Resolving Uncertainty about the Number of Bidders in Independent Private- Value Auctions: An Experimental Analysis
Author(s): Douglas Dyer, John H. Kagel, Dan Levin
Source: *The RAND Journal of Economics*, Vol. 20, No. 2 (Summer, 1989), pp. 268-279
Published by: Blackwell Publishing on behalf of The RAND Corporation
Stable URL: http://www.jstor.org/stable/2555693
Accessed: 05/04/2009 05:47

RAND Journal of Economics
Vol. 20, No. 2, Summer 1989

Resolving uncertainty about the number of bidders in independent private-value auctions: an experimental analysis

Douglas Dyer*

John H. Kagel**

and

Dan Levin***

Results from first-price, sealed-bid auctions, in which there is uncertainty regarding the number of bidders, are reported. Consistent with recent theoretical findings, concealing information regarding the number of bidders raises more revenue for the seller than revealing information. Individual bids show that (1) narrowly interpreted, the Nash equilibrium bidding theory underlying these theoretical predictions is rejected, as less than 50% of all bids satisfy the strict inequality requirements of the theory, but (2) a majority of the deviations from these inequality requirements favor the revenue-raising predictions of the Nash model, and, in a large number of cases, involve marginal violations of the theory.

1. Introduction

■ In most theoretical auction models, the number of competing bidders, n, is assumed to be fixed and known to all agents. (See Vickrey (1961), McAfee and McMillan (1987a), and Milgrom and Weber (1982) for examples of this literature.) In a sealed-bid auction this is rarely the case: the number of bidders will, in general, not be known until after bids have been collected.[1]

McAfee and McMillan (1987b) generalize auction theory to allow for uncertainty about the number of bidders (hereafter numbers uncertainty). They show that in a first-price, sealed-bid auction with independent private values, if the number of bidders is unknown and bidders have constant absolute risk aversion (CARA), then the expected revenue to the seller is greater if the actual number of bidders is concealed rather than revealed. Matthews (1987) extends McAfee and McMillan's result to the case in which buyers have decreasing absolute risk aversion (DARA). In contrast, if bidders are risk neutral, the expected

* Memphis State University.
** University of Pittsburgh.
*** University of Houston.

Financial support for this research was provided by the Information Science and Technology and the Economics Divisions of the National Science Foundation, the Alfred P. Sloan Foundation, and the Energy Laboratory at the University of Houston. The article has benefitted from discussions with Ron Harstad and the comments of Paul Milgrom and the referee. Susan Garvin provided able research assistance. We remain responsible for all errors. Copies of the instructions are available from the authors.

[1] This may result from a number of reasons. For example, bidding may be by invitation only, and the seller does not reveal the list of invitees.

revenue to the seller is the same whether the actual number of bidders is revealed or concealed (McAfee and McMillan, 1987b; Matthews, 1987; Harstad, Levin and Kagel, 1986; Ortega-Reichert, 1968).

There are instances in which the seller may know in advance the actual number of bidders, such as auctions where bidding is by invitation only. Even if this information is not known in advance, however, a contingent bidding procedure provides a relatively simple mechanism that permits each bidder to act as if he knew the actual number of rivals he faces. Under contingent bidding, each buyer submits a vector of bids, where each bid in the vector corresponds to a specific number of bidders. That is, each bidder submits one bid which will be in effect should there be n bidders, a second bid, which will be in effect should there be $n + 1$ bidders, and so on. Under this procedure, the n used in determining the actual bid corresponds to the total number of bidders actually submitting bid vectors. In this way the seller is able to gain any benefits from revealing the number of bidders, even though the actual number of bidders remains unknown prior to the auction.

This article reports the results of a series of laboratory experiments designed to examine the theory's predictions about market outcomes in a first-price, sealed-bid auction with independent private values and uncertainty regarding the number of bidders. We do this by comparing the revenue-raising properties of a contingent bidding procedure to those of a noncontingent procedure.[2] Previous experimental studies of independent private-value auctions have shown that bidders' behavior is consistent with their being risk averse (Cox, Roberson and Smith, 1982; Cox, Smith and Walker, 1985; Kagel, Harstad and Levin, 1987; and Battalio, Kogut and Meyer, forthcoming). If we assume that absolute risk aversion is nonincreasing—a belief that most economists hold as a working hypothesis (Arrow, 1971; Machina, 1983)—we would expect average revenue (prices) to be greater under the noncontingent bidding procedure.

Consistent with these theoretical predictions, noncontingent bidding procedures raise more revenue for the seller than contingent bidding in our experiments. Furthermore, our examinations of individual bidding patterns, in terms of inequality restrictions on the relationship between contingent and noncontingent bids, show that (1) we can reject the hypothesis that the narrowly interpreted Nash equilibrium bidding theory underlying these theoretical predictions is satisfied, as fewer than 50% of all individual bids satisfy the strict inequality requirements of the theory, but (2) a majority of the deviations from these strict inequality requirements favor the revenue-raising predictions of the Nash model, and, in a large number of cases, involve marginal violations of the theory. When considered together, these results suggest that bidders are responsive to the economic forces underlying the Nash model, although they do not respond *exactly* as the theory predicts.

