Collusive Bidder Behavior at Single-Object Second-Price and English Auctions

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Models of collusive bidder behavior at single-object second-price and English auctions are provided. The independent private values model is generalized to permit the formation of coalitions and a strategic response by the auctioneer. Cooperative strategies are found to be dominant in these models: coalitions of any size are viable, and the payoff to each member increases with the size of the coalition. In addition, the collusive strategies of the coalition represent a noncooperative equilibrium. The optimal response of the auctioneer is to establish a reserve price that is a function of the coalition's size. These and other features of the model are found to be consistent with the essential features of actual behavior.

I. Introduction

Auctions account for a surprisingly large volume of economic activity in the United States. The federal government sells timber rights, offshore oil leases, Treasury notes and bills, and many other commodities at auctions. In the private sector, antiques, artwork, and capital equipment are just a few of the items sold at auctions. The recent theoretical work on auctions has greatly increased our understanding of bidder behavior and the auctioneer's strategy within the context of the four main auction schemes: the first-price, second-

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price, English, and Dutch auctions.[1] This research has been conducted within the framework of both the independent private values (IPV) model and the common values model.[2] One assumption regarding bidder behavior is common to nearly all theoretical work: bidders act noncooperatively.[3] Yet allegations of collusion at auctions are widespread. Both Englebrecht-Wiggans (1980) and Milgrom and Weber (1982) have noted the need for a better understanding of the behavior and effects of bidder coalitions at auctions.

This research focuses on bidder coalitions at English and second-price auctions. Intuitively, such a coalition, known as a "ring" or "kipper," which contains K of the N bidders ($2 \leq K \leq N$) at an auction, gains in expected terms by removing $K - 1$ bidders from the competitive bidding. If the coalition does not contain the two bidders with the two highest valuations from the N bidders, then the ring realizes no gain beyond what each member could have obtained acting noncooperatively. However, when the ring contains the two bidders with the highest and second-highest valuations from the N bidders, then the coalition in fact realizes a gain.

Interviews of bidders who regularly participate in rings and of auctioneers who regularly fight rings have provided us with valuable insights regarding collusive behavior at auctions. First, a member of a ring never enters a truly competitive bid against another ring member. Second, rings employ procedures that ensure that the ring will win an item more highly valued by a ring member than by any non-

[1] The four auctions are characterized as follows. English auction: In this oral auction the auctioneer initially solicits a bid at a low price and then gradually increases the price until only one bidder remains active. Dutch auction: In this oral auction the auctioneer initially solicits a very high price and then gradually lowers the price until some bidder stops the auction and claims the item for the current price. First-price auction: This is a sealed-bid auction in which the buyer that submits the highest bid wins the item and pays the amount he has bid. Second-price auction: This is a sealed-bid auction in which the buyer submitting the highest bid wins the item but pays the amount of the second-highest bid.

[2] The independent private values and common values models are characterized as follows. Independent private values: In this model a single, indivisible item is to be sold to one of N risk-neutral bidders. Each bidder knows the value of the item to himself but not to the other bidders. These "private values" are modeled as being independently drawn from some continuous probability distribution that is known to all bidders and to the auctioneer. Common values: In this model the item to be auctioned has a true value, V, that is common but unknown to the risk-neutral bidders. These bidders make estimates of this common value, and these estimates are modeled as being independently drawn from a single underlying probability distribution parameterized by V.

[3] An exception is Robinson (1985), who considers the profitability of cartel membership under the special assumption that cartel membership is given exogenously and that members truthfully reveal their private information to one another. As will be shown, the "incentive-compatibility" problem and the determination of membership are important problems for the cartel. Robinson also ignores the strategic response of the auctioneer and the information distinction between the English and second-price auctions when a cartel is present.

ring bidder. Third, an item won by a ring becomes the property of the ring itself; the ultimate ownership of the item is determined in a secondary auction commonly known as a "knockout," which is separate from the main auction and involves only ring members. Fourth, the gains obtained by the coalition are shared by all ring members rather than accruing to only the winning bidder or some subset of ring members. Fifth, auctioneers respond to the presence of coalitions by establishing higher reserve prices.

In this paper we first develop a model of cooperative bidder behavior at a single-object (nondivisible) second-price auction within the IPV framework assuming risk neutrality of the bidders and the auctioneer. A central issue of the modeling is the determination of a scheme, consistent with the stylized facts, that is used by the coalition to formulate a bid or bids to be submitted at the main auction. In Section III we identify such a scheme, which we call the second-price preauction knockout (PAKT). The second-price PAKT, unlike standard cartel schemes, has a very desirable property: it is an incentive-efficient and durable mechanism. It provides incentives for the members of the cartel to cooperate. Two results emerge from our second-price model. First, the auctioneer's reserve price is an increasing function of the size of the coalition. Second, the expected payoff to a ring member is a strictly increasing function of the size of the coalition, implying that for a given reserve price bid by the auctioneer the optimal ring contains all bidders.

In addition, under the same assumptions, a model of cooperative bidder behavior at a single-object (nondivisible) English auction is presented. The English differs from the second-price in that the auctioneer can observe the bids before taking any action. Specifically, he observes the highest bid. When bidders act noncooperatively, this information plays no role in the auctioneer's strategy at an English IPV auction. The central issue of the English auction model is the determination of Nash equilibrium strategies when the auctioneer can observe the high bid before acting.

An initial understanding of the problem can be attained by comparing two cases. First, assume that all bidders act noncooperatively but that there are only $N - K + 1$ bidders, $N - K$ of whom draw their valuations from a distribution $F(\cdot)$ and one who draws from $F(\cdot)^K$, which corresponds to the distribution for the largest of K observations. Second, assume that there are N bidders who all draw their valuations from $F(\cdot)$, but K are in a ring while $N - K$ are nonring bidders. As we describe in Section IV, these two cases appear superficially to be identical, but there is a crucial difference: the ring has the capacity to have more than one of its members actively bidding, while a similar situation is not possible in case 1 for the $F(\cdot)^K$

bidder. This apparently insignificant difference results in completely different Nash equilibrium strategies for the two cases.

Two main results emerge from our cooperative English auction model. First, for all K, the auctioneer's reserve price does not depend on the highest observed bid, and it is identical to the corresponding reserve price at a cooperative second-price auction. Hence, the revenue equivalence of the second-price and English auctions, which is a well-known result for noncooperative behavior, extends to cooperative behavior. Second, as expected, the two results for the second-price mentioned above hold for the English without modification.

The paper proceeds as follows. In Section II, we present several stylized facts of cooperative bidder behavior and the response to such behavior by the auctioneer. In Sections III and IV, we present models of cooperative behavior at single-object second-price and English auctions, respectively. The concluding discussion in Section V points out some of the unresolved issues regarding bidder coalitions.

