# Multiple Model Estimation :
# The EM Algorithm & Applications

Princeton University
COS 429 Lecture

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# Recapitulation

- Problem of motion estimation
- Parametric models of motion
- Direct methods for image motion estimation
- Camera models & parametric motion
- Image & video mosaicing as an application
- Quasi-parametric model-based motion & structure estimation : Depth and Pose
- Image-based Rendering
- Hi-res stereo sequence synthesis as an IBMR application

# Plan

- Motivate Image-based Modeling & Rendering (IBMR)
  - Change in viewpoint, IMAX app
- Parameterize motion & structure for video
  - Euclidean case
  - Direct Estimation
- Plane+Parallax
  - Formulation
  - Direct Estimation
- IMAX app.
- Tweening app.
- Model-to-video pose estimation
- Video Flashlights

# Real-World Apps of IBR

- The Matrix
- What Dreams May Come
- Titanic

# Application : Dynamic New View Rendering

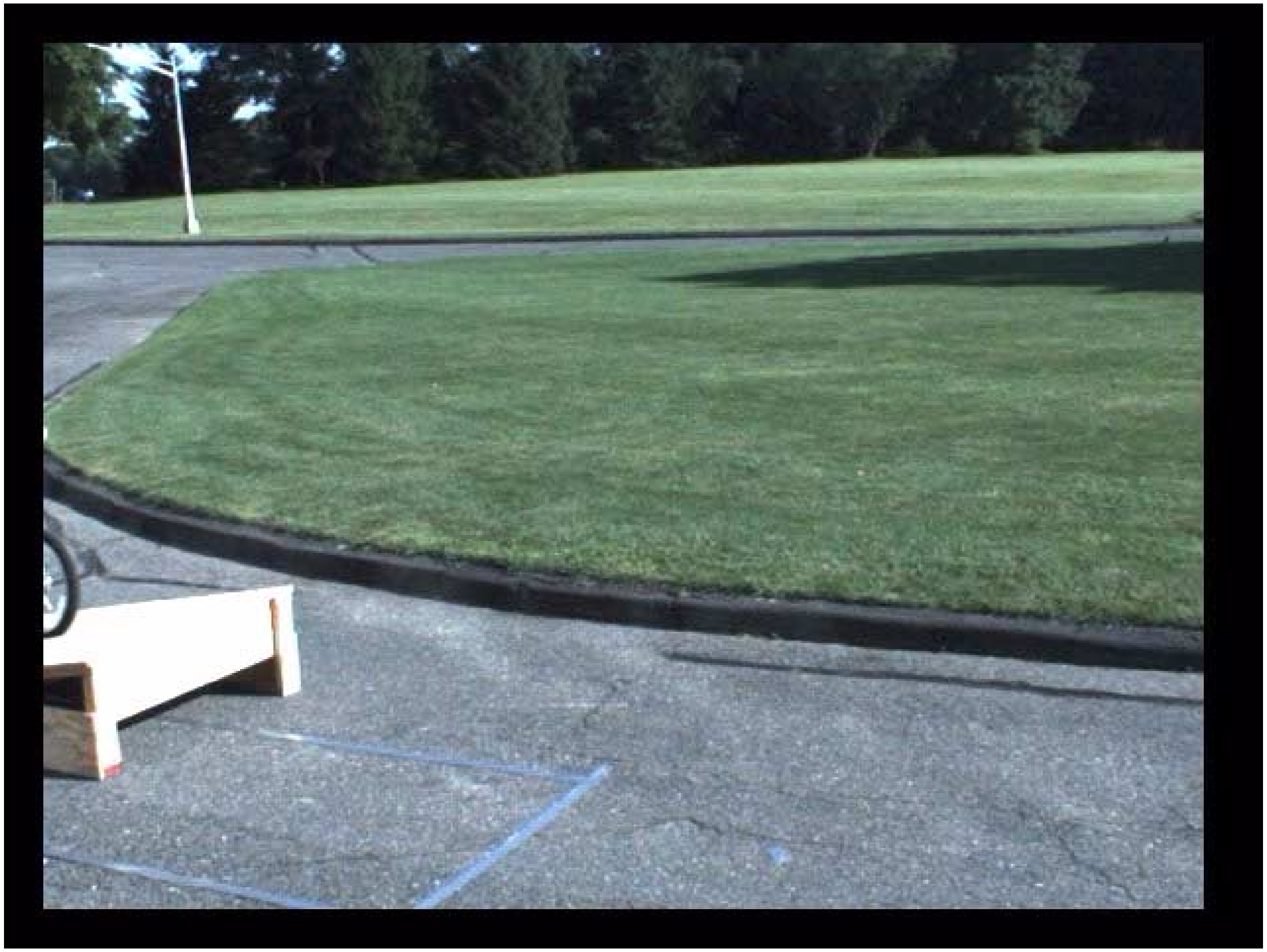
The Matrix

# Flow-based New View Rendering

Original 8 frames

Tweened 71 frames

# Enhanced Visualization

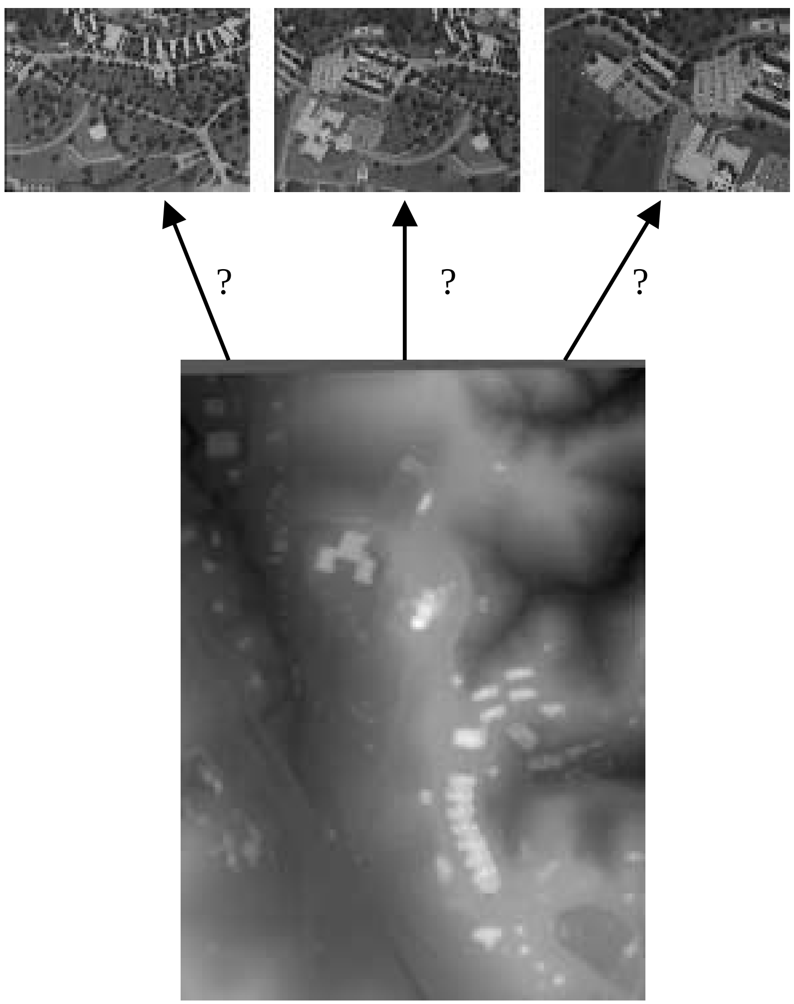


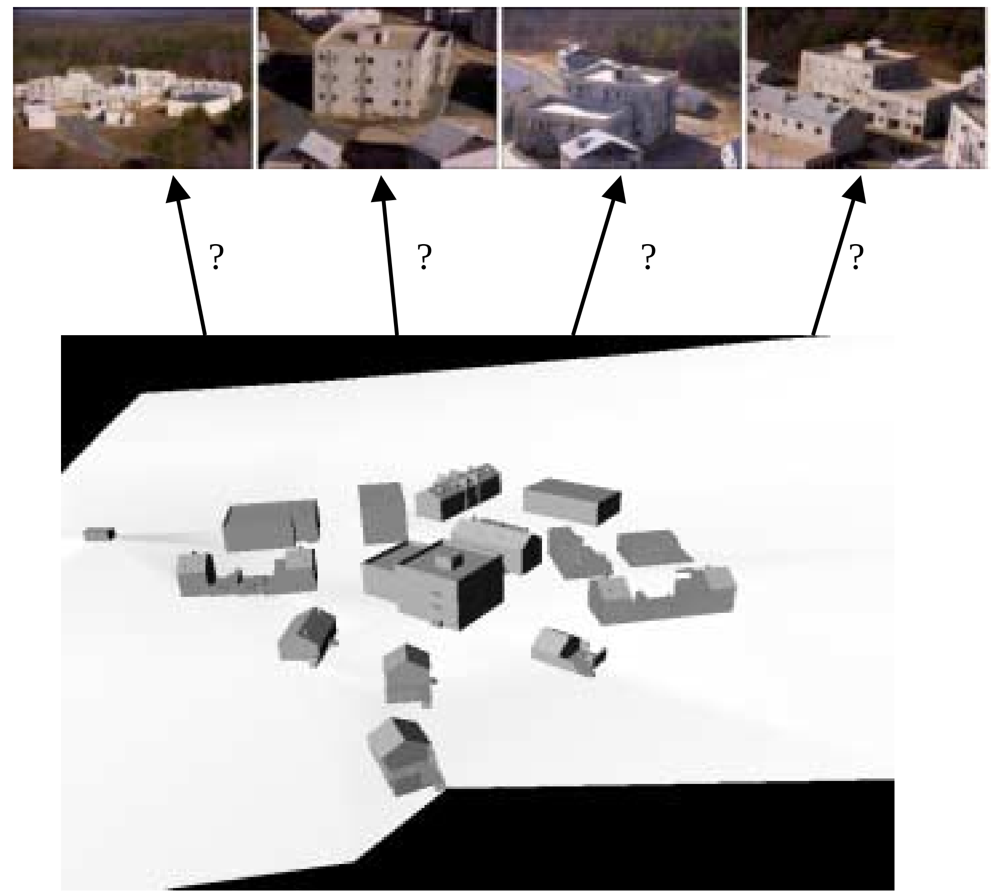
# 3D Model-based Direct Camera Pose Estimation and Video Visualization

[Hsu et al. '00]

