$\boxed{\#6}$ Ausubel & Cramton, "Demand Reduction & Inefficiency in Multi-Unit Auctions," (98)

also J. Morgan, "Efficiency in Auctions: theory & Practice," (01)

General Model: quantity 1, possibly infinitely divisible good (sometimes integer),
n bidders, $i=1,\dots,n$
c_i = capacity to consume
s_i = signal (sometimes consider this value v)
q_i = allocation ($\sum q_i = 1$)
x_i = payment

utility $\quad \min(q_i, c_i) \sum_{j=1}^{n} \underbrace{a_{ij}}_{\text{shared valuation}} s_j + x_i$

Can cover many standard cases:

Independent Private Values —

$a_{ii} = 1, \quad a_{ij} = 0 \quad i \neq j$

First Price 1 indivisible good, $x_i = -b_i$ when $q_i = 1$, else 0

Second Price 1 indivisible good, $x_i = -\max\limits_{j \neq i}(b_j)$, else 0

(forget ties)

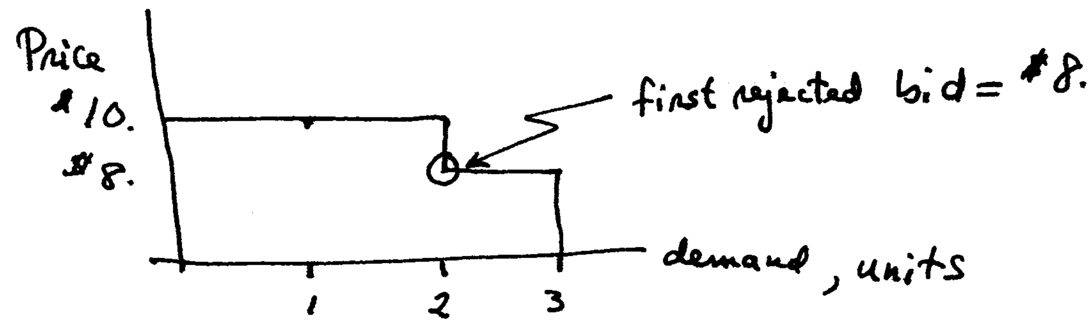
Uniform-Price Auction faulty analogy to Vickrey in multi-unit case. Use "market clearing" price for everyone $\cong$ first rejected price.

Example 1 (Morgan) Two identical units

Bidder #1, capacity 2, values \$10., \$10.
#2, capacity 1, value \$8 (or \$8, \$0)

Uniform-price auction

Suppose bidders bid their values ("sincere")

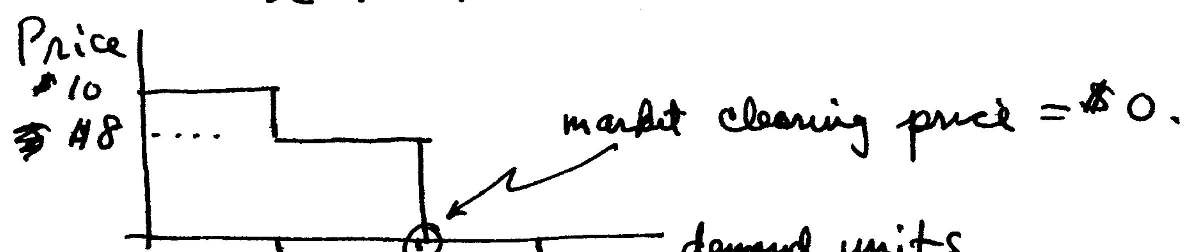


surplus of bidder 1, who gets both units @ \$8., is
$2 \times (10-8) = \$4.$
revenue to seller = \$16.

bidder 2 can do no better.

But suppose bidder 1 bids \$10. for first unit, and \$0. for second: [Verify: equilibrium strategy]

"Demand Reduction"



bidder 1 gets 1 unit at \$0, surplus = \$10.
bidder 2 gets 1 unit at \$0, surplus = \$8.
revenue to seller = \$0.

Inefficient → bidder 2 values object less, gets one!

<u>the point</u>: in single-unit case, price is determined by competitors' bids.
in mult-unit case, not so!

A&C 98 show this not pathological.

