


Computer Animation


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Computer Animation

- What is animation?
 - Make objects change over time according to scripted actions
- What is simulation?
 - Predict how objects change over time according to physical laws



Pixar



University of Illinois

Simulation

- Dynamics
 - Considers underlying forces
 - Compute motion from initial conditions and physics
- Kinematics
 - Considers only motion
 - Determined by positions, velocities, accelerations

Dynamics

Passive--no muscles or motors

```

    graph TD
      User1[user] -- initial conditions --> Model1[model]
      Model1 -- state --> Graphics1[graphics]
      Model1 --- NI1[numerical integrator]
      NI1 --- Model1
      Model1 --- Text1["particle systems  
leaves  
water spray  
clothing"]
  
```

Active--internal source of energy

```

    graph TD
      User2[user] -- desired behavior --> Control[control]
      Control -- forces and torques --> Model2[model]
      Model2 -- state --> Graphics2[graphics]
      Model2 --- NI2[numerical integrator]
      NI2 --- Model2
      Model2 --- Text2["running human  
trotting dog  
swimming fish"]
  
```

Hodgins

Passive Dynamics

- No muscles or motors
 - Smoke
 - Water
 - Cloth
 - Fire
 - Fireworks
 - Dice




Passive Dynamics

- Physical laws
 - Newton's laws
 - Hook's law
 - Etc.
- Physical phenomena
 - Gravity
 - Momentum
 - Friction
 - Collisions
 - Elasticity
 - Fracture

Particle Systems



- A particle is a point mass
 - Mass
 - Position
 - Velocity
 - Forces
 - Color
 - Lifetime
- Use lots of particles to model complex phenomena
 - Keep array of particles
 - Newton's laws



Particle Systems



- For each frame:
 - Create new particles and assign attributes
 - Delete any expired particles
 - Update particles based on attributes and physics
 - Render particles



Particle Systems



- For each frame:
 - ∅ Create new particles and assign attributes
 - ∅ Delete any expired particles
 - Update particles based on attributes and physics
 - Render particles



Creating/Deleting Particles



- Where to create particles?
 - Around some center
 - Along some path
 - Surface of shape
 - Where particle density is low
- When to delete particles?
 - Where particle density is high
 - Life span
 - Random

This is where user controls animation



Example: Wrath of Khan

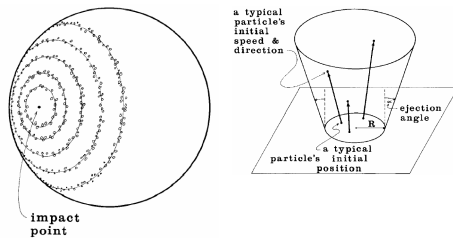


Fig. 2. Distribution of particle systems on the planet's surface.

Reeves

Example: Wrath of Khan



Reeves

Example: Wrath of Khan

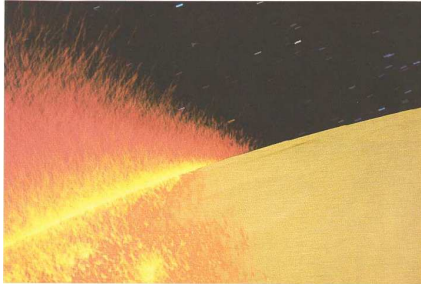


Fig. 7. Wall of fire about to engulf camera.

Reeves

Particle Systems



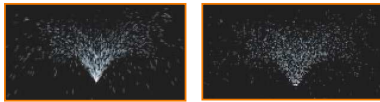
- For each frame:
 - Create new particles and assign attributes
 - Delete any expired particles
 - Update particles based on attributes and physics
 - **Render particles**



Rendering Particles



- Volumes
 - Ray casting, etc.
- Points
 - Render as individual points
- Line segments
 - Motion blur over time



Example: Fire



Allan

Particle Systems



- For each frame:
 - Create new particles and assign attributes
 - Delete any expired particles
 - **Update particles based on attributes and physics**
 - Render particles



Equations of Motion



- Newton's Law for a point mass
 - $f = ma$
- Computing particle motion requires solving second-order differential equation

$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$

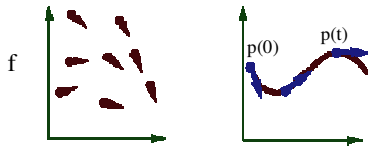
- Add variable v to form coupled first-order differential equations

$$\begin{cases} \dot{x} = v \\ \dot{v} = \frac{f}{m} \end{cases}$$

Solving the Equations of Motion



- Initial value problem
 - Know $p(0)$, $v(0)$, $a(0)$
 - Can compute force at any time and position
 - Compute $p(t)$ by forward integration

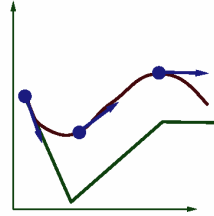


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Solving the Equations of Motion



- Euler integration
 - $p(t+\Delta t) = p(t) + \Delta t f(x,t)$

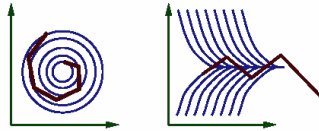


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Solving the Equations of Motion



- Euler integration
 - $p(t+\Delta t) = p(t) + \Delta t f(x,t)$
- Problem:
 - Accuracy decreases as Δt gets bigger

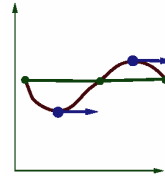


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Solving the Equations of Motion



- Midpoint method (2nd order Runge-Kutta)
 - Compute an Euler step
 - Evaluate f at the midpoint
 - Take an Euler step using midpoint force
 - » $p(t+\Delta t) = p(t) + \Delta t f(p(t) + 0.5 \Delta t f(t), t)$

