COS 423 Problem Set No. 1 Due: Wed. 2/11/09
Spring 2009

No collaboration.

All these problems concern list rearrangement, discussed in class on 2/2 and 2/4. See also the class notes. In all problems, assume that the initial list order for both the on-line algorithm and the off-line algorithm is by first access. (The first item accessed is at the front, followed by the second distinct item accessed, followed by the third, and so on.)

1. Consider the move-half-way strategy, which, after an access, moves the accessed item halfway to the front. (If the item is at position $2k$, it moves to position k; if it is at position $2k+1$, it moves to position $k+1$.) Is this strategy k-competitive with the optimum dynamic off-line strategy for some constant k? Justify your answer.

2. Consider the MTF half the time strategy, which moves an accessed item to the front every other time it is accessed (the second time, the fourth time, etc.); on odd accesses (the first, the third, etc.) it does not move the item. Is this strategy k-competitive with the optimum dynamic off-line strategy for some constant k? Justify your answer.

3. (Extra credit) Suppose the cost of a swap is s, for some $s \geq 1$, instead of 1. Can you devise an on-line strategy that is k-competitive with the optimum dynamic off-line strategy, for some k independent of n (the number of items), m (the number of accesses), and s? Justify your answer. The algorithm is allowed to know s, so the strategy can depend on s, but its competitive factor should not.