The article is organized as follows. In Section 2 we describe the experimental design and procedures. Theoretical predictions for both market revenue results and individual bids are characterized in Section 3. We report the experimental results in Section 4, and Section 5 summarizes and concludes the article.

2. Experimental design

■ Subjects were recruited primarily from MBA classes at the University of Houston to participate in experimental sessions lasting approximately two hours each. In each experiment six subjects acted as bidders, competing for the right to buy a single item in each of a series of trading periods. Each period consisted of either two small ($n = 3$) markets or one single large ($n = 6$) market.

---

[2] It should be emphasized that the theoretical predictions of McAfee and McMillan, and of Matthews, are based on the more general concept of revealing versus concealing the actual number of bidders. The contingent bidding institution examined here is a particular method of operationalizing the notion of revealing the actual number of bidders.

Each bidder was randomly assigned a private resale value for the item. Resale values were random draws from a uniform distribution defined on $[VL, VH] = [\$0.00, \$30.00]$.[3] Bidders were told that "this value may be thought of as the amount you would receive if you were to resell the item." New resale values were assigned each period. Bidders had no knowledge of each other's resale values.

□ **Contingent bidding.** Each bidder was told that while he would know the resale value of the item assigned to him with certainty, he would not know, prior to bidding, the size of the market in which he was competing. He was told that it was equally likely that he would participate in a market of size $n = 3$ or a market of size $n = 6$. However, he would be allowed to submit two bids, one contingent on $n = 3$ and the other contingent on $n = 6$. It was explained that these contingent bids would allow him to bid as if he knew the actual number of bidders, since only that bid corresponding to the actual number of bidders would be used. After each of the bidders had submitted his vector of bids, the actual size of the market was determined randomly. Furthermore, if the event $n = 3$ occurred, the composition of the markets (i.e., which three subjects would be in each small market) was also randomly determined.[4]

After the size and composition of the markets were determined, the bids were opened and posted. If there were six bidders, the high bidder earned profits equal to the difference between his private resale value and his bid contingent on $n = 6$. If there were three bidders, the bidder who submitted the highest bid contingent on $n = 3$ in *each* small market earned profits equal to the difference between his private resale value and his bid contingent on $n = 3$. All other bidders earned zero profits.

Each subject was told that he was under no obligation to submit either two equal or two different contingent bids.

□ **Comparing contingent with noncontingent bidding.** After several periods of contingent bidding, simultaneous bidding under two procedures was introduced. Each subject was informed that while he would continue to submit two contingent bids (the contingent procedure), he would now also submit a single bid, which would be in effect regardless of the number of active bidders (the noncontingent procedure). The noncontingent bids were submitted after the contingent bids were accepted, but before they were opened.

Subjects were told that the high bidder (or bidders, if n was three) would earn profits under only one of the two bidding procedures, to be determined randomly by a coin flip after both sets of bids were accepted. Following the coin flip, market size and composition were determined as before, with a 50% chance of two markets of size $n = 3$ and a 50% chance of a single market of size $n = 6$. Subjects were told that they were under no obligation to submit the same or different bids under the two procedures.[5]

---

[3] These experiments were conducted by hand. In each auction period, a subject would select a bag containing a set of resale values and then draw his resale value from among those in the bag. There were always more bags than auction periods, and more resale values than subjects in each bag. Also, a copy of the computer program used to generate the resale values was maintained, in case subjects wished to verify the random determination of the resale values.

[4] Market size and composition were determined by subjects drawing from a deck of cards, with one card for each possible market of size $n = 3$, and an equal number of cards for markets of size $n = 6$. In experiment 3 the composition of the small markets was predetermined and remained constant throughout the experiment. The deck of cards was adjusted accordingly.

[5] This dual-market bidding procedure directly controls for between subject variability, as well as for differences resulting from variations in randomly drawn private values. It may appear that the dual-market bidding procedure introduces a portfolio effect, whereby the optimal bid under contingent bidding affects the optimal bid under noncontingent bidding, and *vice versa*. However, assuming expected utility maximization, and using a coin flip to randomly determine in which market to pay, this is not the case. Further, Battalio, Kogut and Meyer (forthcoming) and Kagel, Harstad and Levin (1987) report no systematic behavioral differences under dual-market as compared to single-market procedures.

Bidding continued in this manner for at least thirteen trading periods. Each period, after the bids were opened, all bids and corresponding private values for the markets in which profits were paid were posted on the blackboard. At the end of the experiment, subjects were paid their earnings plus a $4.00 participation fee in cash.