# Pose Estimation

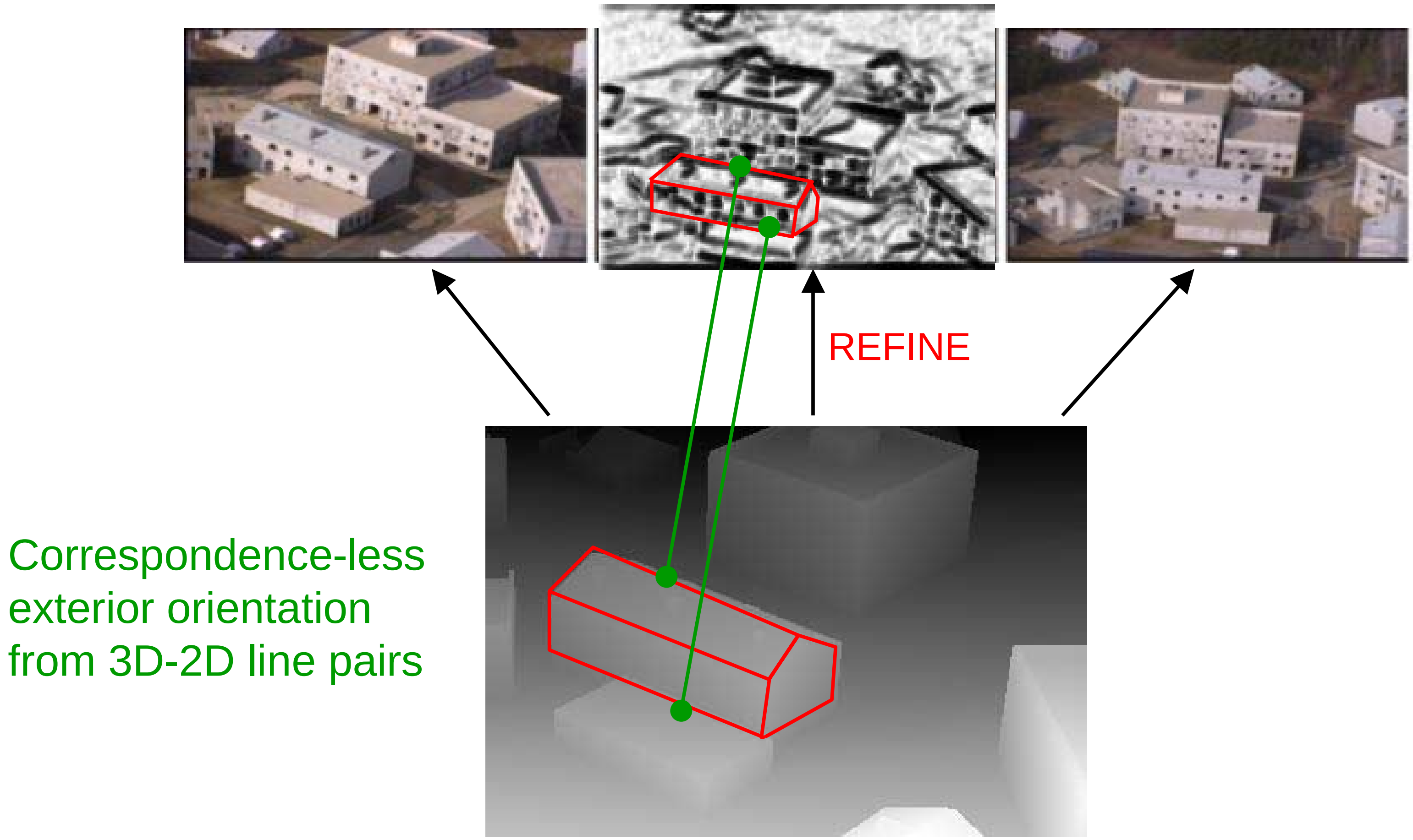
*...when only shape of 3D scene is known ...*



