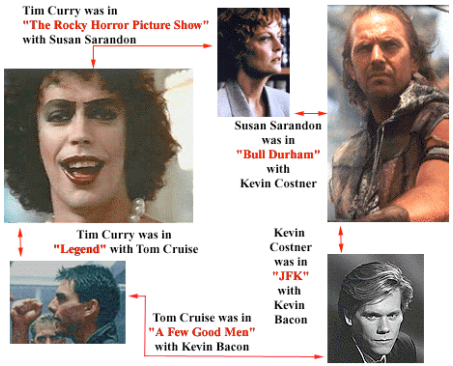
Lecture 17: Small World Phenomenon



Small World Phenomenon

Small world phenomenon.

- Six handshakes away from anyone else in the world.
- Long a matter of folklore.
- "It's a small world after all."

Stanley Milgram experiment (1960s) quantified effect.

- You are given personal info of another person in US, e.g., occupation.
- Goal: deliver message.
- Restriction: can only forward to someone you know by first name.
- Outcome: message delivered with average of 5 intermediaries.

Application demands new ADT.

- Graph = data type that represents pairwise connections.
- Vertex = element.
- Edge = connection between two vertices.

Applications of Small World Phenomenon

Sociology applications.

- Looking for a job.
- Marketing products or ideas.
- Formation and spread of fame and fads.
- Train of thought followed in a conversation.
- Defining representative-ness of political bodies.
- ➡ Kevin Bacon game (movies, rock groups, baseball teams, ICQ buddies, etc.).

Other applications.

- Electronic circuits.
- Synchronization of neurons.
- Analysis of World Wide Web.
- Design of electrical power grids.
- Modeling of protein interaction networks.
- Phase transitions in coupled Kuramoto oscillators.
- Spread of infectious diseases and computer viruses.
- Evolution of cooperation in multi-player iterated Prisoner's Dilemma.

Reference. Duncan J. Watts, *Small Worlds The Dynamics of Networks between Order and Randomness* Princeton University Press, 1999.

Applications of Graphs

Graph	*Vertices*	*Edges*
communication	telephones, computers	fiber optic cables
circuits	gates, registers, processors	wires
mechanical	joints	rods, beams, springs
hydraulic	reservoirs, pumping stations	pipelines
financial	stocks, currency	transactions
transportation	street intersections, airports	highways, airway routes
scheduling	tasks	precedence constraints
software systems	functions	function calls
internet	web pages	hyperlinks
games	board positions	legal moves
social relationship	people, actors	friendships, movie casts
neural networks	neurons	synapses
protein networks	proteins	protein-protein interactions
chemical compounds	molecules	bonds

Internet Movie Database

Queries about actors and movies.

- Given an actor, find all movies that they appeared in.
- Given a movie, find all actors.

Input format. Movie followed by list of actors, separated by slashes.

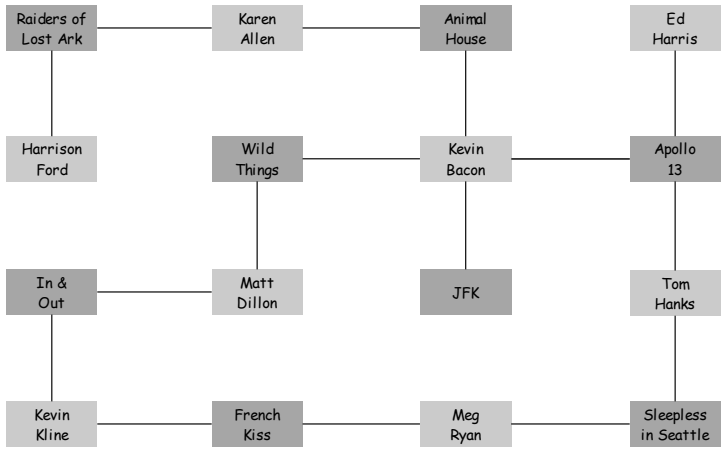
```
Wild Things (1998)/Bacon, Kevin/Campbell, Neve/Dillon, Matt/Murray, Bill/Richards, Denise
JFK (1991)/Asner, Edward/Bacon, Kevin/Costner, Kevin/Jones, Tommy Lee/Grubbs, Gary
Braveheart (1995)/Gibson, Mel//Marceau, Sophie/McGoohan, Patrick/Hanly, Peter
. . .
```

How to represent the actor-movie relationships.