II. Collusive Behavior in Practice

The character of a bidder coalition depends on the type of object being sold. Cassady (1967, chap. 13) discusses the many types of rings found throughout the world. Below we provide the stylized facts of cooperative behavior at an English auction where nondivisible items are being sold. This account is not conjecture, but is based, in fact, on information provided by both auctioneers and ring members. For further details we refer the interested reader to Graham and Marshall (1985).

FACT 1. Rings exist and have a stable form of organization over time.

FACT 2. Rings adopt strategies that eliminate meaningful competition among members at the main auction and yet ensure that no item will be sold to a nonring bidder or be retained by the auctioneer at a price below the maximum of the individual ring members' personal valuations.

The ring appoints a sole bidder who bids on behalf of the coalition at the auction. If the sole bidder withdraws from the bidding, then any ring member can enter the bidding, but no two ring members will ever meaningfully bid against one another. When the bidding begins, ring members will compete against one another in an attempt to disguise their presence from the auctioneer. The rationale for these "meaningless" bids will be made clear in Section IV.

FACT 3. The benefits of ring formation are shared among members rather than, for example, accruing entirely to the ring member who ultimately obtains possession of the item.

FACT 4. Rings have open membership policies in the sense that bidders who are expected to be competitive at the main auction are invited to join.

In fact, if two or more distinct coalitions appear at the same auction, they will invariably merge to form a single coalition.

After the main auction the ring conducts an auction among its members to determine individual ownership of the items. The difference between the price paid at the ring's auction, which is called a knockout, and the price for which the ring obtained the item at the main auction is the gain to coalition formation, which is divided equally among all members.

FACT 5. The auctioneer responds strategically to the existence of a ring.

FACT 6. Rings attempt to conceal their existence from the auctioneer.

Auctioneers set higher reserves when a ring is present. These reserves vary proportionately with the size of the ring. Furthermore, auctioneers use a strategy known as "lift-lining," which is nothing more than the creation of phantom bids. Specifically, when the bidding stops, the auctioneer will generate a false or phantom higher bid if he feels that the high bidder is "good for another bump."

Before we progress to the models, one obvious problem with the facts above needs clarification. If membership is open and all ring members share gains equally, how do coalitions avoid the problem of free riders? In fact, rings address this problem by nesting, or forming rings within rings. Graham and Marshall (1985) discuss the actual nesting procedure in detail. For the following analysis, free riders do not exist ex ante since the IPV model assumes that all bidders draw their valuations from the same distribution.

Finally, it is important to realize that rings need not have a very formal organization. Informal arrangements are often made, for example, by strangers at a local estate auction who, on learning in casual conversation of one another's interests in particular items, agree not to bid against one another. So prevalent are rings, in fact, that a retired auctioneer once noted that in 40 years of auctioneering, he had yet to attend an auction at which a ring was not present.

The models presented in Sections III and IV will explain the stylized facts enumerated above within the context of a single-object second-price and English auction, respectively.

III. A Model of Bidder Collusion at Single-Object Second-Price Auctions

In this section a model of collusive behavior is presented within a relatively simple context: a single-object second-price auction. The rules for this auction are as follows.

The auctioneer chooses and announces a reserve price. Bidders then submit sealed bids. If the highest bid exceeds the reserve price announced by the auctioneer, then the individual who submitted the highest bid will win the item and will pay to the auctioneer either the second-largest bid or the reserve price, whichever is greater. If the highest bid is less than the reserve price, then the item is retained by the auctioneer.

We retain the following features of the IPV model.

ASSUMPTION 1. (i) Bidders are risk neutral. (ii) Each bidder's valuation is modeled as being independently drawn from a continuous distribution—with cumulative $F(\cdot)$, density $f(\cdot)$, and with $F(0) = 0$—that is common knowledge. (iii) Each bidder's valuation for the item to be auctioned is private information known only to himself. (iv) Bidders adopt noncooperative strategies in choosing their bids. (v) The auctioneer's objective is to maximize the expected value of the revenue resulting from the sale of the item to be auctioned.

A great deal is known about this model under the additional assumption that there are no choices available to bidders other than the selection of the bid to be submitted. The auction is equivalent to an English auction. In either auction the dominant strategy of each bidder is to bid his true valuation,[4] and the equilibrium strategy of the auctioneer is to announce a reserve price, S^*, determined implicitly by the equation (see Myerson 1981, 1983; Riley and Samuelson 1981)

$$S^* = V_a + \frac{1 - F(S^*)}{f(S^*)},$$

where V_a is the auctioneer's personal valuation of the item. Without loss of generality we hereafter assume that $V_a = 0$, thus obtaining

$$S^* = \frac{1 - F(S^*)}{f(S^*)}. \qquad (1)$$

If $F(V) \equiv V$ and $f(V) = 1$, for example, then S^* equals ½. Neither of these two behavioral results, however, is robust to a departure from the assumption that the strategies of bidders are limited to the submission of bids.

A. *Ring Behavior: The Preauction Knockout*

Collusive agreements depend critically on the information that will be available to the ring for use in specifying contractual provisions and the information that will be available to the auctioneer for use in

[4] This is precisely correct at the second-price auction (see Vickrey 1961). At the English auction the dominant strategy of a bidder is to remain active until the bidding reaches his true valuation.

combating the ring. We make the following assumptions regarding the initial information state.

ASSUMPTION 2. The identity of the winning bidder and the price paid (the amount of the second-highest bid) are common knowledge. All other information regarding bids submitted in the second-price auction is private information to the person submitting the bid and to the auctioneer.

ASSUMPTION 3. The membership of the ring is common knowledge to the members of the ring but is not known either to the auctioneer or to nonring bidders.

These assumptions refer only to the initial information state, which could, of course, be subsequently modified by the participants. The auctioneer could, for example, announce the identity and bid submitted by each participant. Such information would obviously be valuable to the ring since it could be used to provide better incentives for cooperative conduct and/or penalties for cheating.

The formation of a ring involves elements of both cooperative and noncooperative behavior. A mechanism (see Holmstrom and Myerson 1983) must be chosen for conducting the ring's business. The selection of this mechanism involves cooperative behavior. Consider a mechanism that requires the individual members of the ring to make prior, not necessarily truthful, reports regarding their private information (their valuations for the item to be auctioned) to the ring center. Then, for any given vector of reports of private valuations from ring members, the mechanism must determine, perhaps randomly, (1) the recommended bid for each member to submit at the main auction (these recommended bids can depend only on the reported bids), (2) the ring member who will ultimately receive an item that is won by the ring at the main auction (this can depend on the identity of the winning bidder and the price paid as well as the reported bids), and (3) the payments to collect from and/or make to each member of the ring (these can also depend on the identity of the winning bidder and the price paid as well as the reported bids).

