Efficiency in Auctions: Theory and Practice*

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Abstract

In many contexts, efficiency is an important consideration in structuring auctions. In this paper, we survey several sources of inefficiency arising in auctions. We first highlight how demand reducing incentives, both in theory and in practice, affect the efficiency of multi-unit auctions. Next, we study inefficiencies arising from interdependence in bidder valuations. Again, we highlight both theoretical insights as well as how these translate in practice. Finally, we present an impossibility theorem for attaining efficiency in sufficiently rich auction contexts. An auction form suggested by Klemperer is discussed as a means of ameliorating inefficiencies arising in practice.

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1 Introduction

In the context of conducting monetary policy through open market operations, the main objective of the Eurosystem[1] is to maintain price stability. In achieving this policy goal, a key consideration is to choose procedures that favor efficient allocation of resources and free competition. Officially, the objectives of the Eurosystem are described thus:

> "The primary objective of the Eurosystem is to maintain price stability, as defined in Article 105 of the Treaty... In pursuing its objectives, the Eurosystem has to act in accordance with the principle of an open market economy with free competition, favouring an *efficient* allocation of resources." (p. 4 of "The Single Monetary Policy in Stage Three." Italics added.)

While decisions as to the timing and amount of liquidity to provide in open market operations go to the heart of maintaining price stability, the auction procedures used in the Eurosystem have a significant impact on whether resources are allocated efficiently or not. This is the primary focus of the paper. To address questions of efficiency, we survey a number of important findings, both theoretical and empirical, about the efficiency of various auction forms, some of which are currently used by Eurosystem in open market operations, others of which might be adopted. Our hope is that by providing a taxonomy of some of the causes of and solutions to efficiency problems as well as assessing the effectiveness of these solutions in the field, this survey will serve as a useful guide in structuring open market operations so as to achieve one of the primary objectives of the Eurosystem.

It should be emphasized, however, that the list of inefficiencies detailed in this survey is by no means complete. Rather, the sources of inefficiency highlighted here represent important new findings in auction theory over the last few years. Moreover, from a policy perspective, efficiency, while important, is not the sole goal of the Eurosystem in determining auction forms. Additional considerations such as simplicity, the transparency of the auction form as a means of signaling a monetary policy stance, the effectiveness of an auction form in managing liquidity, and the perceived fairness of auction

[1] The term "Eurosystem" is a shorthand for the composition of banks through which the European System of Central Banks performs its main tasks.

forms may all play a role in guiding decisions. For the most part, these additional motives are beyond the scope of the present paper.

In many respects, the Eurosystem is in a unique position to "customize" its auction forms to address problems specific to the environment in which it operates. The dramatically declining cost of information technology now makes possible the creation of a virtual marketplace whose design can be strongly guided by efficiency concerns. Moreover, physical barriers to coordinating geographically disparate parties is much less an obstacle than in the past due to the ubiquitousness of electronic communication. Finally, information technology allows for the possibility of quick and transparent diffusion of information in auctions. One compelling example of customized auction design is the US electromagnetic spectrum auction. In allocating spectrum licenses, policy makers, assisted by academicians, developed new auction forms specifically designed to address problems unique to each market. The auction form that was developed, known as the simultaneous-ascending auction, blended design elements of a number of different traditional auction forms into a unique hybrid.

The remainder of the paper proceeds as follows: In section 2, we introduce a model that is sufficiently flexible to be adaptable to all of the scenarios considered in the paper. Section 3 considers the simple case where bidders have private valuations for the units being auctioned and multi-unit demand. The main result in this section is to show that the single and multiple rate auction types used in the Eurosystem can result in efficiency losses.[2] An alternative auction form that is efficient is proposed. In section 4, we consider the case where there is interdependence of bidder valuations. That is, one bidder may have some information that affects how the object is valued by some other bidder. The main result in this section is to show that sealed bid auctions such as those used in the Eurosystem are often inferior in terms of efficiency to open auction forms. In section 5, we highlight the fact that in sufficiently rich auction contexts, the goal of full efficiency may be impossible with any auction form. The Anglo-Dutch auction, an auction form suggested by Klemperer (2000a) that combines both open and sealed bid aspects, is discussed as a means of overcoming a variety of other possible sources of inefficiency, such as collusion, which arise in practice. Section 6 presents a

[2]For future reference, a single rate auction in the Eurosystem is commonly known as a "uniform price auction" in the standard auction theory references. Likewise, a multiple-rate auction in the Eurosystem is commonly know as a "discriminatory auction."

brief discussion relating the main results to the Eurosystem and offers policy suggestions.

2 The Model

To study efficiency in the context of auctions, we begin by presenting a simple model that is nonetheless flexible enough to cover a wide variety of settings. An auctioneer is auctioning off a quantity of some (possibly divisible) good. We normalize the quantity available to be 1.[3] There is a set $N = \{1, 2, ..., n\}$ of bidders competing for the good. An allocation consists of a partition of the good among the bidders. A bidder's payoff depends only on the proportion of the good allocated to her. Let q_i denote the portion of the good allocated to bidder i.

Each bidder i receives a signal s_i drawn from a distribution $F_i(s_i)$ whose support is contained in the bounded interval $\left[\underline{s_i}, \bar{s}_i\right]$. Signals are independent across bidders. Bidders have quasi-linear utilities of the following form:

$$V^i = \min(q_i, c_i) \sum_{j=1}^{n} a_{ij} s_j + x_i.$$

The parameter $c_i > 0$ is the bidder's "capacity" to consume or utilize the units obtained in the auction. That is, for the portion of the good in excess of c_i allocated to bidder i,she obtains zero marginal utility. The valuation coefficient, $a_{ij} \geq 0$, reflects the impact of bidder j's signal on i's valuation for a share of the object. Both c_i and the a_{ij} coefficients are assumed to be common knowledge. The variable x_i represents the money transfer to/from bidder i. Typically, this will be a function of i's bid and those of other bidders.

For economy of notation, let A be the $n \times n$ matrix of valuation coefficients for all bidders with characteristic element a_{ij}. Similarly, let $\mathbf{c} = (c_1, c_2, ..., c_n)$, $\mathbf{s} = (s_1, s_2, ..., s_n)$, $\mathbf{q} = (q_1, q_2, ..., q_n)$, and $\mathbf{x} = (x_1, x_2, ..., x_n)$. Finally, let $\Pr(q_i = z_i)$ denote the probability that bidder i receives a share z_i of the object.

To get a flavor of the model, it is helpful to consider some standard cases in auction theory. Suppose that there is a single indivisible unit of the good being auctioned and that capacities are unconstrained, i.e. $c_i \geq 1$ for all i.

[3]This does not mean that we have restricted attention to single unit auctions. If one thinks of the good as partially or completely divisible, one obtains the multi-item case up to a continuum of items.

At one extreme, suppose that each bidder has an independent private value for the object. In this case, for each each bidder i, $a_{ii} = 1$, and $a_{ij} = 0$ for all $j \neq i$. That is, the matrix of valuation coefficients, A, is the identity matrix. Clearly, efficiency requires that the bidder with the highest signal get the object. In section 3, we study this simple case where the good is perfectly divisible and where capacities can be constraining.

At the other extreme, suppose that the value of the object is common to all bidders. A simple way of modeling this in our framework is to let the A matrix have characteristic element $a_{ij} = 1$ for all i, j. In other words, the value of the object to any bidder is simply the sum of the signals received by all bidders. It is clear in this case that bidders have interdependent valuations in that bidder i's valuation of the object is affected by the signal received by bidder j. Clearly, there are no efficiency considerations in a pure common value case. In sections 4 and 5, we study variations of this case where bidders have interdependent but not common values. Efficient allocations in these situations are more complicated – now it is no longer sufficient to award the object to the bidder with the highest signal.

When there is a single indivisible unit of the good being auctioned, the analog of a single rate auction employed in the Eurosystem has an allocation and payoff rule described as follows:

Letting b_i be the bid of bidder i, then

$$\begin{aligned}
\Pr(q_i = 1) &= 1 \text{ if } b_i > b_j \text{ for all } j \neq i \\
\Pr(q_i = 1) &= \frac{1}{m} \text{ in an } m\text{-way tie for highest} \\
\Pr(q_i = 1) &= 0 \text{ otherwise.}
\end{aligned}$$

and $x_i = -\max_{j \neq i}(b_j)$ if $q_i = 1$ and $x_i = 0$ otherwise.

In a first-price sealed bid auction (the single unit analog to the multiple rate auction in the Eurosystem), the allocation rule is the same – the high bidder gets the object. The payment rule under this auction form becomes: $x_i = -b_i$ if $q_i = 1$ and $x_i = 0$ otherwise.