- Vertices: actors, movies.
- Edges: connect actor with any movie in which they appear.
- Use a graph.

Reference: http://www.imdb.com/interfaces

Actor-Movie Graph (Partial)



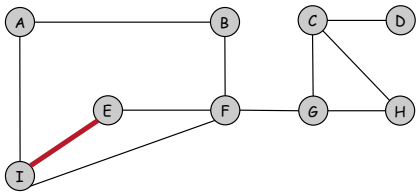
Graph Representation

Graph representation: symbol table of lists.

- Key = name of vertex (e.g., movie or actor).
- Value = adjacency list of neighbors.

Graph operations.

- Add connection `v-w` : `addEdge(v, w)`.
- Return neighbors of `v` as array: `neighbors(v)`.



Symbol Table

Key	Value
A	B I
B	A F
C	D G H
D	C
E	I F
F	E B G
G	C F H
H	C G
I	A E F

String AdjList

Adjacency List Implementation

Adjacency list implementation. No surprises.

```
public class AdjList {
   private Node first;

   private static class Node {
      String name;
      Node next;
      Node(String name, Node next) {
         this.name = name;
         this.next = next;
      }
   }

   public void insert(String s) {
      first = new Node(s, first);
   }

   public String[] toArray() { }
}
```

Graph Implementation

```
public class Graph {
   private SymbolTable st = new SymbolTable();

   public void addEdge(String v, String w) {
      if (st.get(v) == null) addVertex(v);
      if (st.get(w) == null) addVertex(w);
      AdjList vlist = (AdjList) st.get(v);
      AdjList wlist = (AdjList) st.get(w);
      vlist.insert(w);    ⇐ add w to v's list
      wlist.insert(v);    ⇐ add v to w's list
   }

   public void addVertex(String v) {
      st.put(v, new AdjList());     ⇐ add new vertex v
   }                                   with no neighbors

   public String[] neighbors(String v) {
      AdjList adjlist = (AdjList) st.get(v);
      return adjlist.toArray();
   }
}
```


Graph Client Warmup: Movie Finder

Movie finder. Given actor, find all movies in which they appeared.

```
public class MovieFinder {
   public static void main(String[] args) {
      Graph G = new Graph();                                  build graph
      In data = new In(args[0]);  ⇐ file input
      String line;
      while ((line = data.readLine()) != null) {
         String[] names = line.split("/");  ⇐ tokenize input line
         String movie = names[0];
         for (int i = 1; i < names.length; i++)
            G.addEdge(movie, names[i]);     ⇐ movie-actor edge
      }

      In queries = new In();                  print all of actor's movies
      String actor;
      while ((actor = queries.readLine()) != null) {
         String[] neighbors = G.neighbors(actor);
         for (int i = 0; i < neighbors.length; i++)
            System.out.println(neighbors[i]);
      }
   }
}
```


Graph Client Warmup: Movie Finder

```
% java MovieFinder top-grossing.txt
Bacon, Kevin
Animal House (1978)
Apollo 13 (1995)
Few Good Men, A (1992)

Roberts, Julia
Hook (1991)
Notting Hill (1999)
Pelican Brief, The (1993)
Pretty Woman (1990)
Runaway Bride (1999)

Tilghman, Shirley
```

```
% java MovieFinder mpaa.txt
Bacon, Kevin
Air Up There, The (1994)
Animal House (1978)
Apollo 13 (1995)
Few Good Men, A (1992)
Flatliners (1990)
Footloose (1984)
Hero at Large (1980)
Hollow Man (2000)
JFK (1991)
My Dog Skip (2000)
Novocaine (2001)
Only When I Laugh (1981)
Picture Perfect (1997)
Planes, Trains & Automobiles (1987)
Sleepers (1996)
Tremors (1990)
White Water Summer (1987)
Wild Things (1998)
. . .
```