Such mechanisms require individual members (i) to participate voluntarily, (ii) to make reports of their private valuations, and (iii) to submit bids at the main auction. These choices involve noncooperative behavior. A mechanism will be called "incentive compatible" iff it is a Nash equilibrium in the resulting game for each member of the ring to participate (the expected payoff to each member in this equilibrium is at least as great as the payoff that member could expect by not participating), to report his private valuation truthfully, and to submit the recommended bid at the main auction. A given mechanism dominates another if the expected payoff to each and every ring member is at least as great in the former mechanism as in the latter one, regardless of the ring members' private valuations. An incentive-compatible mechanism is incentive efficient iff it is not dominated by any other incentive-compatible mechanism. An incentive-compatible mechanism is durable iff the ring members would never unanimously approve a change to another mechanism even if they knew more than just their own valuations (i.e., communication had occurred).

With these preliminaries in mind, consider the following mechanism, which we call the second-price preauction knockout (PAKT). The rules of the second-price PAKT are as follows.

Prior to the main auction, the risk-neutral "ring center" makes a fixed payment (defined subsequently), P, to each of the ring members. Each of the K members of the ring then submits a sealed "reported bid" to the ring center, who determines the highest and second-highest of these reported bids. The member of the ring who submitted the highest bid is then selected by the ring center as the sole bidder and is advised to submit this highest reported bid at the main auction. Ring members other than the sole bidder are advised to submit no bid or a zero bid at the main auction. Should the sole bidder win the item at the main auction, he would pay the auctioneer the second-highest of all bids submitted at the main auction. He would additionally pay the ring center the difference between the second-highest reported bid from the ring and the second-highest of all bids at the main auction provided that this difference is positive.

Note that the second-price PAKT is effectively a second-price auction with modified rules. Ring members must now choose the bid to be reported to the ring center as well as the bid to be submitted at the main auction. The winning bidder, whether a ring member or not, will pay a price equal to the second-highest of the bids/reported bids submitted. Note also that the ring center is an agent selected from outside the auction who acts as both mediator and banker for the ring. As demonstrated momentarily, the fixed payments, P, are such that the risk-neutral ring center residually claims an amount that has an expected value of zero.

When we consider mechanisms that are "balanced budget" in expected value, our interest in the second-price PAKT stems from the following theorem.

THEOREM 1. The second-price PAKT is both an incentive-efficient and durable mechanism.

Additionally, the second-price PAKT corresponds well to stylized facts 1, 2, and 3 of Section II.

Proof. The three components of the theorem are demonstrated sequentially. In the event the ring wins the item, the member submitting the highest reported bid is awarded the item and pays a total price equal to the second-highest bid/reported bid of the $N + 1$ bids ($N - K$ nonring bids at the main auction, K reported bids from the ring members, and the reserve price bid of the auctioneer). The payment to each ring member is a fixed noncontingent constant and therefore cannot affect incentives. Since the second-price PAKT requires the winning bidder to pay the second price of all bidders and payments to ring members do not affect incentives, it follows from the logic of Vickrey's (1961) seminal paper that it is a Nash equilibrium strategy for each member of the ring to report his or her valuation truthfully to the ring center and to follow his bidding recommendations at the main auction. In fact, truthful revelation and compliance are not only a Nash strategy but a dominant strategy as well.

Voluntary participation is also advantageous. The fixed noncontingent payment, P, is determined as follows. Let the random variable δ be either the second-highest valuation within the ring minus the highest valuation outside the ring or zero, whichever is greater. The expected value of δ is both the expected payoff to the ring and, for a given reserve price of the auctioneer, the expected cost in lost revenues to the auctioneer from collusive behavior. Since all ring members share this payoff equally, the fixed payment made to all ring members is $P \equiv E(\delta)/K > 0$. The expected value of the amount residually claimed by the risk-neutral ring center thus equals zero.[5] Ring membership then entails three possible outcomes for a given ring member. First, if the ring does not acquire the item, membership is advantageous since the ring member receives P and would have obtained nothing acting individually. Second, if the ring wins the item but the item is awarded to another member, membership is still advantageous since once again the member receives P and would have obtained nothing acting individually. Third, if the ring wins the item and it is awarded to the ring member, then membership is still advantageous since the member pays precisely the same price that would have been necessary if he had acted individually but again receives P. Consequently, voluntary participation is also a dominant strategy.[6]

[5] The second-price PAKT is a scheme that involves "budget breaking" in the sense of Holmstrom (1982). If the ring does not win the item, then the ring center, who is a risk-neutral residual claimant, will pay out KP and receive nothing. On the other hand, if the difference between the second-highest valuation within the ring and the highest outside is very large, the ring center, after making payments totaling KP, will claim an amount in excess of KP.

[6] We assume that, except for formation of the ring, the behavior of all participants is noncooperative. This is an important assumption since the second-price PAKT does not itself preclude the formation of subcoalitions within the ring or coalitions that include the auctioneer as well as members of the ring. Two or more bidders might, e.g., privately agree to report only their highest valuation to the ring center or the sole bidder might agree to provide the auctioneer with information regarding the membership and reported bids of ring members. Such nesting and overlapping of coalitions are an important problem for future research since (i) the coalition structures involved do not constitute a partition of the set of players and do not therefore fit well within existing cooperative game theory and (ii) such behavior apparently does take place.

These equilibrium strategies assure that the ring will win the item in every circumstance in which some member has a valuation exceeding the cost of acquiring the item. Further, in every case in which the ring wins the item, it is awarded to that member with the highest valuation. The mechanism, in short, assures the ring members of the greatest possible joint payoff in *each* contingency. Consequently, there exists no other "balanced budget" mechanism, whether incentive compatible or not, that dominates the second-price PAKT. Therefore, the second-price PAKT is incentive efficient.

Since the second-price PAKT is incentive efficient and since participation and truthful revelation of valuations in the second-price PAKT are a dominant strategy, it follows that the second-price PAKT is a durable mechanism (by theorem 2 of Holmstrom and Myerson [1983]). Q.E.D.