3 Private Values and Multi-Unit Demand

In conducting open market operations, the standard auction held in the Eurosystem is one in which there are multiple, identical units of a good. The allocation of debt securities in the US is another classic example of this situation. Historically, US debt was auctioned using a pay-your-bid (also known as 'discriminatory') auction.[4] In this auction, bidders submit a bid price for various quantities of securities. The Treasury then determines the market-clearing price and all bids exceeding this price are awarded their quantities demanded at the price they offered. Friedman (1960) proposed that this mechanism be replaced by a 'uniform-price auction.' Under this auction, bidders once again submit bids for various quantities at a particular price. The Treasury determines the market clearing price. Bidders making offers above the market clearing price receive their quantities demanded; however, all winning bidders pay the *market clearing price.*

Much of the debate over the merits of these competing auction forms has been guided by observations in the case where bidders have private valuations for a single object being auctioned. Vickrey (1961) made the observation that by having bidders submit sealed bids, awarding the object to the highest bidder but having that bidder pay the second highest bid, it is then in the interests of bidders simply to submit a bid equal to their true valuation of the object being auctioned. A bidding strategy where bids simply reflect a bidder's true willingness to pay for the object is often referred to as *sincere bidding.* Not only is sincere bidding an equilibrium strategy in this auction – it is also a weakly dominant strategy. That is, regardless of what you think your rival bidders will do, a bidder can do no better than to simply bid her value truthfully.

As a consequence, in the private value setting, the object will be efficiently allocated to the bidder valuing it most highly. This auction form, known as the Vickrey auction, represented a successful first attempt to guide the design of auctions through considerations of efficiency. Indeed, professional economists widely believed that the uniform-price auction was the clear, multi-unit analog of that proposed by Vickrey. For instance, in an address to the Treasury, economist Robert Weber (1992) notes: "The second-price [Vickrey] auction naturally generalizes to a uniform-price auction, where all bidders pay an equal price corresponding to the highest rejected bid."[5]

It has been widely argued that the merit of the uniform price auction over the discriminatory auction is a reduction in bid-shading. For instance,

[4] This is a multiple rate or 'American' auction in the Eurosystem.

[5] Taken from the text of Weber's June 3, 1992 opening address at the US Federal Reserve/Treasury's "Forum on Change."

Merton Miller offers the following argument against the discriminatory auction, "People will shave their bids downward. All of that is eliminated if you use the [uniform-price] auction. You just bid what you think it's worth."[6] In a similar vein, Friedman suggests: "[The uniform price auction] has the major consequence that no one is deterred from bidding by being stuck with an excessively high price. You do not have to be a specialist. You need only know the maximum amount you are willing to pay for different quantities."[7] Taken together, these arguments suggest two advantages for the uniform price over the discriminatory auction. The first is that, by reducing distortions in bidding, the uniform price auction improves the allocative efficiency of the auction directly. Second, by simplifying the bidding strategies needed to compete successfully, the uniform price auction might encourage entry – especially among non-specialists. This latter effect has been little studied by auction theorists to date.

The intuition academics rely upon to conjecture the absence of bid-shading in the uniform-price auction appears to derive from the one unit case described above. In contrast, equilibrium in the discriminatory auction entails bid shading. (Indeed, bidding one's value in this auction form is a weakly dominated strategy.) Moreover, the appropriate bid shading strategy for a bidder in this auction depends crucially on his or her conjectures about how aggressive the bids of competitors will be. Thus, computations as to the appropriate amount to bid in this auction are arguably more complicated than in the uniform-price auction. Interestingly, in the case where bidders are symmetric, valuations are independent and private, and there is a single indivisible object being auctioned, there is nothing to distinguish between the two auction forms. Both have equilibria that are fully efficient and both raise exactly the same amount in revenues.[8] When valuations are private but signals are distributed asymmetrically, Vickrey (1961) demonstrates that differential bid shading in a discriminatory auction can lead to an inefficient outcome, even in the single unit case.

[6] As quoted in Ausubel and Cramton (1998), p 3.

[7] *Ibid.*

[8] This follows from the "Revenue Equivalence Theorem." See Klemperer (2000b) for a particularly clear statement.

3.1 Theoretical Results on Efficiency

The Inefficiency of the Uniform-Price Auction

When bidders demand more than one unit, simply bidding one's value is no longer optimal in the uniform-price auction. This can be illustrated by a simple example.

Example 1 Suppose that valuations are common knowledge.[9] There are two identical units being auctioned. Bidder 1 values each unit at \$10 and has a capacity of 2 units. Bidder 2, on the other hand, values a unit at only \$8 and has a capacity of 1 unit.

Suppose that the auctioneer holds a uniform-price auction. If both bidders simply bid their values, then bidder 1 will receive both units and pay the market clearing price. In this case, that price is bidder 2's bid of \$8 for one unit. In that case, bidder 1's surplus is \$4 (\$10 minus \$8 for each unit). The auctioneer earns revenues of \$16.

Clearly, bidder 2 can do no better than to bid his value, but bidder 1 can improve her surplus by deviating.[10] Suppose that instead of bidding her value (\$10) for the second unit, bidder 1 reduces her demand for the second unit by bidding \$0 for it. In this case, bidder 1 receives only one unit, bidder 2 receives one unit, and the market clearing price drops to \$0. This allocation is inefficient in that it allocates one of the units to bidder 2, who values it less than does bidder 1. Further, this auction raises no revenues whatsoever for the seller. It does, however, dramatically increase the surplus to bidder 1 (and to bidder 2 as well). Indeed, it is straightforward to verify that this strategy is an equilibrium for both bidders, whereas sincere bidding is not.

The example starkly illustrates the key difference between the single and multiple unit cases. In the single unit case of the uniform price auction, one's bid never affects the final price paid by a bidder. This is always determined by a competitor's bid. This is not the case when there are multiple units. As the example showed, bidder 1's bid for the second object indeed determined the price she paid for one of the objects. Thus, for exactly the reasons in the discriminatory auction, bid-shading on subsequent objects beyond the first is an appropriate strategy for a bidder in a uniform-price auction.

[9] In terms of our general model, this is the case where each bidders signal is generated from a degenerate distribution with mass 1 on a particular value.

[10] Throughout the paper, odd numbered bidders will be referred to in the feminine and even numbered bidders in the masculine.

Of course, the inefficiency in this auction arises not from bid shading *per se*, but rather from differential bid shading. Neither bidder shades his or her bid for the first unit, but does shade for subsequent units. This creates the possibility that bidder 1's shaded bid for the second object will fall below bidder 2's unshaded bid, thus splitting the allocation of the units between the bidders instead of efficiently awarding them both to bidder 1.

The key intuition from the above example is that the uniform price auction gives each bidder an incentive to try to lower the market price by shading down her bids on successive units of the object. Ausubel and Cramton (1998) show that in fact Example 1 does not represent a pathological case. Indeed, there is no equilibrium in the uniform price auction that is efficient. To show how the argument works, we now consider a version of the model presented in Section 2 that is a special case of their results. Suppose that the object is infinitely divisible and that bidders have identical capacities, $\mathbf{c} = (c', c', ...c')$. Further, suppose that A is the identity matrix, so we are in a pure private values case. Finally suppose that signals for all bidders are drawn from distributions with strictly positive density and support over the unit interval (e.g., the uniform distribution).

In the Ausubel and Cramton (1998) framework, bidder i must submit a bid function $b_i(q)$ that is right continuous and weakly decreasing in q. It is sometimes more useful to express a bidder's strategy as a demand function, $q_i(b)$. Demand functions are left continuous and weakly decreasing in b. Of course, a bidder's strategy may depend on her signal, s.

Let the aggregate demand curve at price b be given by $Q(b) = \sum_{i=1}^{n} q_i(b)$. Define the market clearing price, p,to be $p = \inf\{b | Q(b) \leq 1\}$. All inframarginal agents then receive their demand at price p. If $Q(p) > 1$, then the marginal agents are proportionally rationed. Let marginal agent i's incremental demand at price p be

$$\Delta_i(p) = q_i(p) - \lim_{b \downarrow p} q_i(b).$$

Such an agent receives

$$q_i = q_i(p) - (Q(p) - 1)\frac{\Delta_i(p)}{\sum_{j=1}^{n} \Delta_j(p)}.$$

As usual, an outcome of such an auction consists of an allocation $\mathbf{q}$ and a set of transfers $\mathbf{x}$. An outcome is efficient if the good goes to bidders with the highest valuations up to their capacity constraint. Let m denote the largest integer such that $mc' < 1$. Further, suppose that $c' \times (m+1) > 1$. Thus, an ex post efficient allocation gives c' of the good to the m bidders with the highest valuations and gives a rationed share $1 - mc'$ to the bidder with the $m + 1$st highest valuation.[11]

Ausubel and Cramton (1998) show that necessary conditions for an efficient equilibrium in a "conventional auction," such as the uniform price or discriminatory auction are:

Lemma 1 *Ex post efficiency implies symmetric, flat bidding functions for almost every s_i. Moreover, bid functions must be strictly increasing in signals almost everywhere.*

To obtain some intuition for these results, consider the efficiency implications of downward sloping bid functions. If bidder i upon receiving signal s_i submits a strictly downward sloping bid function, i.e. $b_i(0) > b_i(c')$, then it is possible that another bidder j, receiving a lower signal $s_j < s_i$ will submit a higher bid than i for some portion of the good. That is, $b_j(0) > b_i(c)$. This, however, can be a problem for ex post efficiency in that if s_i is greater than the $m + 1$st highest signal and s_j is lower than the $m + 1$st, efficiency requires that i receive a portion c' of the good and j receive none. This then implies that when bidder i receives signal s_i, if we compute the difference, $d_i(s) = b_i(0) - b_i(c')$, for almost all signals $d_i(s)$ must be zero. For similar reasons, strictly upward sloping bidding functions are also not consistent with efficiency. Thus, for almost all signals, bidders must submit flat demand functions if the uniform price auction is to be efficient.

