What computers just *cannot* do. (Part II)

COS 116: 3/6/2007

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Administrivia

- Midterm - take home during week of 3/12
  - Review session Fri 3/9 3pm Friend 005
  - Last year's exam linked under "extras" on web
- Back-off from pseudocode
  - H2, Q6: now optional
  - On midterm: be able to read, but *not* write it

Epimenides Paradox

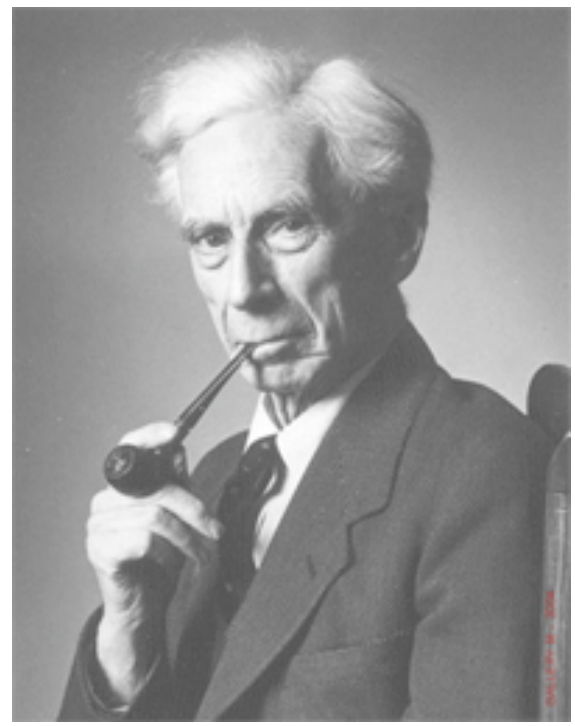
- *Κρῆτες ἀεί ψεύσται*
- “Cretans, always liars!”
- But Epimenides was a Cretan!’

  (can be resolved…)

- More troubling: “This sentence is false.”

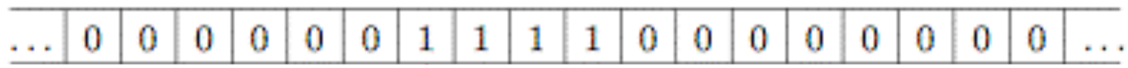
Barber Paradox

- Town with one male barber
- Each man either shaves or sees the barber
- Barber shaves the men who do not shave themselves
- Does barber shave himself? Contradiction either way!



Bertrand Russell (1872 –1970)

Recap from last time



- Turing-Post computational model:
  - Greatly simplified model
  - Infinite tape, each square either 0/1
  - Program = finite sequence of instructions (only 6 types!)
  - Unlike pseudocode, no conditionals or loops, only "GOTO"
  - code(*P*) = binary representation of program *P*

Motivation

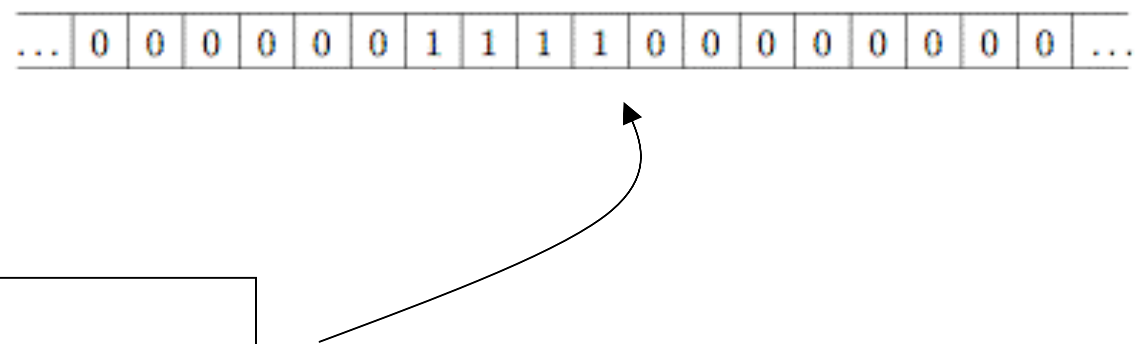
Simplify!

(Get to the heart of the matter)