B. *The Auctioneer's Response*

A final aspect of the model concerns the auctioneer's ability to detect the presence and size of rings. We have assumed that the auctioneer cannot directly determine the size or composition of rings. However, for a given strategy by the auctioneer (i.e., for a given reserve price), he can determine the size of a ring that would be a "best response." We assume that the auctioneer takes this effect into account in selecting his optimal reserve price in a manner to be made more precise momentarily. Attention is restricted to feasible reserve prices, that is, to those values of S for which $F(S) < 1$. These are the reserve prices for which a nonzero probability exists of selling the item and, obviously, the only reserve prices for which the auction exists in any meaningful sense.

The proofs of the following results can be found in the Appendix.

LEMMA 1. The reserve price, $S^*(K)$, that would maximize the expected revenue of the auctioneer for a ring of a given size, K, is an increasing function of K for $1 \leq K \leq N$.

In setting a reserve price, the auctioneer implicitly trades the increased expected revenue from raising the reserve price for the increased probability of retaining the item. As K increases, for a given reserve, the auctioneer's expected revenue falls. The auctioneer is willing to offset this effect and increase the risk of retaining the item by increasing his reserve.

LEMMA 2. For a ring of given size K, where $2 \leq K \leq N$, the expected payoff to an individual ring member is a decreasing function of the reserve price, S^*, over the feasible domain of reserve prices, that is, S such that $F(S) < 1$.

It is a consequence of these two lemmas that it would be advantageous to each member of the ring to "persuade" the auctioneer that the ring is smaller than it actually is or, better still, that the ring does not exist. Such representations, of course, are not credible to the auctioneer.

LEMMA 3. Given any feasible reserve price, the expected payoff to an individual member of a ring is a strictly increasing function of the size of the ring. The "best response" to any given feasible reserve price is thus a ring of size $K = N$.

THEOREM 2. The Nash equilibrium is characterized by a ring of size $K = N$ and a reserve price that is optimal for this ring size.

This equilibrium rationalizes stylized facts 4 and 5 of Section II. It is identical, in fact, to that of a second-price auction with a single bidder whose private valuation is drawn from $F(\cdot)^N$, the distribution for the highest order statistic of the valuations of the N bidders.[7] Since the reserve price is the only bid other than that of the ring, the second-price auction is analogous to a nonnegotiable "take it or leave it" offer on the part of the auctioneer to sell the item to a single buyer (the ring) at this "bilateral-monopoly" price.[8]

IV. A Model of Collusive Behavior at a Single-Object English Auction

Building on the model of Section III, we now provide a model of bidder collusion at a single-object English auction. Assumptions 1 and 3 of Section III are retained here. Without loss of generality we assume that the auctioneer has a zero personal valuation for the item to be auctioned.

The auction scheme considered here is a slight variant of the typical oral English auction, which we call the "English thermometer auc-

[7] Since the N valuations are all obtained from the same cumulative distribution, $F(\cdot)$, these valuations can be treated as order statistics (see David 1981). Recall that if $V_1, V_2, \ldots, V_N$ are a random sample of size N from a cumulative distribution function $F(\cdot)$ and if $Y_1 \leq Y_2 \leq \ldots \leq Y_N$ are the V_i arranged in order of increasing magnitudes, then the Y_i are defined to be the order statistics corresponding to the random sample.

[8] There is a "prisoner's dilemma" aspect to this Nash equilibrium. Will a bidder have a higher expected payoff by being in a ring of size K and facing the optimal reserve for this ring size, S_N, or by acting individually along with all other bidders and facing the lower optimal reserve price for this circumstance, S_1? It can be shown that for the uniform distribution this prisoner's dilemma exists for $K = N$ when $N < 5$. In this case members are worse off in expected terms in an all-inclusive ring than they would be acting noncooperatively. It can also be shown for any distribution, including the uniform, that the auctioneer would prefer to face N bidders acting noncooperatively than to face a ring of any size. Thus cooperation between the auctioneer and the members of the ring, if possible, could benefit all parties at least in the case of the uniform with $N < 5$. It is an unverified conjecture for an arbitrary distribution that this prisoner's dilemma is a phenomenon restricted to small N, i.e., that ring membership entails no prisoner's dilemma for sufficiently large N.

tion." There are two main features that distinguish this auction from the typical oral English auction. First, the bid increments are continuous. Second, bidders cannot reenter the bidding once they have exited, but the auctioneer can freely enter and exit the bidding. Milgrom and Weber (1982, p. 1104) have studied a similar auction in a noncooperative context. They refer to it as the Japanese English auction.[9] The rules of the English thermometer auction are as follows.

A thermometer-like device scaled in dollars is placed in plain view of the N bidders and the auctioneer. When the auction begins, any bidder wishing to submit a nonzero bid must depress a button at his bidding station. The thermometer rises as long as two or more buttons in the auction hall are depressed. At the moment that the number of buttons depressed falls to one, the thermometer stops rising. If the auctioneer is the last agent with a depressed button, then the auction ends and the item is retained by the auctioneer. If one of the N bidders is the last agent with a depressed button, then a brief period of time elapses before the auction ends to allow the auctioneer to reenter the bidding. If the auctioneer does not reenter, then the item is awarded to the last active bidder. Otherwise, the auction continues either until this last bidder removes his finger from the button or, alternatively, until the auctioneer stops bidding and does not reenter.

The strategic similarity of this auction to the second-price auction is apparent. Each bidder must determine a "reserve bid" at which he will release his button if the thermometer has not already stopped. The selection of this reserve bid commits the winning bidder to paying the second-highest reserve bid. It follows from the logic of the second-price auction that it is a dominant strategy for a nonring bidder to choose a reserve bid equal to his personal valuation.

The coalition of size K confronts the same problems enumerated in Section III*A*. Consider the following modification of the second-price PAKT, which we call the English PAKT. The English PAKT is identical to the second-price PAKT with the following exceptions: (i) The sole bidder is instructed to choose a reserve bid equal to his reported bid (the highest reported bid made by any ring member). (ii) Another member will be selected from the ring and instructed to participate in the main auction with a "dummy" reserve bid that depends on the second-highest reported bid from the ring. The precise determination of this dummy bid will be made clear in Section IV*C*. For now it is sufficient to note that it will never exceed either the auctioneer's reserve or the reserve bid of the sole bidder. Since the sole bidder is the

[9] In the Milgrom and Weber (1982) model the price at which each bidder exits is information that is available to the auctioneer and to the other bidders. In our model the only information available to the other bidders and the auctioneer is the price at which the next to the last bidder exits.

only ring member who can win the item and since he will bid up to the highest number reported by the ring members, it is a dominant strategy for each ring member to report a number to the ring center that is equal to the member's personal valuation for the item.