3. Theory

■ **Contingent bidding.** Vickrey (1961) was the first to analyze the independent private-value auction model. He showed that if bidders are risk neutral and private values are independent draws from a uniform distribution defined on $[VL, VH]$, the Nash equilibrium bid function, given that there are n bidders, is

$$b_n(x) = VL + ((n-1)/n)(x - VL), \tag{1}$$

where x is the individual's private value, and n is assumed to be fixed and known by all agents.[6] The Vickrey model in which n is known maps directly into our contingent bidding procedure. If $[VL, VH] = [\$0.00, \$30.00]$, the symmetric risk-neutral bid function (1) simplifies to

$$b_n(x) = ((n-1)/n)x. \tag{2}$$

Even though the actual number of bidders is unknown, the submission of a vector of contingent bids allows agents to bid *as if n* were known. It is clear from (2) that if bidders are risk neutral, there is a strict ordering of bids under the contingent procedure with $b_3(x) < b_6(x)$.

Harris and Raviv (1981), Holt (1980), Maskin and Riley (1980), and Riley and Samuelson (1981) extend the Vickrey model to the case of risk-averse bidders. They show that with risk aversion, the symmetric Nash equilibrium bids will be greater than that given by (1). With or without risk aversion, the assumption of symmetry implies that the highest bid will be submitted by the bidder who has the highest private value so that the auctions are Pareto efficient.

□ **Noncontingent bidding.** Ortega-Reichart (1968) was the first to relax the assumption that n is known. McAfee and McMillan (1987b) show that if buyers have constant absolute risk aversion (CARA), then at the symmetric Nash equilibrium, the expected revenue to the seller is greater if the actual number of bidders is concealed than if it is revealed. They model the number of bidders as an unknown random variable with a known exogenous probability distribution; each bidder was assumed to have the same *ex ante* belief about the probability of a given number of bidders, p_n. Matthews (1987) extends the results of McAfee and McMillan to show that if buyers have decreasing absolute risk aversion (DARA), then the expected revenue to the seller is also greater if the actual number of bidders is concealed.[7]

The intuition underlying the revenue predictions is as follows: McAfee and McMillan show that CARA buyers do not care whether the number of bidders is concealed or revealed, while Matthews shows that DARA buyers prefer that the number of bidders be revealed. Under contingent bidding (i.e., revelation of the number of bidders) there is an element of uncertainty which is absent under noncontingent bidding. If he wins the auction under contingent bidding, a bidder does not know the price he will pay, since he submitted a vector of bids. Under noncontingent bidding, however, a bidder knows with certainty the

[6] Throughout this article, $b_n(x)$ will refer to the contingent bid function given the number of bidders is n, and $b(x)$ will refer to the noncontingent bid function, regardless of the bidders' attitudes towards risk.

[7] With increasing absolute risk aversion, concealing the number of bidders may or may not result in increased average revenue (Matthews, 1987).

price he will pay if he wins, since only a single bid was submitted. If a risk-averse bidder is indifferent between the two bidding procedures (the case of CARA)—in one of which the price he will pay if he wins is a random variable, while in the other it is known with certainty—then the expected payment (i.e., seller's revenues) must be greater under the procedure which lacks the interim uncertainty (i.e., the noncontingent procedure). The same intuition applies for DARA buyers. If a risk-averse bidder *prefers* a procedure in which the price he will pay if he wins is a random variable, to one in which it is not, then the expected payment must be greater under the procedure in which payment is not a random variable.

Ortega-Reichart (1968) and Harstad, Levin and Kagel (1986) explicitly derive the unique symmetric Nash equilibrium bid function with numbers uncertainty for risk-neutral bidders:

$$b(x) = \sum_{n} W_n(x) b_n(x), \tag{3}$$

where $W_n(x) = F^{n-1}(x)p_n / \sum_i F^{i-1}(x)p_i$ is the probability of n bidders given that x is the highest private value observed (i.e., x wins the auction), $F(x)$ is the cumulative probability function of x, and $p_n = nS_n / \sum_k kS_k$ is the *ex ante* probability of there being n bidders conditional on the buyer being an actual bidder, where S_k is the exogenous probability of k bidders participating in the auction.

The noncontingent bid is a weighted average of the contingent (n known) bids, where the weights ($W_n(x)$) are the conditional probabilities of having n bidders. Therefore, a bidder winning with x under noncontingent bidding will pay an amount equal to the expected payment he would make if he won with x under contingent bidding. Hence, with risk neutrality, expected revenue is the same under the two procedures. Furthermore, since $b_3(x) < b_6(x)$, and $b(x)$ is a nondegenerate weighted average of the two contingent bids, the risk-neutral noncontingent bid should lie strictly between the two risk-neutral contingent bids. That is,

$$b_3(x) < b(x) < b_6(x). \tag{4}$$

In the symmetric bidding model, this inequality holds for any concave utility function, as the following proposition indicates.