# Video to Site Model Alignment

- **Model to frame alignment**

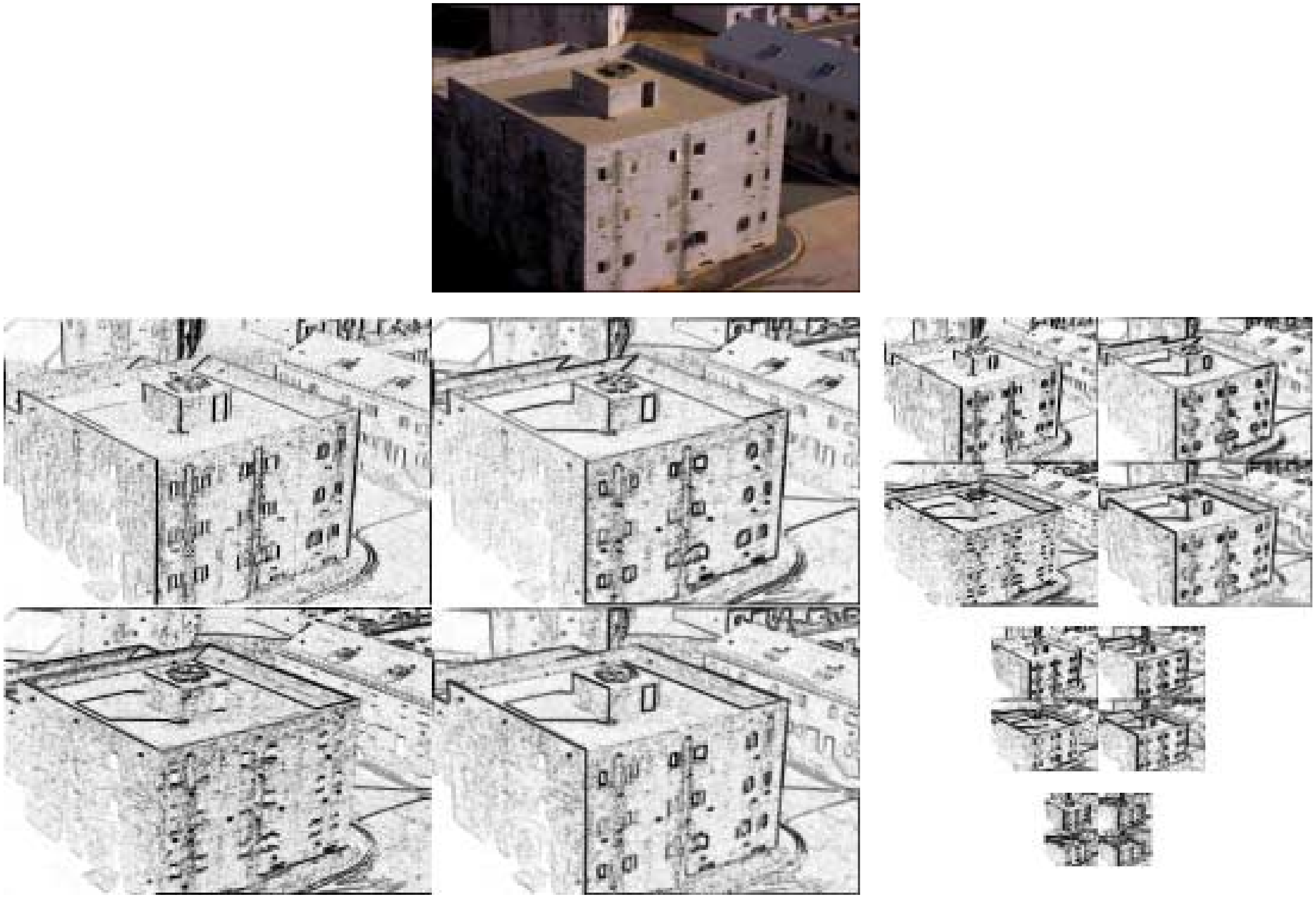


# The REGSITE Algorithm

*... aligning site model edges to image edges...*

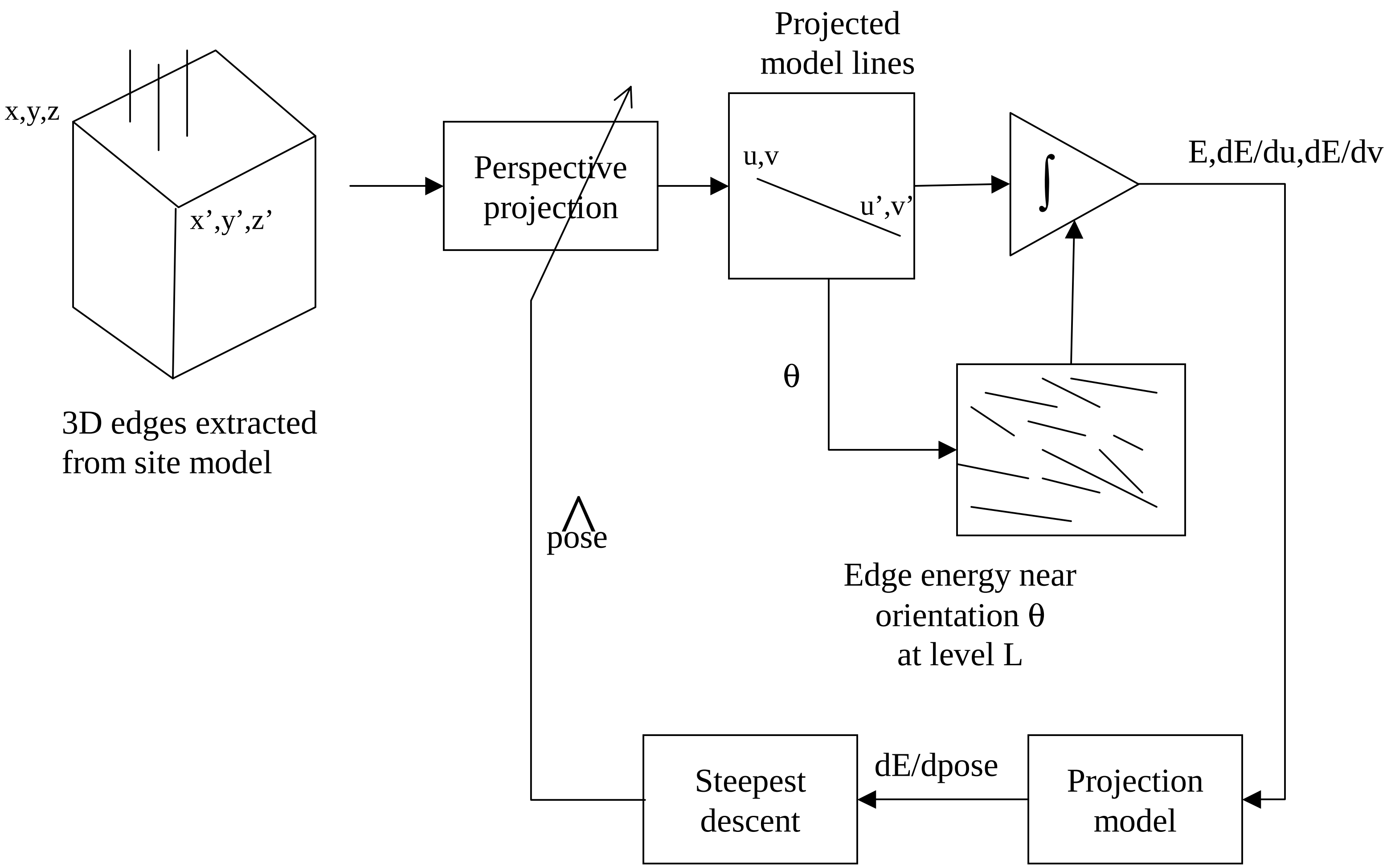
- Inputs:
  - Predicted pose of camera
  - Un-textured (Open Inventor) site model
  - Video frame
- Output:
  - Estimated pose of camera
- Premise
  - Discontinuities in 3D depth are correlated with brightness edges in the video frame (most of the time)
- Approach
  - Oriented energy image pyramids highlight image edges
  - Extract edges (depth discontinuities) from 3D site model
  - Adjust camera pose to maximize overlap of model and image edges
    - refinement is done using coarse to fine strategy over image pyramid