Since in an efficient equilibrium, bidders are submitting flat demand functions, one can express a bidding strategy simply as a cutoff price. That is, a price where demand falls from c' to zero. Let $\phi_i(s)$ denote the cutoff price of bidder i with signal s. Efficiency also requires that bids be symmetric – bidders with the same signals must use the same cutoff prices. To see why this is necessary, suppose that 2 bidders use differing cutoff strategies. Suppose that bidder i receives the mth highest signal, s_i, but has a lower cutoff price than another bidder j, who received a signal s_j that is below the $m + 1$st highest signal. In that case, efficiency requires that i receive c' units and j receive none. If $\phi_j(s_j) > \phi_i(s_i)$ this will not happen. Either j will receive a positive amount of the good, or i will receive none of the good. Thus, for

[11] We ignore the (measure zero) event where there is a tie for the m + 1st highest valuation.

almost all signals, bidders must employ the same cutoff strategies. An almost identical argument implies that outcomes will be inefficient if cutoffs are not strictly increasing in signals.

Recall that in the uniform price auction, a bidder simply pays the market clearing price, p, for each unit she obtains. Hence, $x_i = pq_i$. Next, note that if bidders are choosing efficient bidding strategies in the uniform price auction, then these strategies must entail sincere bidding. This follows from the fact that for any quantity up to $1 - mc'$, bidder i's bid almost never determines the market clearing price. Thus, for the same reasons as in the single unit auction, it is a weakly dominant strategy to bid one's signal for these units. Since efficiency entails flat bidding strategies, this then implies sincere bidding for all units up to c'.

The problem, however, is that sincere bidding is *not* optimal when faced with sincere bidding by all other bidders. To see this, consider the following deviation by bidder i receiving a signal s'

$$q_i(b) = \begin{cases} c' & \text{if } \; b \in [0, b'] \\ 1 - mc' & \text{if } \; b \in (b', s'] \\ 0 & \quad \text{otherwise} \end{cases}$$

where $b' \leq s'$. Obviously, sincere bidding is a degenerate case of the above strategy when $b' = s'$. The key is to show that expected payoffs are decreasing in b' when evaluated at $b' = s'$.

There are three regions to consider:

1. Suppose that the mth highest signal of the other bidders is less than b'. In this case, bidder i gets c' and demand reduction via b' does nothing to expected payoff.

2. Suppose that the $(m+1)$st highest signal exceeds b'. In this case, bidder i gets nothing and demand reduction via b' does nothing to expected payoff.

3. Suppose that b' is between the mth and $(m+1)$st valuations of the other bidders. (This happens with positive probability.) Then, bidder i obtains quantity $1-mc'$ of the good and lowers her price by decreasing b' below s'. In this case, demand reduction strictly increases her expected profits.

Hence, reducing demand on units above $1 - mc'$ is a profitable deviation. As a consequence, efficiency is impossible in the uniform-price auction. Ausubel and Cramton show how this result may be generalized to the case of different capacities, correlated values, and downward sloping demands. In all of these cases, the intuition that bid shading leads to inefficiency plays a crucial role.

To summarize:

Theorem 1 *There is no equilibrium in the uniform price auction that is efficient.*

Implications for the Eurosystem The discriminatory auction was long known to be inefficient, and the uniform price auction was thought to be a remedy. Theorem 1 shows that even in simple private values settings, incentives for demand reduction eliminate the theoretical possibility of efficient allocations with this auction form. Thus, from a theoretical standpoint, neither of the variable rate tender procedures currently in use in the Eurosystem achieves efficiency even in simple private values settings. In the next part of this paper, we offer the appropriate generalization of Vickrey's original auction, and propose this auction form as an alternative to existing variable rate tender procedures. This auction form has the advantage of being theoretically efficient, although it may be less transparent in signaling interest rates than the uniform price auction.

The Vickrey Auction

A key insight in Vickrey's (1961) work on auction design is the observation that by having a winning bidder pay an amount unaffected by her own bid, sincere bidding becomes incentive compatible. Moreover, by making the amount paid by the winning bidder equal to the "externality" imposed on the other bidders (that is, by having the winning bidder pay an amount equal to the surplus in the absence of that bidder), private and social incentives are aligned and efficiency results. While an auction achieving these goals is quite straightforward in the unit demand case, as we saw earlier, the most obvious extension in the multi-unit world, the uniform-price auction, does not share its desirable properties.

To see the appropriate generalization, it is useful to return to Example 1. Recall that bidder 1 valued 2 units at \$10 each; whereas bidder 2 had values of \$8 and \$0. Suppose that bidders bid sincerely and we order the bids from highest to lowest. In this case, the order is \$10, \$10, \$8, and \$0. The highest two bids are awarded units – in this case both units go to bidder 1.

For sincere bidding to be in the interests of the bidders, it is necessary that what you pay should be unaffected by what you bid. Hence, bidder 1's payments will only be affected by bidder 2's bids and vice-versa. Next, to achieve efficient allocations, we would like each bidder to pay an amount equal to the surplus that would have been achieved had that bidder been absent. For instance, absent bidder 1, both objects would have been allocated to bidder 2. Bidder 2 valued these objects at \$8 and \$0; hence his surplus is \$8. Equivalently, since bidder 1 displaced the bids \$8 and \$0 by bidder 2, she pays \$8 for the two units.

More generally, consider a $k-$object auction with the following rules: Bidders each submit bids for all objects.. Bids are arranged highest to lowest. The k highest bids are awarded the objects. Each bidder pays an amount equal to the bids *that she did not submit* that were displaced from winning.

Clearly, bidder 2 can do no better than to bid his values since bidding above bidder 1 simply results in losses. Likewise, since 1's bids do not affect her payments, she can do no better than to bid sincerely as well. Hence, sincere bidding is an equilibrium.

While this design admirably solves the efficiency problem, the transparency of prices as they relate to valuations can be murky. Consider the following example.

Example 2 There are three units being auctioned. Suppose that bidder 1 values the first two units at \$10 each and values the remainder at \$0. Bidder 2 values the first unit at \$8 and all remining units at \$0, and bidder 3 values the first unit at \$6 and the remainder at \$0. With sincere bidding, the Vickrey auction correctly allocates two units to bidder 1 and one unit to bidder 2. Bidder 1 displaces the bid of bidder 3 (at \$6) and one of the \$0 bids. Thus, bidder 1 receives two units at \$6 or, equivalently, pays \$3 per unit. Bidder 2 displaces bidders 3's bid of \$6, so 2's price for the single unit is \$6. That is, bidder 2 is paying twice as much per unit as bidder 1 despite having valuations that are below those of bidder 1. In the context of the Eurosystem, interpreting the results of this auction as a signal of monetary policy stance is likely to be problematic.

Notwithstanding this difficulty, we now return to the model and offer a formal definition for allocation and payment rules in a Vickrey auction. This formulation was first offered in Ausubel and Cramton (1998). Suppose that bidders have private values and capacities c_i and that a perfectly divisible good of unit mass is being auctioned.

Define the aggregate demand of all bidders except i to be

$$Q_{-i}(b) = \sum_{j \neq i} q_j(b).$$

Define p^0 to be the market clearing price as above and define $p^{-i} = \inf\{p : Q_{-i}(p) \geq 1\}$. That is, p^{-i} is the market clearing price in the absence of bidder i. Ignoring rationing, the Vickrey auction awards a quantity $q_i(p^0)$ to bidder i and requires i to pay

$$x_i = q_i\left(p^0\right) p^0 - \int_{p^{-i}}^{p^0} (1 - Q_{-i}(t))\, dt.$$

This is illustrated in the following diagram.

Figure 1 here.

It is straightforward to show that sincere bidding is an equilibrium in this auction mechanism. To summarize:

Proposition 1 *With private values, sincere bidding is an efficient equilibrium in the Vickrey auction.*

3.2 Efficiency in the Field

Recall that demand reduction was the key factor leading to inefficiency in the uniform-price auction. Demand reduction was, of course, a strategic response to the possibility that one's own bids for succeeding units would affect the price paid for earlier units. While theoretically correct, from a policy perspective one may well wonder how important this factor is in affecting actual bidding behavior. Indeed, it seems plausible that once there are sufficiently many bidders in the market, the possibility of one's own bid affecting the price becomes negligible and hence the efficiency gains from the Vickrey auction become quite small.