Proposition. Assuming all bidders have the same concave utility function (symmetric bidding model), then (i) an increase in risk aversion unambiguously raises noncontingent bids, and (ii) equation (4) holds for *any concave* utility function.

Proof. See the Appendix.

Finally, the revenue-raising theorems in McAfee and McMillan (1987a) and Matthews (1987) imply that the following inequality holds with respect to individual bids for CARA and DARA bidders:[8]

$$b(x) \geq \sum_{n} W_n(x) b_n(x). \tag{5}$$

Equation (4), in conjunction with (5), provides the basis for our tests of the theory using individual bids. Namely, we test whether the following inequality holds:

$$b_6(x) > b(x) \geq [W_3 b_3(x) + W_6 b_6(x)]. \tag{6}$$

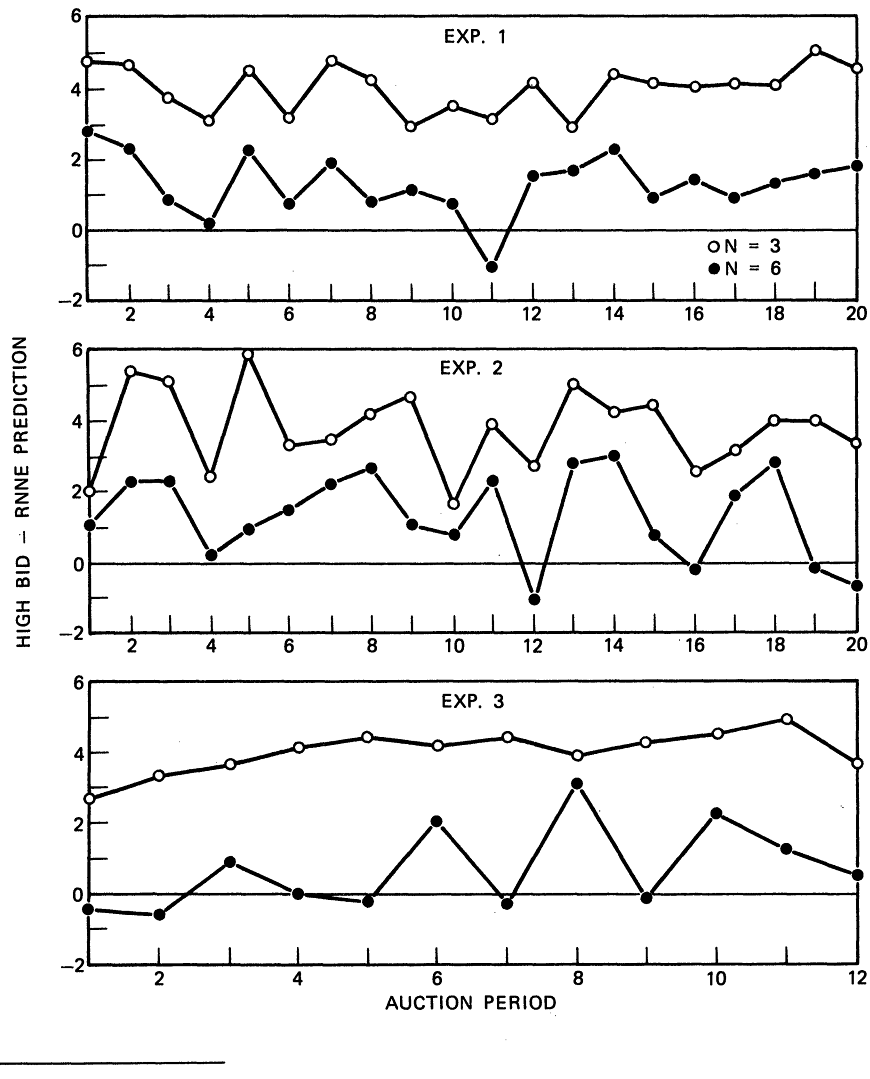
[8] See equations (30) and (12) in Matthews (1987), where the term $G(\theta \mid n)/G(\theta)$ corresponds to the W_n term here. The weak inequality in (5) and (6) (rather than the strict inequality implied by Matthews) is intended to include the risk-neutral case as well.

4. Experimental results

■ Figure 1 provides data for market outcomes, under contingent bidding, for those periods with bidding under both contingent and noncontingent procedures. For each period the difference between the high (winning) bid and the risk-neutral Nash equilibrium (RNNE) bid for the high bidder in both the $n = 3$ and $n = 6$ contingent markets is shown.[9] The high bid consistently lies above the RNNE prediction.