The essential difference between the second-price auction and the English thermometer auction is that the auctioneer can observe the highest bid, b, before taking any action. Within the IPV framework, this difference is irrelevant when bidders act noncooperatively. However, when a ring is present, the auctioneer will use the additional information provided at an English auction to infer the probable source of the highest valuation, namely ring or nonring, and react accordingly. The ring will be penalized if such an inference is possible, and, consequently, the ring will attempt to limit the informativeness of the highest bid, b. These are the critical issues in determining the equilibrium strategies of the ring and the auctioneer.

Assume initially that the auctioneer knows the size of the ring that will operate at the English thermometer auction. This assumption will be relaxed later and an equilibrium size for the ring identified, but for now assume that the auctioneer believes that there is a coalition of size K where $1 \leq K \leq N$. In the following two subsections we will determine equilibrium strategies for the cases $K = N$ and $2 \leq K \leq N - 1$.[10]

A. *The Auctioneer's Optimal Reserve for* K = N

Here we must determine the conditional density of the highest valuation from N bidders given that the highest valuation is greater than or equal to the highest observed bid, b. This density is

$$t(V|b) = \frac{NF(V)^{N-1}f(V)}{1 - F(b)^N}.$$

Again, the first-order condition from the auctioneer's expected revenue problem yields an implicit solution for the optimal reserve price, $S^*(N, b)$:

$$1 - F(S^*(N, b))^N - S^*(N, b)NF(S^*(N, b))^{N-1}f(S^*(N, b)) = 0.$$

Note that $\partial S^*(N, b)/\partial b = 0$ so $S^*(N, b)$ is not a function of b. This result is intuitively obvious since b is a meaningless bid from an all-inclusive coalition and could not possibly convey any relevant information. The

[10] For noncooperative behavior, $K = 1$, it is well known that the auctioneer's optimal reserve, $S^*(1, b)$, is given implicitly by

$$1 - F(S^*(1, b)) - S^*(1, b)f(S^*(1, b)) = 0$$

and that $\partial S^*(1, b)/\partial b = 0$.

equilibrium strategies here are for the coalition to remain active up to the highest personal valuation from their N members and for the auctioneer to respond with a fixed reserve of $S^*(N, b)$.

B. The Auctioneer's Optimal Reserve for $2 \leq K \leq N - 1$

This is the most interesting case since b can be informative regarding the source of the highest valuation, namely ring or nonring.[11] As mentioned above, the ring can limit the informativeness of b by generating more than one bid. To understand this we juxtapose the case presented here with another. Specifically, consider an English thermometer auction attended by $N - K + 1$ bidders who act noncooperatively. Furthermore, assume that all valuations are independent and private but that $N - K$ of the bidders draw their valuations from $F(\cdot)$ while one bidder draws his valuation from $F(\cdot)^K$. This is very similar to our cooperative setting since the ring's reserve bid can be modeled as a draw from $F(\cdot)^K$. However, there is a major distinction: the ring can have up to K active bidders at the auction whereas the one bidder who draws from $F(\cdot)^K$ must act individually by assumption. In what follows we will first determine the equilibrium strategies for the heterogeneous noncooperative case and then focus attention on our cooperative case.

For the heterogeneous noncooperative case, the density of the highest valuation given b is

$$g(V|b) = \frac{[KF(V)^{K-1} + (N - 1)F(b)^{K-1}]f(V)}{1 + (N - 1)F(b)^{K-1} - NF(b)^K}.$$

The first-order condition from the auctioneer's expected revenue maximization problem yields an implicit solution for the optimal reserve price, $S^*(b)$:

$$\begin{aligned} &\{1 - F(S^*(b))^K + (N - 1)F(b)^{K-1}[1 - F(S^*(b))]\} \\ &- S^*(b)[KF(S^*(b))^{K-1} + (N - 1)F(b)^{K-1}]f(S^*(b)) = 0. \end{aligned} \tag{2}$$

A theorem emerges from this implicit solution.

THEOREM 3. If $2 \leq K \leq N - 1$ and $b \leq S^*(b)$, then $dS^*(b)/db < 0$ if $S^*(b)$ is differentiable.

Proof. See the Appendix.

Intuitively, the auctioneer is able to use b to infer the probability that the highest valuation comes from the $F(\cdot)^K$ bidder rather than

[11] We are indebted to an anonymous referee for pointing out an error in an earlier version of the argument of this section.

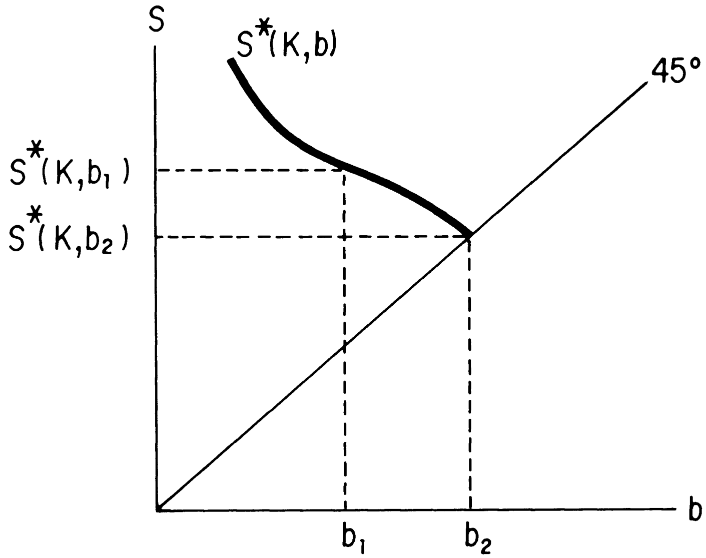

FIG. 1

one of the $N - K$ bidders who draw their valuations from $F(\cdot)$. The fact that $S^*(b)$ is a monotonically decreasing function of b reflects the opportunity costs faced by the auctioneer.[12] When b is low, the auctioneer gambles little by maintaining a high reserve, inferring that the highest valuation comes from the $F(\cdot)^K$ bidder. However, when b is large, it is likely that one of the $N - K$ bidders has the high valuation, and therefore the establishment of a high reserve will increase the chance of the auctioneer's winning the item and forgoing b.[13]

The equilibrium strategies are easily described for the heterogeneous noncooperative case by means of figure 1. First, each of the $N - K + 1$ bidders will remain active up to his own personal valuation. If the bidding stops at a point below b_2, such as b_1, then the auctioneer responds with a reserve bid of $S^*(b_1)$. If the bidding stops at or above b_2, where $S^*(b_2) = b_2$, then the auctioneer does not enter the bidding.

The lift-lining strategy discussed in Section II is easily interpretable within this context. The auctioneer can preannounce a reserve price of $S^*(K, b_2)$. If the bidding stops below b_2, such as at b_1, he generates phantom bids up to the point $S^*(K, b_1)$.