List and Lucking-Reiley (2000) and Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) examined these issues in a series of field experiments. In the US, there is an existing active market in the trading of sports memorabilia, such a sports cards, autographs, and so on. A common way in which buyers and sellers meet in this market is through sports card trading shows. Card dealers typically set up stalls displaying their cards at these shows and negotiate sales prices with buyers. The details of how these negotiations come to pass are variable.

The field experiments take advantage of this existing market and compare allocations under a uniform-price auction and a Vickrey auction as the number of bidders varies. Specifically, two-unit auctions consisting of 2, 3, and 5 bidders were run using each of the auction formats. The "subjects" of the experiment consisted of either only trading card dealers or only non-dealers. The objects being auctioned consisted (mainly) of three different versions of a Cal Ripken, Jr. rookie card.[12] Three companies manufactured Ripken rookie cards: *Topps*, whose card is considered the most valuable, and *Fleer* and *Donruss*, whose cards are less valuable than *Topps*, but equal to one another. The *Topps* card has a "book" value of $70, while the others have a book value of $40.

In each of the experiments, bidders were taken through the following four step experimental procedure:

1. A subject was invited to participate, told that the auction would take about five minutes, and given an opportunity to examine the cards being auctioned.

2. The subject was given the instructions for the auction and asked to compute an example to demonstrate knowledge of the auction rules. The subject was also told that she would be randomly matched with a set number of other bidders of her type (dealers if she was a dealer and non-dealers if she was a non-dealer).

3. Subjects submitted bids.

4. The monitor explained the ending rules for the auction.

Note that winning bidders were required to make payments in accordance with the auction rules and received the card or cards. Bidders were segregated into dealer and non-dealer groups since the experimenters believed that the motivation for demand by dealers (*i.e.,* the possibility of resale) caused their

[12] In one of the treatments, a *Score* Barry Sanders rookie card was auctioned. Its estimated value is comparable to the *Topps* Ripken card.

valuations to differ substantially from the non-dealers. Participation in the experiments was, of course, entirely voluntary.

Table 1 below summarizes the treatments:

Insert Table 1 here.

Since the valuations that bidders place on each of the cards are unobservable, it is not possible to directly estimate whether bidders are employing sincere bidding strategies; however, by comparing across auction types, it is possible to test for the existence of demand reduction effects. Further, by comparing allocations under the two auction forms, it is again possible to infer some rough measure of the impact of demand reduction on efficiency in allocations. Note, however, that all of these measures take as given that bidders are employing something close to sincere bidding in the Vickrey auction. We shall return to this assumption later.

Table 2 below presents summary statistics of the results of the auctions.

Insert Table 2 here.

We now highlight the key aspects of Table 2. First, in the case of the two bidder treatments, there is strong evidence consistent with demand reduction. More formally, one can reject the null hypothesis that the unit 2 bids equal one another in favor of the one-sided alternative that the Vickrey bids lie above the Uniform bids at the 1% significance level. Further evidence of this occurs in the form of zero bids, which are a much higher percentage of the bids in the uniform-price auction as compared to the Vickrey auction. Finally, demand reduction appears to have a strong impact on allocations. There are far more split allocations under the uniform price auction as compared to the Vickrey auction. Thus, for the case of two bidders, it appears that strategic bidding does occur in the uniform-price auction and that this has an effect on efficiency.

Turning to the three and five bidder treatments, it appears that the introduction of even a small number of additional bidders substantially reduces demand reduction effects. In a five bidder treatment, ND5, mean second unit bids in the Vickrey auction are in fact lower than in the uniform-price auction. In the remaining treatments, the results are consistent with demand reduction; however, none of these differences is statistically significant at conventional levels. The proportion of zero bids still tends to be higher in uniform auctions; however it is difficult to see an economic impact from the small amount of demand reduction. Split allocations are largely the same in both auction forms. This proxy for efficiency, however, becomes extremely coarse as the number of bidders increases. To sum up, the field experiments suggest that demand reduction becomes less of an issue with even modest increases in the number of bidders.

A clear limitation in this experimental design is that, to make sensible comparisons between the auction forms, one must assume that equilibrium strategies are a good description of bidder behavior. In the case of the Vickrey auction, this may appear to be an unproblematic assumption in that it is a weakly dominant strategy to bid sincerely in private values settings. However, there may be good reason to doubt this characterization of behavior. Returning to the two bidder treatments in Table 2, notice that mean bids for the first unit are much lower than those under the uniform-price auction despite the fact that, in a private values setting, it is a weakly dominant strategy in both auctions to bid sincerely. The theoretical prediction of no difference between first unit bids across auction forms is rejected at conventional significance levels. In the three and five bidder treatments, there are no significant differences in first unit bids. Thus, one is left to wonder whether the differences in the allocations in the two bidder treatment derive from demand reduction or from anomalous bidding behavior in one or both auction forms.

One can get a clearer sense of bidding behavior in Vickrey auctions through laboratory experiments. In these experiments, it is possible to observe a bidder's valuation for each unit as well as to ensure that the conditions specified in the model are satisfied. The drawback of these experiments relative to field experiments is that the environment is somewhat less natural and thus behavior may not be reflective of what one sees in practice. Nonetheless, it is useful to highlight two sets of relevant laboratory experiments.

Kagel and Levin (1999) use an innovative laboratory design to make comparisons between uniform price and a dynamic analog to the Vickrey auction, called the Ausubel auction (see Ausubel (1997)), that is also theoretically efficient under the conditions derived above for the Vickrey auction.[13] In Kagel and Levin's design, a single human subject with induced flat demand for 2 units competes against a variable number of computerized bidders with single unit demand in a two unit auction. The computerized bidders bid sincerely for their single unit demand and the human bidder is made aware of the fact that the computerized bidders are employing this strategy. This design has

[13] To be more precise, they compared a number of variations of the uniform-price auction with the dynamic Vickrey mechanism proposed by Ausubel. In describing their results, we focus on standard sealed bid versions of the uniform price auction.

the advantage of eliminating strategic uncertainty over the actions of rival bidders as an explanation of observed bidding behavior. In the uniform-price auction, it is optimal to bid sincerely for the first unit and to bid zero for the second unit. In the Ausubel auction, sincere bidding for both units is optimal.

The results for the auction forms were quite interesting. In the uniform price auction, the modal bid for the first unit was equal to value; however, a substantial proportion of bidders selected bids in excess of valuations. When competing against three computer rivals, bids exceeded valuations more than one-fourth of the time. When competing against five computer rivals, bids exceeded values more than 40% of the time. As predicted by the theory, bids on second units exhibited substantial demand reduction (more than 61% of bids were below value), but the extreme prediction of zero bids was borne out relatively infrequently. Thus, while demand reduction was observed, there were substantial departures from theoretical predictions in bidding in this auction.

In the Ausubel auction, behavior was closer to equilibrium predictions. In both three and five rival treatments, over 85% of all first unit bids were within 5 cents of a bidder's valuation. Likewise, there was considerably less demand reduction in second unit bids under the Ausubel auction.

In Table 3, we compare the efficiency of both types of auctions. Efficiency in Table 3 is measured by comparing the realized surplus with the maximum potential surplus in the auction. As the table makes clear, both auction forms achieved high levels of efficiency – in every session, over 95% of the available surplus was realized. Consistent with the theoretical prediction, the Ausubel auction consistently achieved higher surplus than the uniform price auction.

Insert Table 3 here

Manelli, Sefton, and Wilner (2000) compare bidding behavior in multi-unit sealed bid Vickrey auctions with the Ausubel auction. In one treatment of interest, they induced private valuations for bidders and had them compete in a Vickrey-type auction. Specifically, three bidders competed for three units of a good. Each bidder had a flat demand for up to two units and valued the third unit at zero.

Insert Table 4 Here

As Table 4 shows, more than a third of all bids made for the first two units exceed the bidders' values. Thus, in both the sealed bid uniform price auctions studied in Kagel and Levin (2000) and the sealed bid Vickrey auctions studied in Manelli, Sefton, and Wilner, there is substantial overbidding on the first unit despite the fact that sincere bidding is a weakly dominant strategy. Unlike the Kagel and Levin studies, overbidding in the Manelli, Sefton, and Wilner experiments translates into significant welfare losses: The Vickrey auction achieves on 88% efficiency as compared to the theoretical prediction of 100%. Indeed, almost 11% of the time, a single bidder receives all three units even though the third unit is completely worthless to her.

3.3 Summary

From a theoretical perspective, the Vickrey auction is preferred to the uniform-price auction on efficiency grounds. The reason for the inefficiency in the uniform-price auction is that, since sometimes a bidder's payments for objects won are affected by her bids for "later" units, there is a strategic incentive to engage in bid shading or demand reduction in multi-unit auctions. As a consequence of this differential bid shading – more shading for more units – there is no efficient equilibrium in the uniform-price auction. The Vickrey auction remedies this defect in two ways. First, by making a bidder's payments independent of her bids sincere bidding becomes weakly incentive compatible. Second, by linking payments to the "externality" each bidder exerts on the other bidders through her participation in the auction, private and social incentives are aligned. As a consequence, full efficiency is obtained in weakly dominant strategies when bidders have private values.