Moreover, in all three experiments and in all periods, the difference between the high bid and the RNNE bid is substantially larger in the market with fewer bidders. This empirical regularity is consistent with some types of risk-averse bidding. For example, in our design

FIGURE 1



[9] All bids are included in our analysis regardless of the market in which profits were actually paid. In each auction period the difference between the high bid and the RNNE bid of the high bidder contingent on $n = 3$ is computed by averaging these differences for all possible permutations of three out of six bidders for that auction period. Thus, our results are independent of the particular subsets of three bidders realized in cases where we paid off in the two contingent markets of size $n = 3$, and do not depend on an arbitrary allocation of the six subjects into two groups of three in cases where we paid off in the market of size $n = 6$.

the Nash contingent bid function for bidders with constant relative risk aversion (CRRA) is $b_n(R; x) = [(n-1)/(n-R)]x$, where R is the coefficient of CRRA, and $R = 0$ is the RNNE case. Let $D_n(R; x) \equiv b_n(R; x) - b_n(0; x) = [(n-1)R/n(n-R)]x$. However, the average highest $x \equiv \bar{X}_n$ is increasing in n. In our design, $\bar{X}_n = [n/(n+1)]VH$, so that straightforward calculations show that the

$$\text{sign}\,\{D_n(R; \bar{X}_n) - D_{n+1}(R; \bar{X}_{n+1})\} = \text{sign}\,\{n^2 - n - 2 + 2R\} > 0$$

for all $n > 1$. When $n = 3$ or $n = 6$, these differences can be quite large. For example, $\{D_3(R; \bar{X}_3) - D_6(R; \bar{X}_6)\}$ equals \$0.54 if $R = .3$, equals \$1.05 if $R = .5$, and equals \$2.16 if $R = .8$.

Table 1 provides summary statistics of the market outcomes for both the contingent and noncontingent markets. The difference between the high bid and the RNNE bid is calculated for $n = 3$ and $n = 6$ under the contingent procedure. For the noncontingent procedure, we report the difference between the actual bid and the RNNE bid for the highest of the six bidders. In all cases the average actual bid price (high bid) exceeds the RNNE prediction and is significantly different from zero at the 5% level or better. Bidding in excess of the RNNE bid is consistent with risk aversion, which is a necessary condition for the noncontingent bidding procedure to raise more revenue.[10]

Revenue comparisons under the contingent and noncontingent bidding procedures are presented in Table 2.[11] In all three experiments average revenue was greater under the noncontingent procedure, as predicted for CARA or DARA buyers. In experiments 1 and 3 the revenue difference is significantly different from zero at better than the 1% level.

TABLE 1 **Differences between Actual Bid Prices and Risk-Neutral Nash Equilibrium Bid Prices^**

Experiment (# OBS)	Contingent Procedure $n = 3$	Contingent Procedure $n = 6$	Noncontingent Procedure: For the Highest of the 6 Bidders
1 (20)	4.00 (0.15)	1.32 (0.19)	3.69 (0.26)
2 (20)	3.76 (0.26)	1.30 (0.28)	3.15 (0.31)
3 (12)	4.00 (0.18)	.68* (0.35)	3.26 (0.32)
Combined (52)	3.91 (0.12)	1.16 (0.15)	3.38 (0.17)

* Significantly different from 0 at the 5% level.
^ Mean values with standard errors of the mean in parentheses.
All other differences are significantly different from 0 at the 1% level.

[10] In over 80% of the auctions the highest bid was submitted by the holder of the highest private resale value, as predicted by all symmetric Nash equilibria. This result is robust both across procedures and across numbers of bidders within the contingent procedure. The frequency of Pareto-efficient outcomes is comparable to that reported in other private-value auction experiments (Cox, Roberson and Smith, 1982; Kagel, Harstad and Levin, 1987).

[11] Under contingent bidding, revenue for an auction period is calculated on the basis of the simple average of the price in the contingent market with $n = 3$ (where, as in footnote 9, this price is calculated as the average contingent on $n = 3$ for each possible permutation of three out of six bidders) and the contingent market with $n = 6$. For the noncontingent market, we computed an average price for markets of size $n = 3$ using the same procedure as in the contingent market, and took the simple average of this number and the price with $n = 6$.

TABLE 2 **Comparison of Revenue Raised by the Contingent and Noncontingent Procedures**

Experiment (# OBS)	Average Revenue Contingent (in dollars)	Noncontingent	Revenue Difference Contingent − Noncontingent (Paired t-Statistics)
1 (20)	20.44	20.94	−0.50 (6.54)
2 (20)	20.48	20.62	−0.14 (1.05)
3 (12)	20.68	20.96	−0.28 (3.19)
Combined (52)	20.51	20.82	−0.31 (4.71)

Averaged across all three experiments, the noncontingent procedure raised prices $0.31 more than the contingent procedure, which is significantly different from zero at better than the 1% level.

Table 3 shows the frequency with which the noncontingent procedure raised more revenue. Combined across all experiments, the noncontingent procedure yields a higher average price in 43 out of 52 auction periods. Higher revenue under the noncontingent procedure is clearly not the result of a few outlying observations, but the result of consistently higher bidding with numbers uncertainty rather than without.