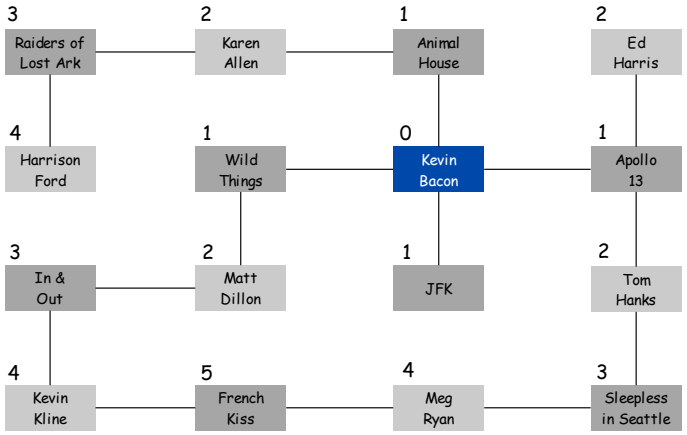

Kevin Bacon Game

Game. Given an actor or actress, find chain of movies connecting them to Kevin Bacon.

Actor	Was in	With
Whoopi Goldberg	Ghost	Patrick Swayze
Patrick Swayze	Dirty Dancing	Jennifer Gray
Jennifer Gray	Ferris Beuller's Day Off	Matthew Broderick
Matthew Broderick	The Road to Wellville	John Cusack
John Cusack	Bullets Over Broadway	Dianne West
Dianne West	Footloose	Kevin Bacon
Kevin Bacon		

Bacon Numbers

Bacon number: length of shortest such chain to Kevin Bacon.
How to compute: find shortest path in graph, and divide length by 2.



Solving the Kevin Bacon Problem: Java Implementation

```
public class Bacon {
   public static void main(String[] args) {
      Graph G = new Graph();
      In data = new In(args[0]);
      String line;
      while ((line = data.readLine()) != null) {
         String[] names = line.split("/");
         String movie = names[0];
         for (int i = 1; i < names.length; i++)
            G.addEdge(movie, names[i]);
      }

      BFSearcher bfs = new BFSearcher(G);
      bfs.search("Bacon, Kevin");

      In queries = new In();
      String actor;
      while ((actor = queries.readLine()) != null)
         bfs.showPath(actor);
   }
}
```

build graph (identical to warmup)

preprocess graph

process queries

Kevin Bacon: Sample Output

```
% java Bacon top-grossing.txt
Goldberg, Whoopi
Sister Act (1992)
Grodénchik, Max
Apollo 13 (1995)
Bacon, Kevin

Stallone, Sylvester
Rocky III (1982)
Tamburro, Charles A.
Terminator 2: Judgment Day (1991)
Berkeley, Xander
Apollo 13 (1995)
Bacon, Kevin

Tilghman, Shirley
```


Breadth First Searcher ADT

Goal: given one vertex `s` find shortest path to every other vertex `v`.

BFS from source `s`: `search(s)`.

- Put `s` onto a FIFO queue.
- Repeat until the queue is empty:
  - remove the least recently added vertex `v`
  - if `v` has not yet been visited add all of its neighbors `w` to the queue and set `visited[w]= v`

implement using symbol table

Key observation: vertices visited in increasing order of distance from `s` because we use FIFO queue.

Breadth First Searcher: Preprocessing

Goal: given one vertex `s` find shortest path to every other vertex `v`.