[12] The function $S^*(K, b)$ is a monotonically decreasing function of b (for $2 \leq K \leq N - 1$) regardless of the value of b. It is meaningless within the context of the English thermometer auction, however, to enter a reserve bid for values of $b > S^*(K, b)$.

[13] This result should be contrasted with that of Myerson (1983), who considers the case of heterogeneous bidders but assumes that each bidder's type (distribution) is known to the auctioneer. We assume instead that this information is private to the respective bidders.

Why is the same equilibrium not applicable to the ring of size K where $2 \leq K \leq N - 1$? Since the ring can have more than one active bidder, it would never allow the thermometer to stop below b_2 if the ring's reserve bid exceeds b_2. Specifically, the ring would have two active bidders when the auction starts, the sole bidder and a dummy bidder. The sole bidder would remain active until the thermometer reaches the highest valuation of the K ring members (the sole bidder's valuation), while the dummy bidder would always exit the bidding at b_2 and thereby eliminate the potential of a high reserve bid by the auctioneer. However, since this strategy is enormously informative to the auctioneer, it cannot be an equilibrium. If the thermometer stopped at b_2, the auctioneer would know with probability one that the coalition had the high valuation. He would then be playing a game solely against a K-member coalition that was willing to pay at least b_2 for the item. It is easily shown that the auctioneer's reserve price response in this situation exceeds $S^*(K, b_2)$. Therefore, these strategies do not constitute an equilibrium.

Recall that the auctioneer knows the size of the ring. Suppose he adopts a fixed reserve price strategy; that is, it does not depend on b. The ring knows that if its bidding strategy is informative, the auctioneer will abandon this strategy. Consider the highest and second-highest valuations from within the ring, V_K and V_{K-1}, respectively. If V_{K-1} is not greater than the auctioneer's fixed reserve, S, then the coalition loses nothing by having the ring's dummy bidder remain active up to V_{K-1}. By doing so the auctioneer can obviously infer nothing from b. Alternatively, if $V_{K-1} > S$, then the ring would eliminate the gain to cooperative behavior by having the dummy bidder remain active up to V_{K-1}. Consequently, in this situation the ring disguises V_{K-1} by generating a false valuation from the distribution $F(x)^{K-1}/F(S)^{K-1}$. This is the cumulative distribution for the highest order statistic from $K - 1$ given that it is less than S. Consequently, for $V_{K-1} > S$, let D_{K-1} be this false valuation, but for $V_{K-1} \leq S$, let $D_{K-1} = V_{K-1}$. The dummy bidder remains active up to D_{K-1}. This sometimes false valuation is completely uninformative to the auctioneer if the thermometer stops at $b = D_{K-1}$ since D_{K-1} is observationally equivalent to V_{K-1}. Specifically, with the exception of a truncation term, D_{K-1} and V_{K-1} are generated by the same distribution. Since b is uninformative, when the ring adopts this strategy the auctioneer determines an optimal fixed reserve price (not dependent on b) for N bidders, K of which belong to a coalition. Given this optimal fixed reserve, the ring's strategy is still optimal. Hence, these strategies form an equilibrium. This intuitive argument leads to a theorem.

THEOREM 4. At an English thermometer auction where $N - K$ of the N bidders act noncooperatively while K bidders are in a ring that uses the English PAKT and where the assumptions stated in this section hold, equilibrium strategies are (i) for the ring's sole bidder to remain active up to V_K while the ring's dummy bidder remains active up to D_{K-1} (as defined above) and (ii) for the auctioneer to quote a fixed reserve, namely, one that does not depend on b, the highest observed bid.

Proof. See the Appendix.

Since b is uninformative for the auctioneer, the only information on which he can base a reserve price is K. This leads to a corollary.

COROLLARY 1. The auctioneer's optimal fixed reserve is identical to the fixed reserve at a second-price auction.

Proof. See the Appendix.

Corollary 1 and theorem 4 immediately yield another interesting result.

COROLLARY 2. Expected seller revenue is identical for the second-price and English auctions.

Therefore, within the IPV framework, the revenue equivalence of the second-price and English auctions holds not only for noncooperative behavior but for cooperative behavior as well.

Note that the "meaningless" coalition bids discussed in Section II can now be understood. Specifically, they are represented in our model by D_{K-1}. But why generate meaningless bids? Suppose that the coalition enters a bid only through the sole bidder. Knowing this, the auctioneer would respond with a reserve bid of $S^*(b)$, which is a monotonically decreasing function of b. Knowing this, the coalition would generate a false bid up to b_2, and the equilibrium unravels as described above. The meaningless coalition bid, D_{K-1}, sustains the equilibrium described in theorem 4.

C. *The Equilibrium Size of the Ring*

Since we have identified the auctioneer's reserve price for all values of K, we need only determine the optimal coalition size to characterize the Nash equilibrium.

LEMMA 4. The expected payoff to an individual member of a ring at an English thermometer auction is an increasing function of the coalition's size for a given reserve price of the auctioneer. The "best response" to any given feasible reserve price is thus a ring of size $K = N$.

Proof. Given the results of Sections IV*A* and IV*B* and corollary 1, the proof is identical to that of lemma 3 (see the Appendix).

Of course, the auctioneer's optimal response to a ring of size N is $S^*(N, b)$, which equals $S^*(N)$. Thus theorem 2 applies without alteration for the English thermometer auction. Again, the auctioneer makes a nonnegotiable "take it or leave it" offer to sell the item to a single bidder (the all-inclusive coalition) at the reserve price.

V. Discussion

Models of single-object second-price and English auctions have been proposed in which cooperative behavior is permitted and in which the auctioneer is allowed to respond strategically to such behavior. These relatively simple models are fully consistent with the stylized facts of actual ring and auctioneer behavior as outlined in Section II. Equilibrium for these cooperative games entails an all-inclusive ring and the choice of a reserve price by the auctioneer that exceeds the optimal noncooperative reserve price. Cooperative behavior strictly dominates noncooperative behavior, and it is rational for the auctioneer to respond strategically.

Furthermore, for a given ring size where $1 \leq K \leq N$, the equilibrium reserve price of the auctioneer is identical for the two auctions. Therefore, the revenue equivalence result for the second-price and English auctions within the IPV context extends to cooperative behavior. In addition, the generation of meaningless bids by the coalition and the lift-lining strategy of the auctioneer, which were discussed among the stylized facts of Section II, have been shown to be natural results of our English thermometer auction model.

One obvious issue for future research is the modeling of cooperative behavior at Dutch and first-price auctions. Dutch auctioneers of fruit in Belgium have informed us that coalitions are indeed a problem. In addition, we have evidence of rings at first-price auctions in the United States. However, these rings are invariably all-inclusive and appear to be rather unstable.

