

No Collaboration

1. Suppose you are given a connected graph with edge costs and a minimum spanning tree T for the graph, and a new vertex is added, which is connected to each old vertex by a new edge with given cost. Devise an algorithm to find a minimum spanning tree in the new graph. Your algorithm should run in $O(n)$ time: prove that the only edges you need to consider are the newly added edges and those in the old minimum spanning tree. Prove the correctness of your algorithm and verify its running time.
2. A *hypercube* of dimension k is a graph on $n = 2^k$ vertices and $k2^{k-1}$ edges, constructed as follows. The vertices are the bit vectors of length k ; that is, each vertex is a vector of k bits, each of which is 0 or 1. Two vertices are joined by an edge if their vectors differ in exactly one position. This graph is regular of degree k . Suppose that the edges of a hypercube have costs and we wish to find a minimum spanning tree. Give an algorithm that will compute a minimum spanning tree of a hypercube in $O(kn)$ time; that is, in time linear in the number of edges.
3. (Extra credit) A *cube-connected cycle graph* of dimension k is constructed from a hypercube of dimension k by replacing each vertex by a cycle so that each vertex in the expanded graph has degree 3. The vertices are vectors with $k + 1$ positions. The first k positions are bits and the $(k + 1)$ -st position is an integer from 1 through k , inclusive. There are two kinds of edges. Two vertices are connected by an edge if they agree on the first k positions and differ by one (mod k) in the final position, or if they agree on all positions except one bit position, say position j , and they both have j in the last position. (That is, the last position specifies the value of the unique differing bit.) This graph has $k2^k$ vertices and $3k2^k/2$ edges. Suppose that the edges of a cube-connected cycle graph have costs and we wish to find a minimum spanning tree. Devise, if you can, a linear-time deterministic minimum spanning tree algorithm for cube-connected cycle graphs.