There are, however, a number of drawbacks to the Vickrey scheme. First, it is not obvious how prices relate to valuations. It could well be that a bidder who values the object more pays less for it than some other bidder. It is also possible that a bidder who obtains multiple units will end up paying less than a bidder obtaining one unit. It may also not be palatable to have two bidders, each of whom receive exactly one unit, pay different prices, where the high bidder ends up paying *less* than the lower bidder. Second, in laboratory settings, the optimal strategy in the Vickrey auction is far from transparent – even to experienced bidders. As a result, there may be a wide gap between the theoretical and actual efficiency properties of this auction. Finally, as the number of bidders grows larger, the possibility of affecting the price with one's own bids diminishes. This ameliorates demand reduction incentives and may lead to smaller efficiency losses from the uniform-price auction. In field experiments with three or more bidders, there is little to distinguish bids for multiple units in Vickrey versus uniform-price auctions.

4 Interdependent Values

In the previous section, we examined how demand reduction incentives can create inefficiencies even in relatively simple private values contexts. We saw how a sealed bid auction form, the Vickrey auction, offers a solution for these types of inefficiencies. Of course, the situation facing most bidders participating in open market operations is likely to be far more complicated. Information is likely to be dispersed among the bidders and valuations may well depend on information possessed by rivals. In this case, a new force of inefficiency comes into play. In a sealed bid auction, a bidder may simply lack relevant information in making an appropriate bid. Sincere bidding, in this context, is simply impossible due to the uncertainties about just what a bidder's value exactly is. Clearly, auctions that generate information in the course of the bidding process can help to eliminate some of this uncertainty, and, as we shall see, can improve the efficiency of final resource allocations.

To get a sense of this, consider the following example:

Example 3 This example is adapted from Perry and Reny (1999). Suppose that there are three bidders competing for a single object and that capacities are unconstraining. Suppose that the matrix of coefficients is

$$A = \begin{matrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}.$$

Thus, bidders 1 and 3 have private values while bidder 2's valuation depends on his own signal and that of bidder 1.

Efficiency requires that the object be allocated to bidder 1 if $s_1 \geq s_2$ and $s_1 \geq \frac{s_3}{2}$. Bidder 2 should receive the object if $s_2 > s_1$ and $s_1 + s_2 \geq s_3$. If none of these conditions hold, bidder 3 should receive the object.

Suppose that a Vickrey (or uniform price) auction is held. Then, for the usual reasons, it is a weakly dominant strategy for bidder 1 to simply bid twice her signal. Likewise, bidder 3 should simply bid her signal. Bidder 2, on the other hand, cannot simply bid his value since it depends on s_1, which, unfortunately, bidder 2 does not know. For bidder 2 to beat bidder 1 when he should, it must be the case that the bidding strategy 2 is using, $b_2(s_2)$ exceeds 1's bidding strategy whenever $s_2 \geq s_1$. This implies that a necessary condition for efficiency is

$$b_2(s_2) \geq 2s_2. \tag{1}$$

On the other hand, if bidder 1 has a low signal, say ε, then efficiency requires that bidder 3 win when $s_3 \geq \varepsilon + s_2$. This implies that a necessary condition for efficiency is

$$b_2(s_2) \leq s_2 + \varepsilon \tag{2}$$

for arbitrarily small ε. Clearly, equations (1) and (2) are mutually incompatible; thus, even when bidders 1 and 3 are bidding sincerely, efficiency is impossible using this sealed bid auction form.

Now suppose that, instead, an English auction is used. That is, a price is announced and bidders indicate whether or not they would like to remain in the auction. When only one bidder remains, he or she gets the object at that price. The strategies of bidders 1 and 3 are unchanged by this variation. Bidder 2, on the other hand remains in the auction until he is indifferent between staying in and winning at the next instant and dropping out. Thus, prior to anyone dropping out of the auction, bidder 2 will drop out at a price $p = 2s_2$. If bidder 3 is the first to drop out, then bidder 2 holds to this strategy. If on the other hand, bidder 1 is the first to drop out, then bidder 2 is able to infer 1's signal and drops out at a price $p' = s_1 + s_2$.

These strategies lead to the efficient allocation. Since 1 and 3 are both dropping out at their values, it will never be the case that 1 gets the object when 3 should or vice-versa. Moreover, since prior to 1's dropping out of the auction the condition for bidder 2 to beat bidder 1 is $s_2 \geq s_1$, bidder 2 will never receive the object when it should go to 1. Thus, the only remaining case is determining that 2 does not get the object when 3 should and vice versa. Clearly, if 1 drops out first, the allocation will be efficient. Suppose that 3 drops out first. This means that $s_3 \leq 2s_1$, but then it was not efficient to allocate the object to 3.

To summarize, in Example 3, the English auction allocates efficiently when a uniform price auction does not. The remainder of this section shows how this observation may be generalized.

4.1 Theoretical Results on Interdependent Valuations

We begin by studying a specification of the model used in laboratory experiments by Kirchkamp and Moldovanu (2000), the results of which are reported in the next part of the paper. In their setting, valuations are all interdependent but in a simple and symmetric fashion. Detailed derivations of equilibrium behavior for this example are found in Kirchkamp and Moldovanu (2000).

Example 4 Suppose that there are three bidders, each of whom receive uniformly distributed signals. There is a single indivisible unit being auctioned. Bidder i's utility is

$$V_i = q_i \left(s_i + \alpha s_{i+1}\right) + x_i$$

where we use the convention that if $i+1=4$, then $i+1=1$. The coefficient $\alpha \in (0,1)$. It is helpful to think of this situation as one where bidders are seated in a circle. A bidder's valuation is most affected by her own signal, but somewhat affected by the signal of the bidder on her right; this is true for all bidders.

As usual, ex post efficiency requires that we award the object to the bidder who values it most highly. Notice, however, that this bidder does not necessarily have the highest signal. For instance, suppose that $\alpha = \frac{1}{2}$ and that the realized signals are $\mathbf{s} = \left(\frac{3}{4}, 1, 0\right)$. In this case, bidder 1 values the object at 1.25, bidder 2 values it at 1, and bidder 3 values it at $\frac{3}{8}$. Thus, efficiency requires that the object go to bidder 1 even though her signal is only second-highest.

Suppose that we run a Vickrey auction to allocate the object. Consider a symmetric equilibrium where each bidder employs the bidding strategy $b(s)$ and where b is increasing and differentiable. In this case, Kirchkamp and Moldovanu (2000) show that symmetric equilibrium bidding behavior is:

$$b(s) = s\left(1 + \frac{3}{4}\alpha\right).$$

Since this strategy is increasing in the bidder's own signal, it will have the property that the object is misallocated in circumstances similar to the one given above. Note also that, with interdependent values, it is no longer the case that simply bidding one's signal is a weakly dominant strategy of the Vickrey auction. The key difficulty is that in determining her bid a bidder is missing important information – namely the signal of her neighbor. As a consequence, inefficient allocations are possible.

Once again, Kirchkamp and Moldovanu show that running an English auction solves the problem. They first note the following property of ex post efficient allocations in this example.

Lemma 2 *An efficient allocation never awards the object to the bidder with the lowest signal.*

Suppose that bidder i had the lowest signal, s. Compare i's valuation to bidder $i+1$ with signal s'. In this case, i's valuation is $s+\alpha s'$ whereas $i+1$'s valuation is $s'+\alpha s''$ where $s'' \geq s'$. Since $s', s'' \geq s$ then $i+1$'s valuation exceeds i's.

Suppose that bidders initially follow a strategy where the price at which they drop out of the auction is increasing in their own signals. In this case, the bidder with the lowest signal is the first to drop out. Suppose, without loss of generality, that bidder 3 drops out. In that case, bidder 2 knows exactly his valuation for the object and so it is a weakly dominant strategy for him to drop out once this value has been reached. That is, the cutoff price for bidder 2 is

$$p_2 = s_2 + \alpha s_3.$$

At each price, p, bidder 1 performs the following thought exercise: Knowing bidder 3's signal and 2's strategy, 1 can infer 2's signal if he drops out of the auction at the next instant. Thus, conditional on bidder 2 dropping out, 1's value is $s_1 + \alpha(p - \alpha s_3)$. Thus, bidder 1 should drop out when

$$s_1 + \alpha(p - \alpha s_3) = p.$$

Solving yields the cutoff strategy for bidder 1 of:

$$p_1 = \frac{s_1 - \alpha^2 s_3}{1-\alpha}.$$

Using these strategies, bidder 1 wins the auction whenever

$$\begin{aligned} p_1 &\geq p_2 \\ \frac{s_1 - \alpha^2 s_3}{1-\alpha} &\geq s_2 + \alpha s_3 \\ s_1 + \alpha s_2 &\geq s_2 + \alpha s_3. \end{aligned}$$

But this is exactly the condition for ex post efficiency. The distribution of signals was not important to generating this result.