Table 4 checks for the consistency of individual bids relative to the predictions of the theory. Part A of Table 4 shows the relationship between the two contingent bids. Bidders consistently responded in the "right" direction to the strategic forces involved in changing the number of bidders: 73.7% of all contingent bids satisfy the strict inequality $b_3(x) < b_6(x)$. For 2.9% of the bids, $b_3(x) > b_6(x)$—an outright bidding error relative to the Nash equilibrium for which there is no rationalization. In the remaining 23.4% of the observations, the two contingent bids are equal. One possible reason for this is that bidders may not always take the time and effort to carefully calibrate their bids in cases where their private values are so low that they have little chance of winning the auction (Cox, Smith and Walker, 1987; Kagel, Harstad and Levin, 1987). Repeating the same bid is a low-cost way of complying with the experimenters' requirements for submitting contingent bids. To test for this, we repeated the analysis using only private resale values greater than $10.00. In this case, 11.7% of all contingent bids were equal, while the bidding error ($b_3(x) > b_6(x)$) rate remains virtually the same, 2.8%.

TABLE 3 **Number of Periods in which the Noncontingent Institution Raised more Revenue**

Experiment	Total Periods	Periods in Which Noncontingent Raised More Revenue	Percentage of Periods
1	20	18	90%
2	20	15	75%
3	12	10	83%
Combined	52	43	83%

TABLE 4A Individual Bidding Patterns

Ordering of Bids in Contingent Markets			
$b_3(x) < b_6(x)$	$b_3(x) = b_6(x)$	$b_3(x) > b_6(x)$	Total # OBS
73.7%	23.4%	2.9%	312

Part B of Table 4 shows noncontingent bids relative to contingent bids for those cases in which $b_3(x) < b_6(x)$. Of the 230 observations, 33.9% satisfy the complete inequality requirements of the theory as embodied in equation (6). While this is less than a plurality of the individual observations, 71.3% of the noncontingent bids satisfy the requirement that $b(x) \geq [W_3b_3(x) + W_6b_6(x)]$; this requirement underlies the market level prediction that concealing information about the number of bidders will raise average revenue. This result is due to the fact that in 29.1% of the cases $b(x) = b_6(x)$, while in the remaining 8.3% of the cases $b(x) > b_6(x)$; hence, this inequality is automatically satisfied. While $b(x) = b_6(x)$ is a violation of the full inequality requirement of equation (6) and qualifies as a bidding error, it is only a marginal violation and errs in favor of the revenue-raising predictions of the theory. Taken together, we have a majority, 63%, of these individual bids satisfying the somewhat weaker inequality requirement $b_6(x) \geq b(x) \geq [W_3b_3(x) + W_6b_6(x)]$.

Restricting the analysis in Table 4B to those resale values in excess of $10.00 has virtually no effect on these results. Furthermore, of the 73 observations for which bids in the two contingent markets were equal, 95% of the time the noncontingent bid was equal to the two contingent bids as well.

5. Summary and conclusions

■ We conducted a series of first-price, sealed-bid auction experiments in which a single indivisible commodity was auctioned off among buyers with independent private values when there was uncertainty about the number of competing bidders. The actual number of bidders was an exogenously determined random variable with a 50% chance of being either three or six. Revenue-raising properties of a noncontingent bidding procedure (numbers uncertainty not resolved) were compared with those of a contingent bidding procedure (numbers uncertainty resolved).

Looking at individual bids in the contingent markets shows that bidders are sensitive to the most elementary strategic considerations in the auctions, as the Nash equilibrium bidding model predicts the ordering of the overwhelming majority of individual bids (particularly when we restrict our attention to those bids with higher probabilities of winning).[12] While the revenue predictions with respect to contingent versus noncontingent

TABLE 4B Ordering of Bids in Contingent and Noncontingent Markets for those Observations where $b_3(x) < b_6(x)$

(230 OBS Total)				
$b(x) \geqslant [W_3b_3(x) + W_6b_6(x)]$		$b(x) < [W_3b_3(x) + W_6b_6(x)]$		
$b_6(x) > b(x)$	$b_6(x) \leq b(x)$	$b(x) > b_3(x)$	$b(x) \leq b_3(x)$	Total # OBS
33.9%	37.4%	7.0%	21.7%	230

[12] Battalio, Kogut and Meyer (forthcoming) report comparable frequencies of bidding higher in independent private-value auctions with ten as compared to five bidders, holding $[VL, VH]$ constant.

bidding procedures are correct, a strict application of the Nash equilibrium model at the individual bidding level does not do nearly as well; less than a majority of all bids satisfy the full inequality requirement that $b_6(x) > b(x) \geq [W_3b_3(x) + W_6b_6(x)]$, even when we restrict the analysis to those cases in which $b_3(x) < b_6(x)$. A large majority of individual bids, however, satisfy the revenue-raising requirements of the theory (i.e., $b(x) \geq [W_3b_3(x) + W_6b_6(x)]$) and, in a large number of cases, involve marginal violations of the form $b(x) = b_6(x)$.