Another issue concerns the nesting of rings. Rings within rings are quite prevalent. The nested knockout procedure used by coalitions has a very interesting property. Consider an all-inclusive ring of four bidders, {1, 2, 3, 4}, whose net valuations (valuations minus the auctioneer's reserve price) are [10, 7, 6, 2], respectively. Suppose that these net valuations are public knowledge within the coalition and that the item is purchased by the coalition at the auctioneer's reserve. Let X_i be the payoff from the nested knockout to bidder i. Then

$$X_1 = \frac{2}{4} + \frac{6-2}{3} + \frac{7-6}{2} + (10-7),$$

$$X_2 = \frac{2}{4} + \frac{6-2}{3} + \frac{7-6}{2},$$

$$X_3 = \frac{2}{4} + \frac{6-2}{3},$$

$$X_4 = \frac{2}{4},$$

or $X_i = \Sigma_{j=i}^{N}(V_j - V_{j+1})/j$, where $V_{N+1} = 0$. Interestingly, X_i is the Shapley value of bidder i. This result holds for all N and any vector of valuations. Thus rings have adopted a simple scheme to award individual members their average marginal contribution and thus eliminate free-rider problems. This result and the incentive aspects of the nested knockout will be investigated in future work.

Appendix

Proof of Lemma 1

The objective of the risk-neutral auctioneer is to choose the reserve price, $S^*(K)$, that maximizes expected revenue:

$$S^*(K) = \underset{S}{\operatorname{argmax}}\Big(S\{(N - K)[1 - F(S)]F(S)^{N-1} + [1 - F(S)^K]F(S)^{N-K}\} + \int_S^\infty y(N - K)[(N - 1)F(y)^{N-2} - NF(y)^{N-1} + F(y)^{N-K-1}]f(y)\,dy\Big).$$

The first term represents the reserve price times the probability that the reserve price is the price paid, that is, the probability that the reserve price is the second-highest bid. The second term is the probability that the reserve price is neither the highest nor the second-highest bid times the conditional expected value of the second-highest bid given that the reserve price is neither the highest nor the second-highest bid.

The first-order condition for an interior maximum is

$$(N - K)[1 - F(S^*(K))]F(S^*(K))^{N-1} + [1 - F(S^*(K))^K]F(S^*(K))^{N-K} - S^*(K)NF(S^*(K))^{N-1}f(S^*(K)) = 0. \tag{A1}$$

This yields an implicit solution for $S^*(K)$. Note that, for $K = 1$, equation (A1) yields the same solution for $S^*(1)$ as equation (1).

Evaluating the left-hand side of (A1), the derivative of the objective function, at $K + 1$ and $S^*(K)$ yields

$$F(S^*(K))^{N-K-1}[1 - F(S^*(K))][1 - F(S^*(K))^K],$$

which is positive for all K and $S^*(K)$. But this means that $S^*(K + 1)$, the reserve price that is optimal for $K + 1$, must be larger than $S^*(K)$. Q.E.D.

Proof of Lemma 2

The density for the second-highest valuation from the ring is

$$u(x) = K(K - 1)F(x)^{K-2}[1 - F(x)]f(x)$$

while the density for the highest valuation from outside the ring is

$$h(y) = (N - K)F(y)^{N-K-1}f(y).$$

Assuming independence, namely, that ring membership is a random sample of the homogeneous bidders, the expected value of the difference between the second-highest valuation from the ring and the highest outside the ring, given that the difference is positive, is, for $2 \le K \le N - 1$,

$$D(K, S^*) = \int_{S^*}^{\infty}\int_{S^*}^{x} (x - y)u(x)h(y)\,dy\,dx + \int_{S^*}^{\infty}\int_{0}^{S^*} (x - S^*)u(x)h(y)\,dy\,dx.$$

For $K = N$ the corresponding expression is

$$D(N, S^*) = \int_{S^*}^{\infty} (x - S^*)N(N - 1)F(x)^{N-2}[1 - F(x)]f(x)\,dx.$$

The fixed payment to a ring member given a reserve price S^* is then

$$P(K, S^*) = \frac{D(K, S^*)}{K}.$$

Adding the expected "surplus" associated with winning an item at a price below the member's valuation yields a total expected payoff for $2 \le K \le N - 1$ corresponding to

$$E(K, S^*) = \frac{\int_{S^*}^{\infty}\int_{S^*}^{x} (z - y)w(z)h(y)\,dy\,dz + \int_{S^*}^{\infty}\int_{0}^{S^*} (z - S^*)w(z)h(y)\,dy\,dz}{K},$$

where the density for the highest valuation from the ring is

$$w(z) = KF(z)^{K-1}f(z).$$

For $K = N$ the corresponding expression is

$$E(N, S^*) = \frac{\int_{S^*}^{\infty} (z - S^*)NF(z)^{N-1}f(z)\,dz}{N}.$$

Provided that $F(S^*), F(S') < 1$, it is apparent that $S^* < S'$ implies that $E(K, S^*) > E(K, S')$. Q.E.D.

Proof of Lemma 3

The proof is by induction using the notation of the proof of lemma 2. First note that

$$\begin{aligned} E(N, S^*) - E(N - 1, S^*) &= \int_{S^*}^{\infty} (z - S^*)F(z)^{N-2}[F(z) - F(S^*)]f(z)\,dz \\ &\quad - \int_{S^*}^{\infty}\int_{S^*}^{z} (z - y)F(z)^{N-2}f(z)f(y)\,dy\,dz \\ &> \int_{S^*}^{\infty} (z - S^*)F(z)^{N-2}[F(z) - F(S^*)]f(z)\,dz \\ &\quad - \int_{S^*}^{\infty}\int_{S^*}^{z} (z - S^*)F(z)^{N-2}f(z)f(y)\,dy\,dz \end{aligned}$$

provided that $F(S^*) < 1$. But the last expression is nonnegative. A similar argument establishes that $E(K, S^*) - E(K - 1, S^*) > 0$. Q.E.D.

The proof of theorem 2 is an obvious consequence of lemmas 1 and 3.

