$M(v) = v - \frac{1-F}{f} = 2v - 1 \Rightarrow v_* = 1/2$$

with one buyer this is optimal mechanism of any sort.

$$E[\text{revenue}] = \int_{v_*}^{1} (2v-1)\,dv = \int_{1/2}^{1} (2v-1)\,dv = v^2 \Big|_{1/2}^{1} - \frac{1}{2} = \frac{1}{4}.$$

Case II. Sells to two buyers (Same F), no reserve; that is, reserve $= 0$.

$$E[\text{revenue}] = \int_0^1 (2v-1)\,d[v^2] = \int_0^1 (2v-1)\,2v\,dv$$

$$= \frac{4}{3} v^3 \Big|_0^1 - v^2 \Big|_0^1 = \frac{1}{3} \quad > \frac{1}{4}\,!$$

So: the extra bidder is worth more than setting the reserve optimally.

to generalize:

Some facts

1. $$\int_0^1 M(v)\, dF^n(v) = E\left\{\max\left[M(v_1), M(v_2), \cdots, M(v_n)\right]\right\}$$

proof F^n is distr. fctn. of max of n ind. draws

2. $$\int_{v_*}^1 M(v)\, dF^n(v) = E\left\{\max\left[M(v_1), M(v_2), \cdots, M(v_n), 0\right]\right\}$$

proof integrate only where $M(v) \geq 0$, $v_* = M^{-1}(0)$, M mono $\uparrow$ in v

3. $$E\{M(v)\} = 0$$

proof
- expected revenue with one bidder, no reserve $= 0$.
- or, check

$$\int_0^1 \left[v - \frac{1-F}{f}\right] dF = \int_0^1 (vf - 1 + F)\, dv$$

$$= \int_0^1 v\, dF - 1 + \int_0^1 F\, dv = \int_0^1 v\, dF - 1 + 1 - \int_0^1 v\, dF = 0$$

Consider now the comparison between an optimal auction with n bidders, and a no-reserve auction with $n+1$ bidders:

$$\Delta = E\{\overset{\text{no-reserve}}{\text{revenue}} \text{ with } n+1\} - E\{\overset{\text{optimal}}{\text{revenue}} \text{ with } n\}$$

$$= \int_0^1 M\, dF^{n+1} - \int_{v_*}^1 M\, dF^n$$

$$= E\left\{\max\left[M(v_1), M(v_2), \cdots, M(v_{n+1})\right]\right\} - E\left\{\max\left[M(v_1), \cdots, M(v_n), 0\right]\right\}$$

<u>Case 1</u> Suppose in some realization of $v_1, \dots, v_n$ that

$$R_n = \max[M(v_1), \dots, M(v_n)] \geq 0$$

then

$$\Delta = E\{\max[R_n, M(v_{n+1})]\} - E\{\max[R_n, 0]\}$$

$$= E\{\max[R_n, M(v_{n+1})]\} - E\{\max[R_n]\}$$

$$> 0$$

<u>Case 2</u> Suppose $R_n < 0$

$$\Delta = E\{\max[R_n, M(v_{n+1})]\} - 0 \quad > 0$$

recall $E\{M\} \neq 0$ ■