Doubling program

1. PRINT 0
2. GO LEFT
3. GO TO STEP 2 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

Halting



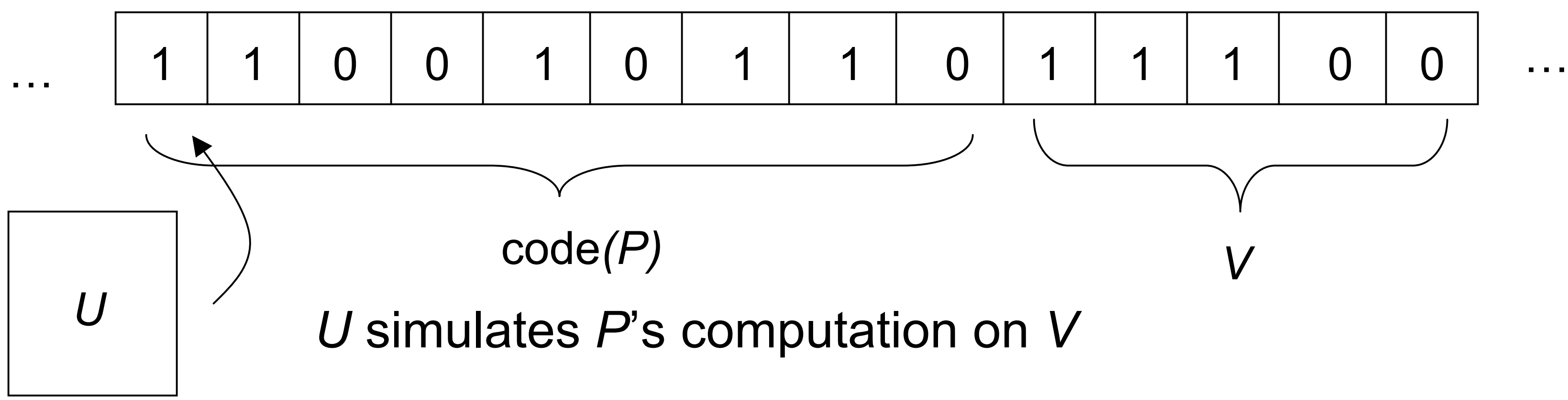
Program

1. PRINT 0
2. GO LEFT
3. GO TO STEP 2 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

Program halts on this input data if STOP is executed in a finite number of steps

Some facts

- Fact 1: Every pseudocode program can be written as a T-P program, and vice versa
- Fact 2: There is a universal T-P program

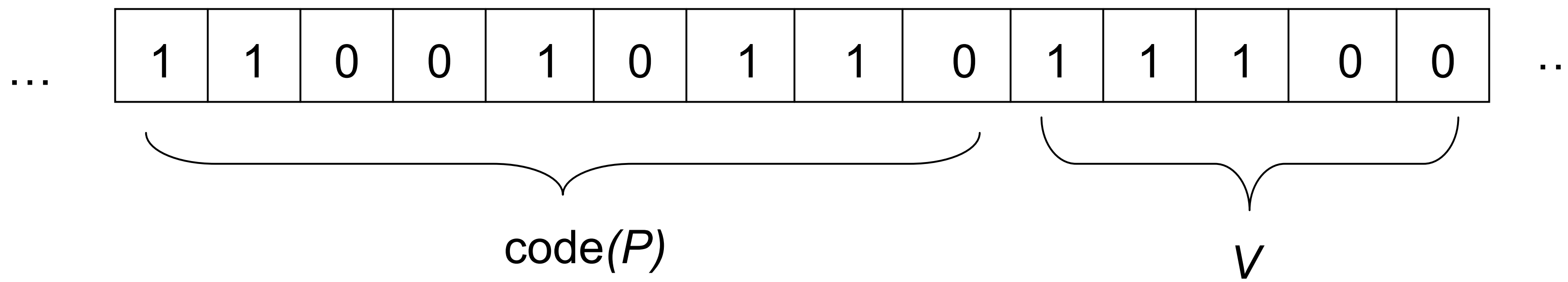


Discussion

Is there a universal pseudocode program ?

How would you write it?

Halting Problem



- Decide whether *P* halts on *V* or not

- Cannot be solved! Turing proved that
  no Turing-Post program can solve Halting Problem

Short detour

- Proof by contradiction…
- Feeding a program to itself…

Proof By Contradiction

- Suppose statement S is true
- Make series of logical deductions from S
- Arrive at deduction that is clearly false
- … therefore S must be false

Feeding a program to itself

- A python program to count lines:

```
import sys
count = 0
for line in sys.stdin.readlines():
    count = count + 1
print count
```

- Run this program using itself as input:

```
% python count_lines.py < count_lines.py
5
```

Proof for halting problem

- Suppose we had a solution:

```
would_it_stop( program, data ):
    if( something terribly clever ) {
        report TRUE;
    } else {
        report FALSE;
    }
```

This version due to Craig Kaplan, U of Waterloo
http://www.cgl.uwaterloo.ca/~csk/halt/

Proof for halting problem

- Feed a program to itself:

```
stops_on_self( program ):
    report would_it_stop( program, program );
```

Proof for halting problem

- Now let's mix things up:

bobs_yer_uncle(program):

```
if( stops_on_self( program ) ) {
    while( TRUE ) { do nothing }        (loop forever)
} else {
    report TRUE;
}
```

Proof for halting problem

- Finally, run bobs_yer_uncle on itself
- Two possible outcomes:
  - Never halts, or
  - Halts and reports TRUE

Proof for halting problem

- Consider case of infinite loop
  - stops_on_self(bobs_yer_uncle) reports TRUE
  - would_it_stop(bobs_yer_uncle, bobs_yer_uncle) reports TRUE
  - … but then bobs_yer_uncle would stop when fed itself
  - … contradiction!