Proof of Theorem 3

Equation (2) can be rewritten as

$$1 - G(S^*(b)|b) - S^*(b)g(S^*(b)|b) = 0,$$

where $G(S^*|b)$ is the cumulative associated with $g(V|b)$ evaluated at $S^*(b)$. Now if $S^*(b)$ is differentiable, it follows from the second-order necessary condition for a maximum that

$$\begin{aligned}\text{sign}\,\frac{dS^*(b)}{db} &= -\text{sign}\left[\frac{\partial G(\cdot)}{\partial b} + S^*(b)\frac{\partial g(\cdot)}{\partial b}\right]\\ &= \text{sign}\,\frac{\partial}{\partial b}\left[\frac{1 - G(S|b)}{g(S|b)}\right]_{S=S^*(b)}\\ &= \text{sign}\,((N-1)(K-1)f(b)f(S)F(b)^{N-2}\\ &\qquad \times \{[1 - F(S)]KF(S)^{K-1} - 1 + F(S)^K\})\end{aligned}$$

and since the expression in braces is negative, we have $\text{sign}[dS^*(b)/db] < 0$. Q.E.D.

Proof of Theorem 4 and Corollary 1

Suppose for the moment that ring members adopt the strategies described in part i of theorem 4. We will show that it is then optimal for the auctioneer to establish a fixed reserve price of $S = S^*(K)$. Given such a fixed reserve price, we will then show that the strategies of part i are optimal for each member of the ring.

Let b denote the highest observed bid and S the reserve price anticipated by the ring. We consider the two cases $b \leq S$ and $b > S$.

Case 1: $b \leq S$

The ring's dummy bid, D_{K-1}, either is V_{K-1} if $V_{K-1} \leq S$ or, alternatively, is an observation from $F(x)^{K-1}/F(S)^{K-1}$ if $V_{K-1} > S$. Let a denote the highest valuation from all N bidders and $c(a, b)$ the joint density:

$$\begin{aligned}c(a, b) &\equiv [N(N-1)F(b)^{N-2}f(a)f(b)]\\ &\quad + \left\{(N-1)KF(b)^{N-K-1}[F(a)^{K-1} - F(S)^{K-1}]\left[\frac{F(b)^{K-1}}{F(S)^{K-1}}\right]f(a)f(b)\right\}\\ &= (\text{term } 1) + (\text{term } 2)\\ &= (N-1)F(b)^{N-2}\{NF(S)^{K-1} + K[F(a)^{K-1} - F(S)^{K-1}]\}\frac{f(a)f(b)}{F(S)^{K-1}}.\end{aligned}$$

Term 1 is the joint density of the two highest valuations from N regardless of their origin (ring or nonring) and thus accounts for all possibilities except for the case in which the ring has the highest valuation and $b = D_{K-1} \leq S < V_{K-1}$. Term 2 accounts for this exception. Then $c(a|b) = c(a, b)/c(b)$ is the conditional density for the highest valuation from the N bidders, a, given b. The auctioneer's optimal reserve price solves

$$S_1^* \equiv \underset{S}{\text{argmax}} \int_S^\infty Sc(a|b)da.$$

After substitution and simplification, the first-order necessary condition for an interior maximum is

$$-S_1^*NF(S_1^*)^{K-1}f(S_1^*) + [1 - F(S_1^*)^K] + (N - K)F(S_1^*)^{K-1}[1 - F(S_1^*)] = 0.$$

Note that this expression does not depend on b, and S_1^* is therefore a constant.

Case 2: $b > S$

In this case the highest observed bid cannot be D_{K-1}. Here b equals either V_K or V_{N-K} (the highest valuation from the $N - K$ nonring bidders) or V_{N-K-1} (the second-highest valuation from the $N - K$ nonring bidders). The joint density of a and b is then

$$\begin{aligned} h(a, b) &\equiv [(N - K)KF(a)^{K-1}F(b)^{N-K-1}f(a)f(b)] \\ &\quad + [(N - K)(N - 1)F(b)^{N-2}f(a)f(b)] \\ &= (\text{term } 1') + (\text{term } 2') \\ &= (N - K)[KF(a)^{K-1}F(b)^{N-K-1} + (N - 1)F(b)^{N-2}]f(a)f(b). \end{aligned}$$

Term 1′ accounts for the possibility that the ring has the highest valuation from all N bidders and b comes from a nonring bidder. Term 2′ accounts for the possibility that the highest valuation comes from a nonring bidder and consequently that b is the second-highest valuation from all bidders. (Note that the factor $N - K$ appears in both terms instead of the factor N since it is not possible in either term that b is the second-highest valuation from within the ring.) The conditional density for the highest valuation given b is $h(a|b) = h(a, b)/h(b)$. In this case the optimal reserve price is given by

$$S_2^* \equiv \underset{S}{\text{argmax}} \int_S^\infty Sh(a|b)\,da.$$

The first-order condition is

$$\begin{aligned} &-S_2^*KF(S_2^*)^{K-1}f(S_2^*) + [1 - F(S_2^*)^K] \\ &+ (N - 1)F(b)^{K-1}[1 - S_2^*f(S_2^*) - F(S_2^*)] = 0. \end{aligned}$$

In this case $dS_2^*/db < 0$. However, when $b = b_1$ where $b_1 \equiv S_1^*$, it is shown below that $S_2^* < S_1^*$. Since $dS_2^*/db < 0$, it follows that $S_2^* < b$ for all values of b that exceed S_1^*. Thus, since it is meaningless for the auctioneer to have a reserve price below the highest observed bid at an English thermometer auction, S_1^* is the sole reserve price of the auctioneer.

To show that $S_2^*(K, b_1) < S_1^*$, we differentiate $\int_S^\infty Sh(a|b)\,da$ with respect to S and evaluate this derivative at S_1^* yielding

$$\begin{aligned} &-S_1^*KF(S_1^*)^{K-1}f(S_1^*) + [1 - F(S_1^*)^K] \\ &+ (N - 1)F(S_1^*)^{K-1}[1 - S_1^*f(S_1^*) - F(S_1^*)]. \end{aligned}$$

Subtracting the first-order necessary condition from case 1 and simplifying yields

$$(K - 1)F(S_1^*)^{K-1}[1 - F(S_1^*) - S_1^*f(S_1^*)].$$

The first-order necessary condition from case 1 and the fact that $[1 - F(S_1^*)^K] > 0$ imply that $[1 - F(S_1^*) - S_1^*f(S_1^*)] < 0$. Thus $S_1^* > S_2^*$ at b_1.

Finally, multiplying the implicit solution for S_1^* by $F(S_1^*)^{N-K}$ yields an expression identical to (A1). Thus $S_1^* = S^*(K)$. This establishes corollary 1.

Finally, since $S^*(K)$ depends only on knowledge of K and $F(\cdot)$, it follows that the ring can, through the generation of the dummy bid D_{K-1}, make b completely uninformative to the auctioneer as regards the source, ring or nonring, of the highest valuation. Consequently, the strategies of part i of theorem 4 for the ring members are optimal. Q.E.D.

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