These results suggest that the basic economic forces underlying the revenue predictions of McAfee and McMillan, as well as Matthews, are at work in these auctions. When we take into account that there are subjective transactions costs associated with making bids and that bidders are rarely observed making detailed calculations, the fact that a large majority of the individual observations satisfies the revenue-raising requirements of the theory, in conjunction with the increase in revenue under noncontingent bidding, provides evidence that the forces specified in the theory were at work. Furthermore, given that the assumption of strict symmetry is violated (somewhat under 20% of all auctions were won by someone other than the highest private value holder; see footnote 10), and the fact that a number of bidders with low resale values did not find it worthwhile to adjust their bids contingent on n, it is clearly too much to expect that point predictions of auction models will be exactly satisfied.

From a broader perspective, our results serve to reinforce the impression that, by and large, Nash equilibrium bidding theory, with some accounting for risk aversion, serves to do a reasonably good job of predicting laboratory market behavior in private-value auction experiments (Battalio, Kogut and Meyer, forthcoming; Cox, Roberson and Smith, 1982; Kagel, Harstad and Levin, 1987). The results also provide an internal consistency test of the hypothesis that the persistent bidding in excess of the RNNE bid found in independent private value laboratory experiments reflects, at least in part, elements of risk aversion.[13]

Appendix

■ The proof of the proposition follows.

Let $1 < \bar{N} < \infty$ be the number of potential bidders with $N \leq \bar{N}$ the number of actual bidders. p_n is the exogenous probability that $N = n$, $n = 1, 2, \ldots, \bar{N}$. All bidders have the same concave von Neuman-Morgenstern utility function in gains, $u(\cdot)$, where $u(0) = 0$. Each actual bidder receives an independent, private value signal, x, $x \in [VL, VH]$, drawn from a distribution function, $F(x)$, with a continuous density, $f(x) > 0$, on the open support of x.

Denote by $b(x)$, with $b'(x) > 0$, the unique, symmetric, Nash equilibrium bid function for this game, and let $\sigma(b) = x$ be the inverse of $b(x)$.

In equilibrium each bidder's strategy is the best response to his rivals' equilibrium strategies; thus, using symmetry, we can write the problem for any actual bidder with a private value x as

$$\max_b u(x - b)G(\sigma(b)), \tag{A1}$$

where $G(\sigma(b))$ is the probability of winning with a bid of b, and $g(\sigma) > 0$ is the density. Clearly $b(x) < x$ for $x > VL$. The first-order condition for maximization in conjunction with symmetry is

$$0 = u(x - b(x))g(x)\sigma'(b(x)) - u'(x - b(x))G(x).$$

Thus,

$$b'(x) = \frac{u(x - b(x))g(x)}{u'(x - b(x))G(x)}. \tag{A2}$$

It is well known that we must have $b(VL) = VL$.

[13] See Kagel and Levin (1988) for a study suggesting that risk aversion is not the sole factor underlying bidding above the RNNE prediction in first-price, private-value auctions.

Proof of Part (i). Suppose that all bidders are more risk averse in the sense of having larger absolute risk aversion, $R(t) \equiv \frac{-u''(t)}{u'(t)}$. Denote variables of the model for more risk-averse bidders using carats.

First we show that for any $t > 0$, $\hat{\theta}(t) > \theta(t)$, where $\theta(t) = \frac{u(t)}{u'(t)}$. $\theta'(t) = 1 + R(t)\theta(t)$ and

$$\hat{\theta}'(t) = 1 + \hat{R}(t)\hat{\theta}(t).$$

Thus, $\hat{\theta}(t) \geq \theta(t)$ and the fact that $\hat{R}(t) > R(t)$ by assumption implies that $\hat{\theta}'(t) > \theta'(t)$. $\hat{\theta}(0) = \theta(0)$ and $\hat{\theta}'(0) = \theta'(0) = 1$, which implies, by the previous argument, that $\hat{\theta}(t) > \theta(t)$ immediately after $t = 0$. If at any $t > 0$, $\hat{\theta}(t)$ gets close enough to $\theta(t)$ from above, then $\hat{\theta}'(t) > \theta'(t)$ (by continuity), and $\hat{\theta}(t)$ stays strictly above $\theta(t)$.

Now, $b'(x) = \theta(x - b(x)) \frac{g(x)}{G(x)}$. Thus for any $x > 0$, $\hat{b}(x) = b(x)$ implies that $\hat{b}'(x) > b'(x)$. $\hat{b}(VL) = b(VL)$, and (it can be shown) $\hat{b}'(VL) = b'(VL)$, so we can repeat the previous argument to conclude that $\hat{b}(x) > b(x)$ for all $x > VL$.