<u>Simplified Version of model for main result</u> (can be extended to more general cases)

- infinitely divisible object
- $c_i = c'$ identical capacities
- $A = I$ pure private values
- $f > 0$, support $[0,1]$

We can think about choosing $b_i(s_i, q_i)$ (bidding function, or sometimes $q_i(s_i, b_i)$ (demand function)

<u>Proposition 1</u> In simplified uniform-price auction,

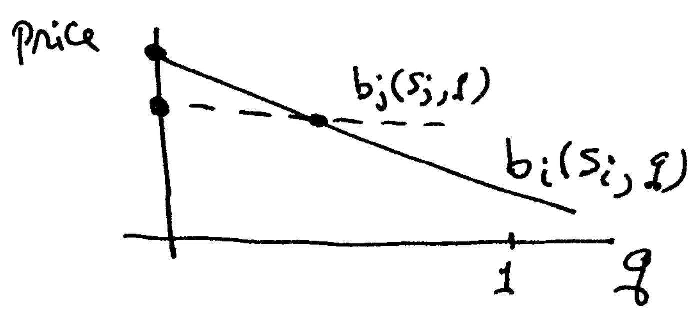
Efficiency $\Longrightarrow$ bidding functions are <u>flat</u>, <u>symmetric</u>, and <u>strictly increasing in signals</u> s_i.

<u>Proof sketch</u>

<u>flat</u>: Suppose (it's possible) that

$s_i >$ (m+1)st highest signal

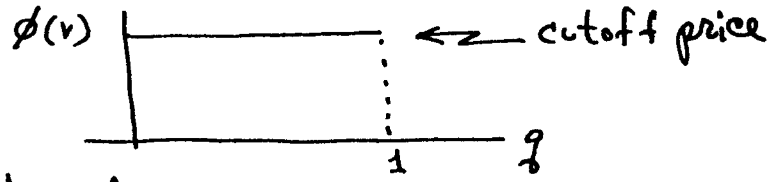
& $s_j <$ (m+1)st highest signal



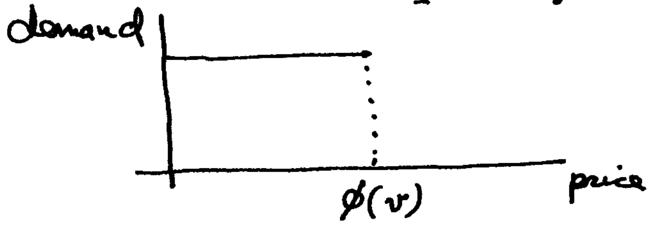
then if b_j intersects b_i, j will receive some of the good, which violate efficiency.

$\therefore$ b's must be flat ■

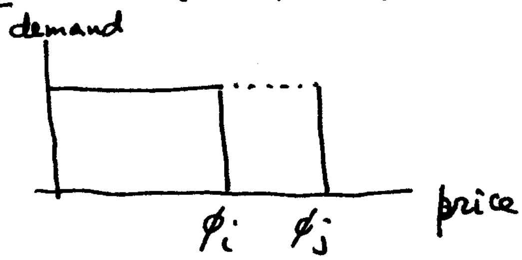
$\therefore$ $b(s, q)$'s are flat fctns of q:



or



<u>Symmetry:</u> We want to show cutoffs are all the same. Suppose not:



Efficiency will be violated if i receives a signal in the winning region, & j a signal in the losing region. ∎

<u>strictly increasing in signals s_i:</u>

Suppose that $v_i > v_j$ & $\phi(v_i) \le \phi(v_j)$. then, again, efficiency can be violated if v_i is in winning region and v_j in losing region. ∎

<u>Proposition 2</u> In simplified uniform-price auction,

$$\text{Efficiency} \Rightarrow \text{sincere bidding} \quad (b(v, q) = v)$$

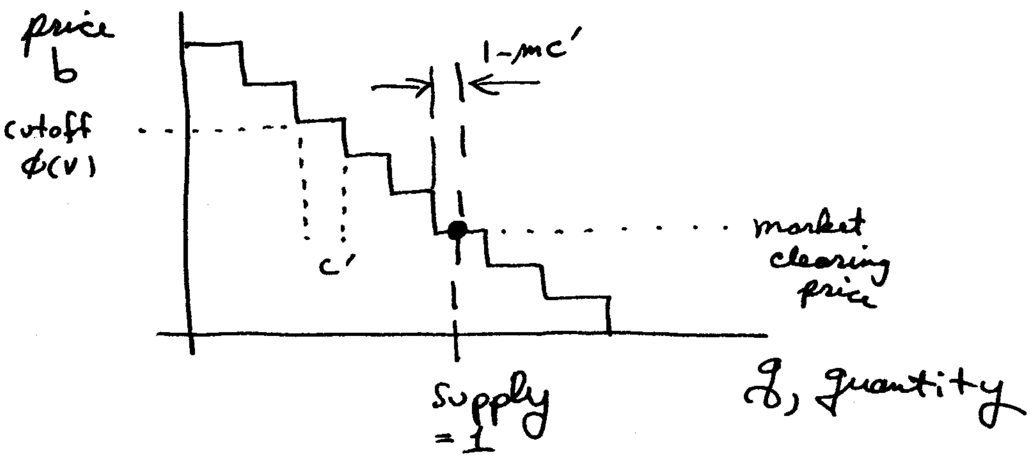
<u>Proof sketch</u> Let m be largest integer such that