Example 3 demonstrates that when one bidder has an interdependent valuation, the English auction can be efficient when a sealed bid auction is not. Example 4 demonstrates that when all bidders have interdependent valuations in a symmetric fashion, the same result holds. A key question, then, is to identify conditions on interdependent valuations where there exists an efficient equilibrium in the English auction. Maskin (1992) first observed that for the case where there are only two bidders, then an English auction is efficient provided bidders' value functions satisfy a single crossing property. In our framework:

1. **Single crossing:** Valuations satisfy the single crossing property if for all $i, j \neq i$, $a_{ii} > a_{ji}$.

That is, an increase in i's signal increases her valuation at a higher rate than it does any other bidder j. Krishna (2000) offers sufficient conditions for English auctions to allocate efficiently in the n bidder case. Translated into our (more restrictive) framework, these sufficient conditions are:

2. **Average crossing:** Valuations satisfy the average crossing condition if for all $i, j \neq i$,

$$\frac{1}{n}\sum_{k=1}^{n} a_{kj} > a_{ij}.$$

In words, for valuations satisfying the average crossing property, the marginal impact of j's signal on any other bidder i's value is less than the marginal impact of j's signal on the average value.

3. **Cyclical crossing:** Valuations satisfy cyclical crossing if for all j,

$$a_{jj} > a_{j+1j} \geq a_{j+2j} \geq ... \geq a_{j-1j}$$

where $j + k \equiv (j + k) \bmod n$.

Notice that, after a relabeling of the bidders, the matrix A used in Example 4 satisfies the cyclical crossing property. The highest marginal impact of a bidder's signal is on her own valuations while the impact on other's valuations may be ordered in a cyclical way. Their example does not, however, satisfy the average crossing property for $\alpha > \frac{1}{2}$. Example 3 (barely) fails to satisfy the average crossing property but does satisfy the cyclical crossing property. Further, notice that conditions 2 and 3 imply single crossing in the two bidder case.

With these conditions in mind, it follows from Krishna (2000) that

Theorem 2 *Suppose that valuations satisfy the cyclical or average crossing property and there is a single indivisible unit for auction, then the English auction is efficient.*

Implications for the Eurosystem From the perspective of the Eurosystem, the lesson here is clear:

A dynamic or open auction is efficient in many circumstances where a sealed bid auction is not.

In other words, all of the auction forms currently used in undertaking open market operations have the drawback that key information possessed by bidders is not incorporated into the final price. This has two consequences. First, as we saw, it can lead to inefficient allocations. Second, it means that the informational value that market price gives to the policy makers in the Eurosystem in determining the appropriate monetary policy stance is needlessly garbled. While it is usual to think of interest rate setting activities of the Eurosystem sending a signal to the market, by employing variable rate auctions, information in this case is in fact bidirectional – the market sends key information to the Eurosystem by its bids in open market operations. Open auction forms let the Eurosystem take better advantage of the informational value of variable rate auctions.

An Important Caveat The assumption that only a single unit is being auctioned in Theorem 2 is surprisingly important – even if all bidders have only unit demand. To see this, consider a variation of Example 3 where there are two units available and each bidder has a capacity for only a single unit. As usual, it is a weakly dominant strategy for bidders 1 and 3 to simply bid their values. Bidder 2, on the other hand, chooses to drop out at a price where, if the other two bidders immediately dropped out, 2 would be indifferent between winning and losing. Thus, $b_2(s_2) = 2s_2$. Notice however, that with two items available once a single bidder drops out the auction is over.

Suppose that $\mathbf{s} = \left(\frac{7}{8}, \frac{1}{8}, \frac{1}{2}\right)$. In this case, it is efficient to give one unit to bidder 1 and one unit to bidder 2. Notice however, that bidder 2 drops out of the auction when the price reaches $\frac{1}{4}$ whereas bidder 3 drops out only when the price reaches $\frac{1}{2}$. Thus, the English auction, in this case, allocates the objects incorrectly to bidders 1 and 3. It is easy to show that a continuum of such cases are possible.[14]

4.2 Laboratory Results on Interdependent Valuations

Until recently, the costs associated with holding oral auctions on a large scale effectively precluded the use of these auction forms in many contexts. However, with the rise of information technology, electronic bidding in real-time is no longer a serious difficulty, nor is it especially costly to implement. Thus, some of the practical advantages of running a sealed-bid auction as opposed to an oral auction have disappeared. Still, before advocating a change to the more efficient, but perhaps more cumbersome English auction over a sealed bid form, it is useful to assess whether the theoretical efficiency gains translate into actual bidder behavior.

A set of experiments, Kirchkamp and Moldovanu (2000) explore bidding in the three bidder, single object context of Example 4 above. Specifically, they compare bidding in a second price sealed bid auction and an ascending auction using a variety of parameter values for α.[15] In assessing behavior in English auctions, it is useful to distinguish the strategic problems facing the three bidders. The first bidder to drop out needs to be playing a monotonic bidding strategy for other bidders to correctly infer her signal. Suppose bidder i drops out first. Then, bidder $i-1$'s problem of determining what to bid is considerably easier than that faced by $i+1$. In equilibrium, $i-1$ can perfectly infer i's signal from her dropout bid, so it is a weakly dominant strategy for $i-1$ to bid up to her value. Equilibrium bidding for bidder $i+1$, on the other hand, requires that he infer his value based on the possibility that $i-1$ will drop out at the next moment. This is obviously a more cognitively difficult strategy.

Kirchkamp and Moldovanu find that the low bidder does indeed follow a monotonic strategy that is relatively close to the equilibrium prediction. The low bidder tends to stay in longer than the equilibrium prediction for

[14] Since in the single unit case, the Ausubel auction and the English auction are identical, the same example demonstrates that the Ausubel auction is inefficient. Essentially, this observation was first made by Perry and Reny (1999).

[15] In these experiments, the implementation of the second price sealed bid auction is non-standard. Kirchkamp and Moldovanu implement this mechanism by running an ascending clock auction where the time at which the first bidder drops out is unobservable. All clocks stop when there is only a single active bidder remaining. As a consequence, the drop out strategy of the high bidder is censored, and the frame in which the auction is conducted differs from most other implementations of the second price sealed bid auction.

low signals and relatively shorter for high signals. Bidders to the "left" of the low bidder likewise play strategies that are close to equilibrium strategies. On the other hand, bidders to the "right" of the low bidder tend to be far from equilibrium strategies. In particular, their bids are relatively insensitive to their *own* signal as well as to the implied signal of the low bidder (which affects them only indirectly). Bids in the sealed bid auction are also relatively close to theoretical predictions.

We now turn to efficiency in the two auctions. In analyzing efficiency, Kirchkamp and Moldovanu find it useful to divide the analysis into two cases: "Easy" cases are those in which the bidder with the highest signal also has the highest value. "Hard" cases are where the bidder with the highest signal does not have the highest value. The theoretical prediction is that there should be no difference in the efficiency of the competing auction forms in easy cases, but that the English auction should dominate the sealed bid auction in the hard cases. Figure 2 illustrates the efficiency outcomes of the Kirchkamp and Moldovanu experiments. In this figure, the circles indicate the mean efficiency for the English auction under various α parameters while the crosses indicate mean efficiency for the sealed bid auction. In all cases for the English auction, the theoretical prediction is 100% efficiency. The figure highlights the fact that the English auction does represent an efficiency improvement over the sealed bid auction; however, because of deviations from equilibrium bidding behavior on the part of the "right" bidder, the efficiency of the English auction is below 100%. Dividing this into easy versus hard cases, one can see that there is little to distinguish the two auction forms in easy cases, but that efficiency in hard cases is much higher with the English auction. Interestingly, deviations in equilibrium bidding behavior in the sealed bid auction result in a much higher frequency of efficient outcomes (about 30% of the time) as opposed to the theory prediction (0% of the time).

Figure 2 here.

As was mentioned in Section 3.2 above, Manelli, Sefton, and Wilner (2000) also compare oral and sealed bid auction forms. In addition to the private values treatment already considered, Manelli, Sefton, and Wilner also study the case where bidders have interdependent valuations. As in the Kirchkamp and Moldovanu experiments, the sealed bid auction form they use is not theoretically efficient, whereas the oral auction is. In the Manelli, Sefton, and Wilner experiments, three bidders compete for three units; however, each bidder only demands two units. A bidder's valuation for the first two units is: $\frac{1}{2}s_i + \frac{1}{4}(s_i + s_i)$ where we use the usual numbering convention.

Notice that in the unit demand case, this interdependence does not affect the efficiency properties of the Vickrey auction; however, with multi-unit demand, interdependencies create demand reduction incentives, which harm efficiency.

Table 5 summarizes some of the main findings of these experiments. First, notice that unlike the sealed bid auction in the Kirchkamp and Moldovanu experiments, the Manelli, Sefton, and Wilner experiments show significant overbidding. This is also true, but less so, for the oral auction. Thus, in both auction forms, there are significant departures from equilibrium bidding. These departures directly translate into efficiency losses. This is may be easily seen by looking at the frequency by which all three units were allocated to a single bidder. In the case of the oral auction, the theoretical prediction is 0%. In contrast, this happened slightly more than 7% of the time in the experiments. Moreover, despite the fact that the oral mechanism is predicted to yield 100% efficiency, in fact, its efficiency of around 84% was slightly lower than for the sealed bid auction.