Proof for halting problem

- Consider case where it reports TRUE
  - stops_on_self(bobs_yer_uncle) reports FALSE
  - would_it_stop(bobs_yer_uncle, bobs_yer_uncle) reports FALSE
  - … but then bobs_yer_uncle would run forever
  - … contradiction!

Lessons to take away

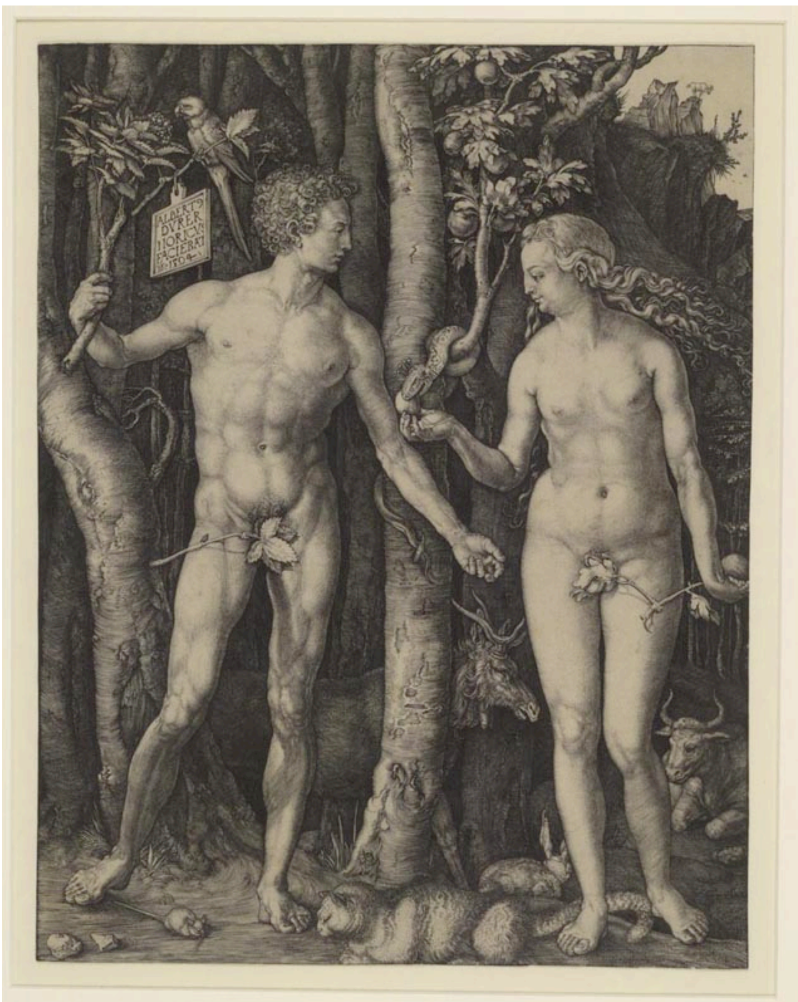
- Computation is a very simple process
  (can arise in unexpected places)

- Universal Program

- No real boundary between hardware, software, and data

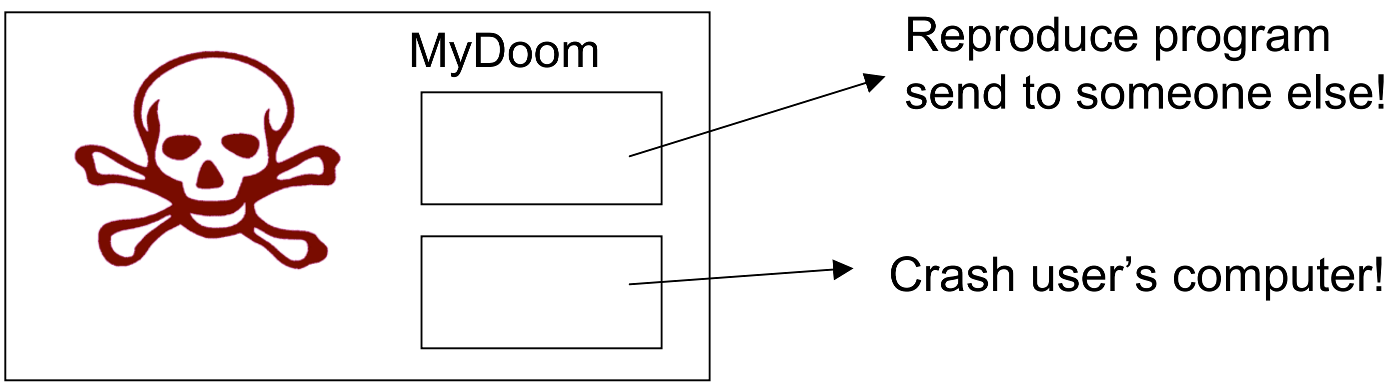
- No program that decides whether or not mathematical statements are theorems.

Age-old mystery: Self-reproduction.



How does the seed encode the whole?

Self-reproducing programs



- Fact: for every program *P*, there exists a program *P'* that has the exact same functionality except at the end it also prints code(*P'*) on the tape

Next time

Graphics…