Note we imposed no restrictions on $F(x)$ except continuity of f and $f > 0$.

Proof of Part (ii). With independent private values,

$$G(x) = \sum_{n=1}^{\bar{N}} F^{n-1}(x)p_n; \qquad \text{thus,}$$

$$\frac{g(x)}{G(x)} = \sum_{n=1}^{\bar{N}} (n-1)W_n(x)\frac{f(x)}{F(x)} = \bar{n}(x)\frac{f(x)}{F(x)}, \tag{A3}$$

where $W_n(x) = F^{n-1}(x)p_n / \sum_i F^{i-1}(x)p_i$. $\bar{n}(x)$ is the weighted average of the number of rivals, with the W's as the weights. Note that $0 \leq W_n(x) < 1$ and $\sum_n W_n(x) \equiv 1$.

In the contingent case (A2) becomes

$$b_n'(x) = \frac{u(x - b_n(x))}{u'(x - b_n(x))}(n-1)\frac{f(x)}{F(x)}. \tag{A4}$$

Thus, by comparing (A2) and (A4) and by using (A3), we see that $b_n(x) = b(x)$ implies that $b_n'(x) \gtreqless b'(x)$ as $(n-1) \gtreqless \bar{n}(x)$. Since $b(VL) = b_n(VL) = VL$ for all n, we have that if $(n-1) \gtreqless \bar{n}(x)$ for all x, then $b_n(x) \gtreqless b(x)$ for all x. The fact that in our design $\bar{n}(x)$ is a nondegenerate weighted average (for $x > VL$) of 2 and 5 implies that $2 < \bar{n}(x) < 5$, which establishes the proposition. *Q.E.D.*

References

ARROW, K.J. *Essays in the Theory of Risk Bearing.* Amsterdam: North Holland, 1971.

BATTALIO, R.C., KOGUT, J., AND MEYER, J. "Individual and Market Bidding in a Vickery First-Price Auction: Varying Market Size and Information." In L. Green and J.H. Kagel, eds., *Advances in Behavioral Economics,* Vol. 2. Ablex Publishing: Norwood, N.J., 1988.

COX, J.C., ROBERSON, B., AND SMITH, V.L. "Theory and Behavior of Single Object Auctions." In V.L. Smith, ed., *Research in Experimental Economics,* Vol. 2. Greenwich: JAI Press, 1982.

———, SMITH, V.L., AND WALKER, J.M. "Experimental Development of Sealed-Bid Auction Theory: Calibrating Controls for Risk Aversion." *American Economic Review,* Vol. 75 (1985), pp. 160–165.

HARRIS, M. AND RAVIV, A. "Allocation Mechanisms and the Design of Auctions." *Econometrica,* Vol. 49 (1981), pp. 1477–1499.

HARSTAD, R.M., LEVIN, D., AND KAGEL, J.H. "A Model of Auctions with an Uncertain Number of Bidders." Mimeo, University of Houston, 1986.

HOLT, C.A. "Competitive Bidding for Contracts Under Alternative Auction Procedures." *Journal of Political Economy,* Vol. 88 (1980), pp. 433–445.

KAGEL, J.H., HARSTAD, R.M., AND LEVIN, D. "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study." *Econometrica,* Vol. 55 (1987), pp. 1275–1304.

——— AND LEVIN, D. "Independent Private Value Auctions: Bidder Behavior in First-, Second- and Third-Price Auctions with Varying Numbers of Bidders." Mimeo, University of Houston, 1988.

MACHINA, M.J. "The Economic Theory of Individual Behavior Toward Risk: Theory, Evidence and New Directions." Center for Research on Organizational Efficiency, Stanford University, Technical Report No. 433, 1983.

MASKIN, E.S. AND RILEY, J.G. "Auctioning an Individual Object." Mimeo, UCLA, April 1980.

MATTHEWS, S.A. "Comparing Auctions for Risk-Averse Buyers: A Buyer's Point of View." *Econometrica,* Vol. 55 (1987), pp. 633–646.

MCAFEE, R.P. AND MCMILLAN, J. "Auctions and Bidding." *Journal of Economic Literature,* Vol. 25 (1987a), pp. 699–738.

——— AND ———. "Auctions with a Stochastic Number of Bidders." *Journal of Economic Theory,* Vol. 43 (1987b), pp. 1–19.

MILGROM, P.R. AND WEBER, R.J. "A Theory of Auctions and Competitive Bidding." *Econometrica,* Vol. 50 (1982), pp. 1089–1122.

ORTEGA-REICHERT, A. "Models of Competitive Bidding Under Uncertainty." Ph.D. dissertation, Stanford University, 1968.

RILEY, J.G. AND SAMUELSON, W.F. "Optimal Auctions." *American Economic Review,* Vol. 71 (1981), pp. 381–392.

VICKREY, W. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance,* Vol. 16 (1961), pp. 8–37.