Table 5 here.

To recap, the theoretical prediction that oral auctions have efficiency advantages over sealed bid auctions receives mixed support in laboratory settings when bidders have interdependent valuations. In the single object case, a dynamic Vickrey (English) auction outperforms a sealed bid Vickrey auction. In multi-unit settings with interdependent valuations, the dynamic Vickrey (Ausubel) auction does slightly worse than a sealed bid Vickrey auction in terms of efficiency.

5 Further Obstacles to Efficiency

In this section, we highlight a number of additional obstacles to efficiency. From a theoretical perspective, we begin by showing that once the determinations of bidder valuations become sufficiently rich, theoretical efficiency becomes impossible. We present a theorem formalizing what "sufficiently rich" means. Next, we consider some practical obstacles to efficiency raised by Klemperer (2000a) and others. Finally, we highlight a "hybrid" auction form suggested by Klemperer as a means of overcoming some of these obstacles.

5.1 Theoretical Obstacles to Efficiency

We previously showed that, with multi-unit demand, efficiency in the uniform-price auction is lost. Further, with certain types of interdependencies among bidder valuations, efficiency in sealed bid auction forms is lost. In each case, we were able to offer alternative auction forms that restored theoretical efficiency. We now show that, in many circumstances, auctions that yield efficient outcomes simply do not exist. We begin with a simple two bidder example.

Example 5[16] Suppose that there are two bidders competing for a single indivisible object and that neither bidder is capacity constrained. Suppose that bidder 1 receives a signal in the interval $[1, 2]$. Bidder 2 receives a constant signal $s_2 = -\frac{3}{2}$. Finally, suppose

$$A = \begin{matrix} 1 & 0 \\ 2 & 1 \end{matrix}.$$

In this example, notice that bidder 1's signal has a greater marginal impact on 2's valuation than it does on 1's valuation. Thus, the single crossing condition is not satisfied. Efficiency requires that the object be allocated to bidder 1 when $s_1 \leq \frac{3}{2}$ and allocated to bidder 2 otherwise.

Maskin (1992) observed that in contexts such as this example, there is no auction that can achieve the efficient allocation. To see this, from the revelation principle, it is known that any auction that achieves an efficient allocation has an equivalent direct revelation mechanism (DRM) whereby bidder 1 sends a message $\hat{s}_1$ as to what her signal is and the mechanism then specifies an allocation $\mathbf{q}(\hat{s}_1)$ and a transfer rule $\mathbf{x}(\hat{s}_1)$.[17] A DRM is incentive compatible if it is optimal for bidder 1 to reveal her true signal. Suppose that bidder 1 receives a signal $s_1' \leq \frac{3}{2}$, then incentive compatibility and efficiency require

$$s_1' + x(s_1') \geq x(s_1'') \tag{3}$$

for all $s_1'' > \frac{3}{2}$.

Next, suppose that bidder 1 receives a signal $s_1'' > \frac{3}{2}$, incentive compatibility and efficiency require

$$x(s_1'') \geq s_1'' + x(s_1'). \tag{4}$$

[16] Thanks to Vijay Krishna for suggesting an example along these lines.

[17] Since bidder 2 receives a constant signal, disclosure on his part is not needed.

But since $s_1'' > s_1'$, then it is obvious that equations (3) and (4) cannot be simultaneously satisfied. Hence, there is *no* efficient auction mechanism.

To recap, if valuations do not satisfy the single crossing condition, then there may be *no* auction that allocates the objects efficiently.

Jehiel and Moldovanu (2000) point out that impossibility can also arise generically when signals are multi-dimensional. To touch on this issue, we extend the model to the case where bidders receive multi-dimensional signals. In particular, suppose that each bidder receives a signal $\mathbf{s}_i = \left(s_i^1, s_i^2, ..., s_i^k\right)$. The characteristic element of the valuation matrix is $\mathbf{a}_{ij} = \left(a_{ij}^1, a_{ij}^2, ..., a_{ij}^k\right)$, where a_{ij}^l describes the impact on bidder i's valuation of the realization of s_j^l.

To see how the presence of multidimensional signals can easily lead to the impossibility of efficient allocation, consider the following example:

Example 6 Suppose that there is a single indivisible object being auctioned between two bidders. Bidder 1 receives two signals, $s_i = (s_1^1, s_1^2)$. The second bidder receives constant signals. The first signal tells bidder 1's valuation for the good, the second signal bidder 2's valuation. Bidder 2 is uninformed about values. Thus, the utilities of each bidder are:

$$\begin{aligned} V^1 &= q_1 s_1^1 + x_1 \\ V^2 &= q_2 s_1^2 + x_2. \end{aligned}$$

Obviously, it is efficient to give the object to bidder 1 when $s_1^1 \geq s_1^2$.

Consider transfer rules when $s_1^1 \geq s_1^2$. Incentive compatibility requires that the transfer rule must be constant in all of these cases. If not, bidder 1 will economize by choosing the pair $(s_1^1,\ s_1^2)$ that maximizes x_1 subject to the constraint that $s_1^1 \geq s_1^2$. The same reasoning implies that in the case where $s_1^1 < s_1^2$, the transfer rule must likewise be constant. Let x_1^1 denote the constant transfer for reports $s_1^1 \geq s_1^2$, and let x_1^2 be likewise defined.

For a given realization of (s_1^1, s_1^2), bidder 1 will choose the larger of $s_1^1 + x_1^1$ and x_1^2. Thus, bidder 1 will report $s_1^1 \geq s_1^2$ when $s_1^1 > x_1^2 - x_1^1$ and will report $s_1^1 < s_1^2$ when the opposite inequality holds. Notice, however, that these reporting strategies are independent of s_1^2 – in other words, incentive compatibility requires an allocation strategy that is independent of s_1^2. However, the relationship between s_1^1 and s_1^2 is absolutely crucial to allocate the objects efficiently. As a result, even in this simple setting, constructing an efficient auction is impossible.

The key intuition here is that bidder 1 has a piece of information, s_1^2, that does not directly affect her valuation but does affect efficiency. The single transfer instrument available to the auctioneer is simply not sufficient to extract this information.

More generally, consider the case of the model where there are $K < \infty$ feasible allocations of the object(s) being auctioned. At its most general, an auction consists of an *allocation rule*, π, and a *payment rule*, x. Let the space of all signals received by bidders be S. Under these circumstances, an allocation rule is a function mapping realized signals into probabilities of each of the feasible allocations. That is $\pi : S \to \Re^K$ where for all k, $\pi_k(s) \in [0,1]$ and where for all s, $\sum_{k=1}^{K} \pi_k(s) = 1$. An allocation rule is efficient if it allocates the objects such that the weighted sum of bidders' valuations is maximized. A direct revelation mechanism (DRM) consists of a pair (π, x) mapping reported signals into allocations and transfers.

With this notation, Jehiel and Moldovanu (2000) offer the following result generalizing this intuition.

Theorem 3 *Let (π, x) be an efficient DRM. Assume that the following are satisfied: 1. For some allocation k, there exist $i \neq j$ such that $q_i > 0$, $\mathbf{a}_{ii} \neq 0$ and $\mathbf{a}_{ji} \neq 0$. 2. There exist open neighborhoods $\Theta^i \in s_i, \Theta_1^{-i}, \Theta_2^{-1} \in S^{-i}$ such that $\pi_k(s_i, s^{-i}) = 1$ for all $(s_i, s^{-i}) \in \Theta^i \times \Theta_1^{-i}$ and $\pi_k(s_i, s^{-i}) = 0$ for all $(s_i, s^{-i}) \in \Theta^i \times \Theta_2^{-i}$. Then (π, x) cannot be incentive compatible.*

In words:

When one bidder possesses information about another bidder's valuation that is central in determining an efficient allocation, designing an efficient auction may be impossible.

5.2 Practical Obstacles to Efficiency

Klemperer (2000a) highlights the fact that collusive behavior among bidders may be an important practical obstacle to achieving efficient allocations. Moreover, he goes on to argue that open auctions may be more susceptible to collusive behavior than sealed bid auctions. The idea is the following: in an open, ascending auction early bids on items may be used to signal an implicitly collusive arrangement for the division of the objects. Cramton and Schwartz (1999) find evidence of this type of signaling in US broadband auctions. In these auctions, the last three digits of (multi-million dollar) early bids corresponded to area codes of particular regions for which bandwidth was being auctioned. Klemperer also highlights observations made by Jehiel and Moldovanu (2000a and 2000b) regarding bidding behavior in German spectrum auctions. In this case, there were ten licenses available. In the initial round of bidding, Mannesman, a key player in this market, made five high bids on licenses and five much lower bids. This was viewed by a rival as an offer to split the licenses between them a five apiece. The result was non-aggressive bidding on the part of both bidders and an even split of the licenses.