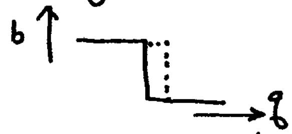
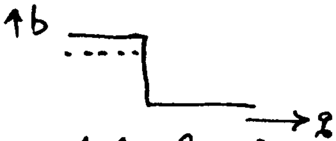
$$mc' < 1$$

and assume $c'(m+1) > 1$ (no exact division).

then c' goes to each of m bidders, and $1 - mc'$ to the bidder with $(m+1)$st highest valuation.

demand curve looks like



Suppose we change bidder i's $q(v,b)$ by a positive amount ϵ  or $b(v,q)$ 

with probability 0 this does not affect market clearing price. Thus, by the same argument as in the single-unit Vickrey auction, sincere bidding is a weakly dominant strategy. But $b(v,q)$ is flat as a function of q, so $b(v,q) = v$ for all q. ∎

In summary, Efficiency ⟹ flat, symmetric, increasing in s, and sincere bidding.

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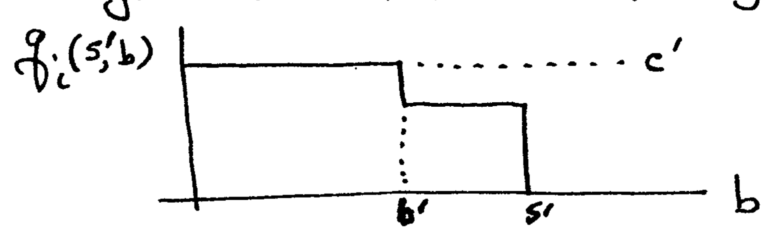
theorem There is no efficient equilibrium strategy in this uniform-price auction.

proof sketch: Suppose $(n-1)$ bidders besides i bid sincerely. Consider this deviation by bidder i

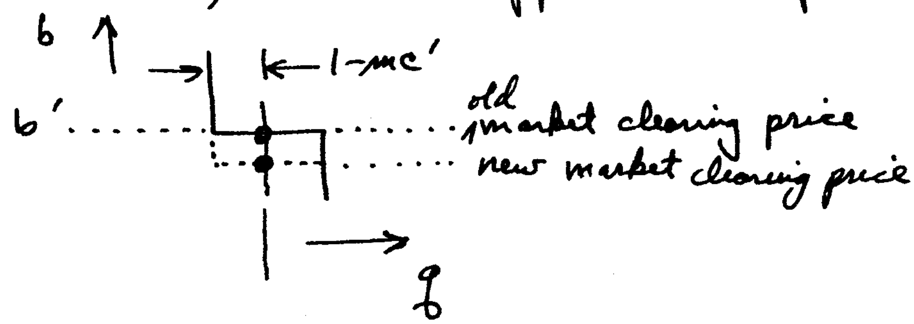


Examine payoff as b' decreases from s'.
that is, back off from sincere bidding, reducing demand. Decrease b' by, say, ε.

<u>Case 1</u> • mth highest signal of other bidders $< b'$.
this has no affect, i gets c', payoff is the same.

<u>case 2</u> • $(m+1)$st highest signal $> b'$. this also has no affect, i gets 0, payoff is the same.

<u>case 3</u> • b' between mth and $(m+1)$st highest valuations, which happens with positive probability.



This keeps i's quantity fixed at $1-mc'$, but lowers the market price, thus increasing i's payoff. This is therefore a <u>favorable deviation</u>, which shows sincere bidding is not a symmetric equilibrium. ∎

Appropriate generalization of Vickrey Auction

Return to example 1

Bidder 1 values each of two identical items at \$10., cap = 2

— 2 \$8., cap = 1

Values #1: 10, 10

#2: 8, 0

Suppose bidders bid sincerely; order bids high to low:

$$\left.\begin{array}{c}10\\10\end{array}\right\}\text{ 2 units to \#1}$$

$$\boxed{\begin{array}{c}8\\0\end{array}} \leftarrow \text{displaced bids.}$$

Payment : criterion: am't paid should be unaffected by bid.

& = amount = surplus that would be achieved if that bidder were absent.

without #1 $\left.\begin{array}{c}8\\0\end{array}\right\}$ → both to #1 ⇒ displaced bids

general algorithm for k objects

1. rank bids
2. award objects to highest k bids
3. bidders pay the displaced bids

But "transparency", relation of prices to valuations, can be "murky" RJM

Example 2 k = 3 objects

#1 10, 10, 0 #2 8, 0, 0 #3 6, 0, 0

rank

10	#1
10	#1
8	#2
6	#3
0	
0	
0	
0	

#1 gets 2 units

#2 gets 1 unit

bidder #1 displaces 6 ⇒ pays \$6 for 2

bidder #2 displaces 6 ⇒ pays \$6 for 1

will bidders accept this?

Result: with private values, the generalized Vickrey auction is efficient