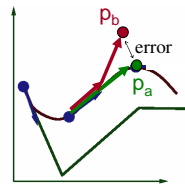


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Solving the Equations of Motion



- Adapting step size
 - Compute p_a by taking one step of size h
 - Compute p_b by taking 2 steps of size $h/2$
 - Error = $|p_a - p_b|$
 - Adjust step size by factor $(\epsilon/\text{error})^{1/n}$

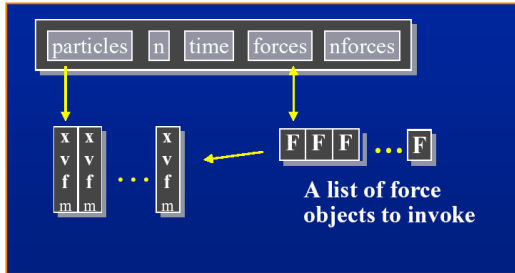


Particle System Forces



- Force fields
 - Gravity, wind, pressure
- Viscosity/damping
 - Liquids, drag
- Collisions
 - Environment
 - Other particles
- Other particles
 - Springs between neighboring particles (mesh)
 - Useful for cloth

Particle System Forces



Witkin

Example: Grass in Wind

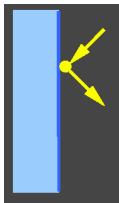


Fig. 12. white.sand.

Example: Bouncing Off Wall



- Requires
 - Collision detection
 - Collision response (dynamic forces)

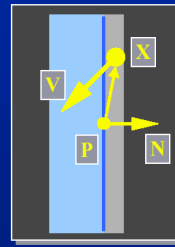


Witkin

Example: Bouncing Off Wall



Collision Detection



$$(X - P) \cdot N < \epsilon$$

$$N \cdot V < 0$$

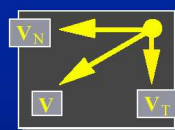
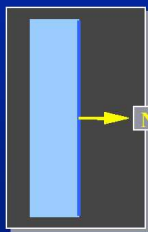
- Within ϵ of the wall.
- Heading in.

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Example: Bouncing Off Wall



Normal and Tangential Components



$$V_N = (N \cdot V)N$$

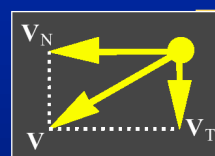
$$V_T = V - V_N$$

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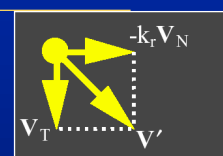
Example: Bouncing Off Wall



Collision Response



Before



After

$$V' = V_T - k_r V_N$$

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Example: Bouncing Off Wall



Contact Force
 $F' = F_T$
 The wall pushes back, cancelling the normal component of F .
 (An example of a *constraint force*.)

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Example: Pool Balls

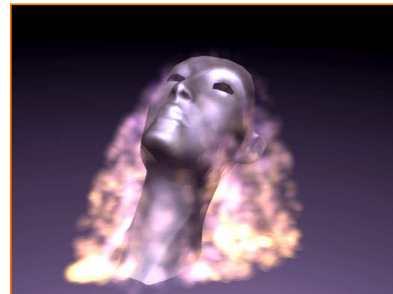


Example: Complex Objects



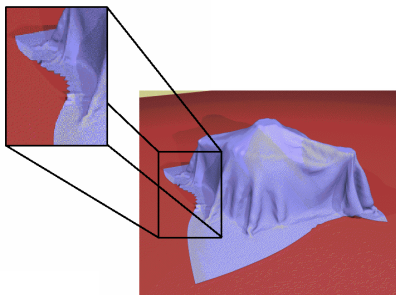
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Example: Complex Objects



Allan

Example: Cloth



Breen

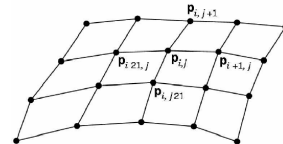
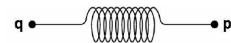
Example: Cloth



- Spring-mass mesh
- Hooke's law

$$f = -k_s(|d| - s) \frac{d}{|d|}$$

f = force
 k_s = spring constant
 $d = p - q$
 s = resting length

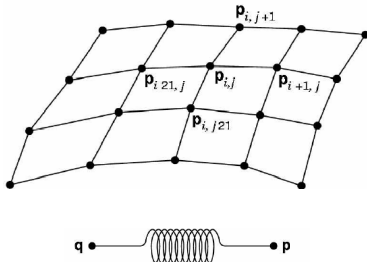


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Example: Cloth



- Spring-mass mesh



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Example: Cloth



- Hooke's law

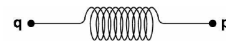
$$f = -k_s(|d| - s) \frac{d}{|d|}$$

f = force
 k_s = spring constant
 $d = p - q$
 s = resting length

- Damping term

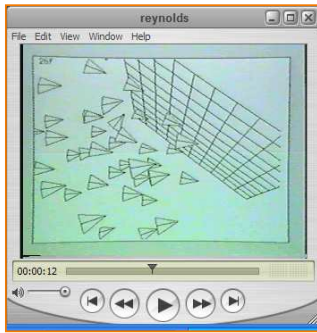
$$f = - \left(k_s(|d| - s) + k_d \frac{\dot{d}}{|d|} \right) \frac{d}{|d|}$$

$$\dot{d} = \dot{p} - \dot{q}$$



Hodgins

Example: Flocks & Herds



Reynolds

Example: Flocks & Herds



Hodgins

Summary



- Particle systems
 - Lots of particles
 - Simple physics
- Interesting behaviors
 - Waterfalls
 - Smoke
 - Cloth
 - Flocks
- Solving motion equations
 - Simplest method is Euler integration
 - Better to use adaptive step sizes