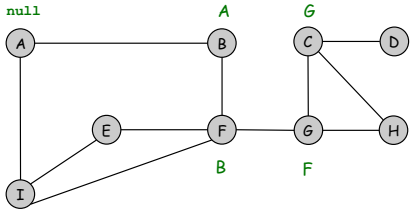
```
public void search(String s) {
   Queue q = new Queue();
   q.enqueue(s);
   visited.put(s, "");
   while (!q.isEmpty()) {
      String v = (String) q.dequeue();
      String[] neighbors = G.neighbors(v);
      for (int i = 0; i < neighbors.length; i++) {
         String w = neighbors[i];
         if (visited.get(w) == null) {
            q.enqueue(w);
            visited.put(w, v);
         }
      }
   }
}
```


Breadth First Searcher: Printing the Path

Print the shortest path from `v` to `s`.

- Follow visited path from v back to s.
- Print v, visited[v], visited[visited[v]], . . ., s.
- Ex: shortest path from `C` to `A`: `C - G - F - B - A` (source)

```
public void showPath(String v) {
   while (visited.get(v) != null) {
      System.out.println(v);
      v = (String) visited.get(v);
   }
}
```



Symbol Table

Key	Visited
A	-
B	A
C	G
D	C
E	I
F	B
G	F
H	G
I	A

Breadth First Searcher ADT

Isolate algorithm from graph data type.

- Keep modules independent.
- Avoid feature creep.

```
public class BFSearcher {
   private Graph G;
   private SymbolTable visited;

   BFSearcher(Graph G) {
      this.G = G;
      this.visited = new SymbolTable();
   }

   public void search  (String s) { }
   public void showPath(String v) { }
   public int  distance(String v) { }
}
```


Running Time Analysis

Analysis. BFS runs in linear time and scales to solve huge problems.

Data File	*Movies*	*Actors*	*Edges*	*Read input*	*Build graph*	*BFS*	*Show*
top.txt	187	8,265	10K	0.10 sec	0.10 sec	0.10 sec	0 sec
mpaa-g.txt	967	13,850	18K	0.16 sec	0.24 sec	0.13 sec	0 sec
y2k.txt	4,754	43,940	57K	0.29 sec	0.56 sec	0.30 sec	0 sec
mpaa.txt	14,192	170,539	383K	0.87 sec	3.4 sec	1.4 sec	0 sec
all.txt	122,812	418,468	1.5M	2.8 sec	14.9 sec	9.4 sec	0 sec

26MB

Perspective: Google indexes 8 billion web pages (50TB), and executes 250 million searches per day!

Data Analysis

Exercise: compute histogram of Kevin Bacon numbers.
Input: 122,812 movies, 418,468 actors.

Bacon #	Frequency
0	1
1	1,494
2	127,778
3	239,608
4	36,455
5	2,963
6	275
7	39
8	47
9	99
10	15
11	2
∞	9,692

Fred Ott, solo actor in *Fred Ott Holding a Bird* (1894) ➡

⬅ Akbar Abdi, star of Iranaian film *Honarpisheh*

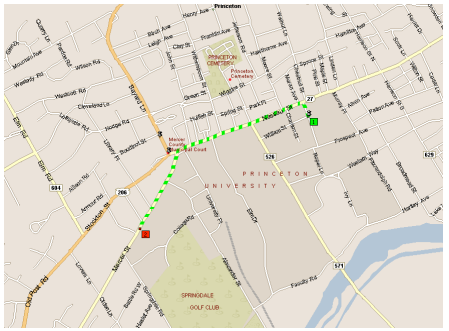
Applications of Breadth First Search

More BFS applications.

- Word ladder: `green` - `greet` - `great` - `groat` - `groan` - `grown` - `brown`
- Shortest number of hops for Internet packet.
- Particle tracking.
- Image processing.
- Crawling the Web.
- . . .

Extensions.

- GPS map directions.
- Google.



Conclusions

Linked list: ordering of elements.
Binary tree: hierarchical structure of elements.
Graph: pairwise connections between elements.

Layers of abstraction.

- Adjacency list: linked list.
- Queue: linked list.
- Symbol table: array of linked lists.
- Graph: symbol table of adjacency lists.
- Breadth first searcher: graph + queue + symbol table.

Importance of ADTs.

- Enables us to build and debug large programs.
- Enables us to solve large problems efficiently.