In circumstances where many of the efficiency problems highlighted in this paper are paramount, Klemperer (2000a) suggests employing an auction form he calls the "Anglo-Dutch" auction. In the simple single object case, this auction consists of an ascending auction which proceeds until only two bidders remain, followed by a sealed bid auction where bids can be no lower that the price level reached during the ascending phase of the auction. First, as we showed in Section 4, interdependencies in bidder valuations require the information aggregation properties of the ascending auction to facilitate efficient allocations. By correctly selecting the form of the sealed bid auction, demand reducing incentives (emphasized in Section 3) may be avoided. Finally, the sealed bid phase of the auction guards against collusive possibilities. It remains to assess, both theoretically and in the lab, the effectiveness of this hybrid auction form in multiple-unit auction settings.

6 Discussion

Allocating resources efficiently is one of the primary goals of the Eurosystem. In conducting open market operations, the form of the auction employed by the Eurosystem can impact how resources are allocated. The focus of this paper is to point out several sources of inefficiency in allocations that can arise from auction forms similar to the variable rate auction procedures being employed in the Eurosystem. First, as we saw in Section 3 of the paper, even in the simple case where bidders have private values, demand reducing incentives in the uniform price auction can lead to inefficient allocations. The Vickrey auction is a simple and workable solution to this problem. Lack of information at the time of bidding is a second source of inefficiency. As we say in Section 4, when bidders have interdependent valuations, sealed bid auction forms can lead to inefficiency by preventing bidders from conditioning bid amounts on information that comes in as part of the auction process. In this case, one solution would be to move to dynamic auction forms, such as the English auction. Indeed, there are many circumstances where the English auction yields an efficient allocation when sealed bid auction forms do not. Finally, in Section 5, we pointed out that when environments are sufficiently rich or valuations ill-behaved, then it may simply be impossible to allocate efficiently. We also highlighted some practical difficulties, such as the possibility of collusion, from the open auction forms we presented. Faced with these obstacles, Klemperer's Anglo-Dutch auction offers one possible solution. This mechanism retains the desirable aspects of open auctions structured in a way that minimizes demand reducing incentives; however, the final phase of the auction is a sealed bid phase, which might make collusive outcomes more difficult to sustain.

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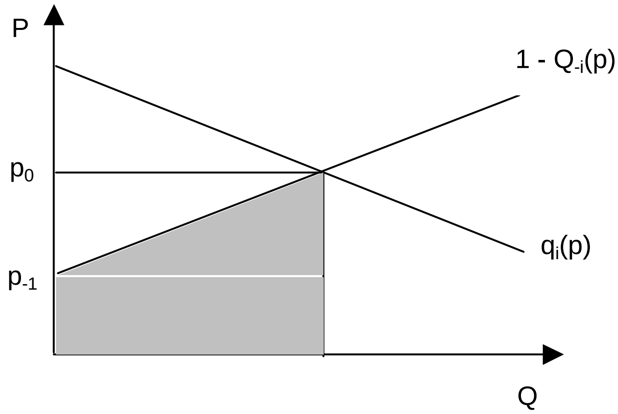
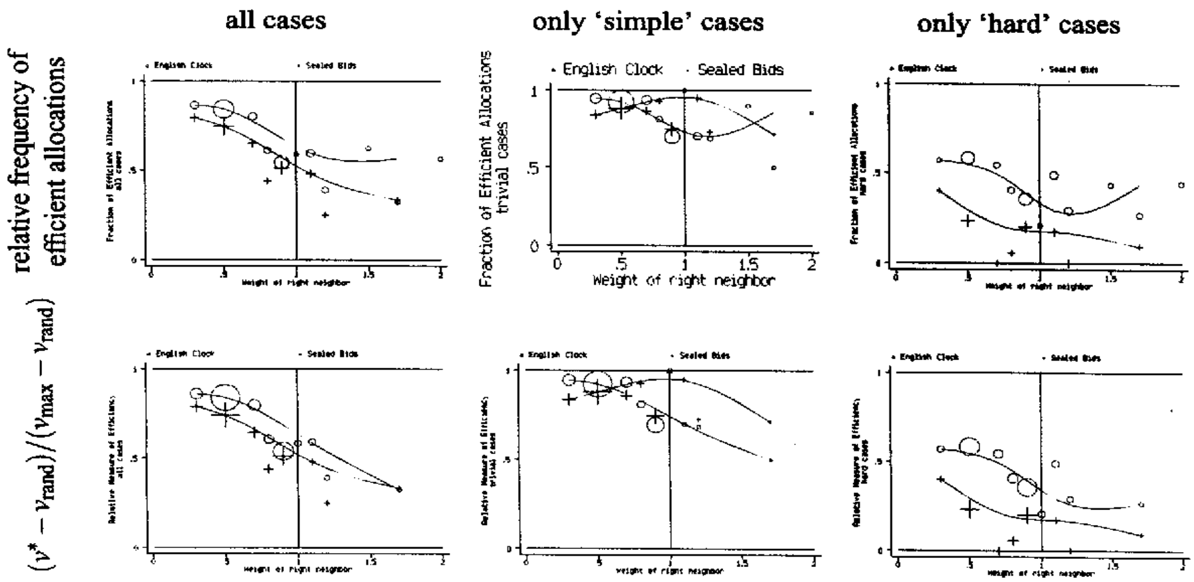
Figure 1: Payment Rule in the Vickrey Auction



Source: Ausubel and Cramton (1998).

Figure 2: Efficiency in Interdependent Value Auctions



Sizes of the symbols are proportional to the number of observations. Splines connect four median bands. The figure shows that the higher efficiency of the English auction is obtained primarily in 'hard' cases.

Source: Kirchkamp and Moldovanu (2000), Figure 7.

Table 1: Summary of Experiments

Treatment	Card Auctioned	Bidder Type	Vickrey Auctions	Uniform Auctions
D2	Ripken Topps	2 Dealers	15	15
ND2	Sanders Score	2 Non-dealers	17	17
D3	Ripken Topps	3 Dealers	9	9
ND3	Ripken Donruss	3 Non-dealers	12	12
D5	Ripken Fleer	5 Dealers	6	6
ND5	Ripken Fleer	5 Non-dealers	6	6

Source: Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) Table 1.

Table 2: Summary Statistics

Treatment	Bid on 1st Unit Vickrey	Bid on 1st Unit Uniform	Bid on 2nd Unit Vickrey	Bid on 2nd Unit Uniform	Split Allocations Vickrey	Split Allocations Uniform	Zero Bids Vickrey	Zero Bids Uniform
D2	49.60	62.67	41.77	30.60	11.7%	43.3%	0.0%	3.3%
	(15.19)	(15.28)	(14.46)	(13.43)				
ND2	51.82	62.21	28.82	16.62	27.9%	42.6%	5.9%	20.6%
	(15.19)	(15.28)	(14.46)	(13.43)				
D3	39.74	38.17	29.26	22.44	86.1%	91.7%	11.1%	14.8%
	(26.87)	(26.44)	(25.05)	(17.81)				
ND3	20.03	20.68	9.69	8.63	74.1%	77.8%	30.6%	41.7%
	(13.60)	(13.46)	(10.46)	(9.94)				
D5	31.12	31.45	20.68	18.63	90.0%	93.3%	6.7%	20.0%
	(22.81)	(15.91)	(14.82)	(15.00)				
ND5	20.77	19.44	9.77	10.48	95.0%	93.3%	26.7%	40.0%
	(14.20)	(15.93)	(11.03)	(11.74)				

Terms in parentheses are standard deviations.

Source: Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) Tables 3 and 4.

Table 3: Summary of Uniform and Ausubel Experiments

Session	Number of Rivals	Auction	Efficiency Actual	Efficiency Predicted
1	3.00	Uniform	98.29	97.30
			(0.72)	(0.25)
2	3.00	Uniform	95.36	96.72
			(0.92)	(0.29)
3	5.00	Uniform	98.19	98.46
			(0.83)	(0.16)
9	5.00	Ausubel	99.90	100.00
			(0.07)	
10	3.00	Ausubel	98.60	100.00
			(0.38)	

Source: Kagel and Levin (2000) Tables 4 and 10.

Table 4: Summary of Vickrey Experiments

Overbid	2.075
	(1.085)
Three objects	0.108
	(0.167)
Efficiency	0.871
	(0.070)

Overbid denotes the number of bids for the 1st or 2nd unit that exceed a bidder's valuation.
Three objects denotes the proportion of the time a single bidder obtained all three objects
Parentheses denote standard deviations.

Source: Manelli, Sefton, and Wilner (2000) Table 2.

Table 5: Summary of Interdependent Values Experiments

	Sealed Bid (Vickrey)	Oral (Ausubel)
Overbid	1.092	0.433
	(0.689)	(0.407)
Three objects	0.017	0.071
	(0.083)	(0.039)
Efficiency	0.853	0.844
	(0.065)	(0.082)

Overbid denotes the number of bids for the 1st or 2nd unit that exceed a bidder's valuation.
Three objects denotes the proportion of the time a single bidder obtained all three objects
Parentheses denote standard deviations.

Source: Manelli, Sefton, and Wilner (2000) Table 2.