COS 423 Spring 2009 Problem Set 6

No Collaboration

1. Consider an unrooted tree such that each edge has a positive length. The edges are undirected, so that they can be traversed in either direction. The *distance* between a pair of vertices is the length of the unique simple path joining them. The *diameter* of the tree is the maximum distance between a pair of vertices. Describe an O(n)-time algorithm to compute the diameter of an *n*-node tree. Hint: Root the tree at an arbitrary vertex. Observe that the path between any pair of vertices consists of two parts: one from one vertex to their nearest common ancestor, and the second from the nearest common ancestor to the other vertex. Consider an algorithm that processes the tree in bottom-up order (from leaves to the root).

2. We say that a directed graph is *semiconnected* if for any two vertices v and w, there is a path from v to w or a path from w to v (or both). Describe an O(n + m)-time algorithm to test whether a directed graph with n vertices and m arcs is semiconnected.