# Oriented Energy Pyramid



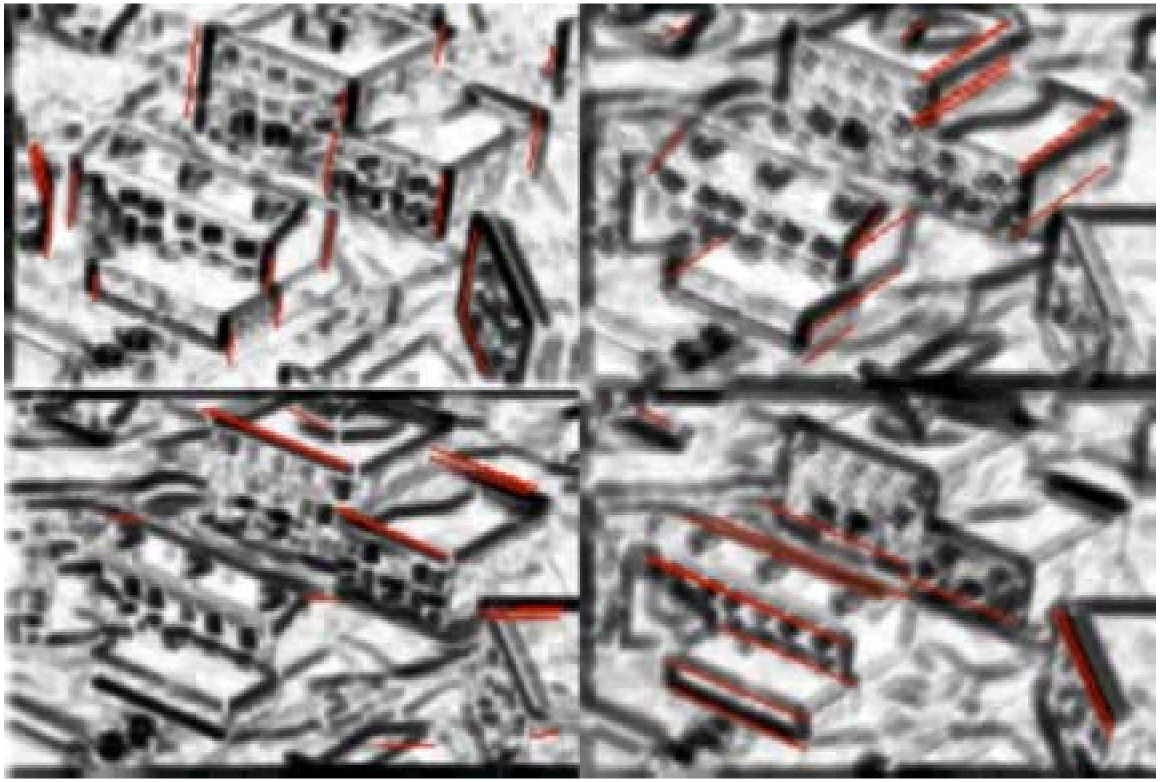
**Oriented Energy Pyramid: 4  Orientation Bands 0 deg., 45 deg., 90 deg., 135 deg.**

# Pose Refinement Procedure



# Pose Refinement Results

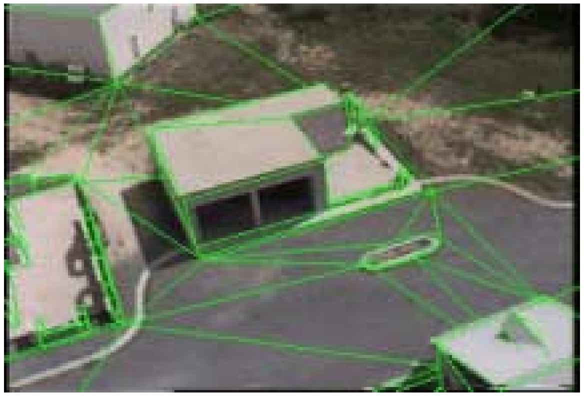
Iterative coarse-to-fine adjustment of pose



# Geo-registration of Video Sequence from Draper Helicopter to 3D Site Models

Original Video

Model rendered from the pose of the helicopter sensor.
Pose recovered after geo-registration process



Overlay of site model on video

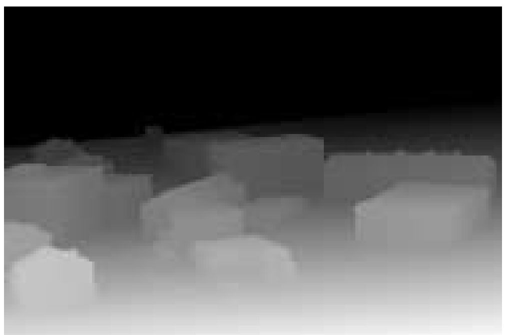
World as seen from the view-point of the runner

# Re-projection & Enhanced Visualization of Video

Geo-registration of video to site models

Original Video

Site model



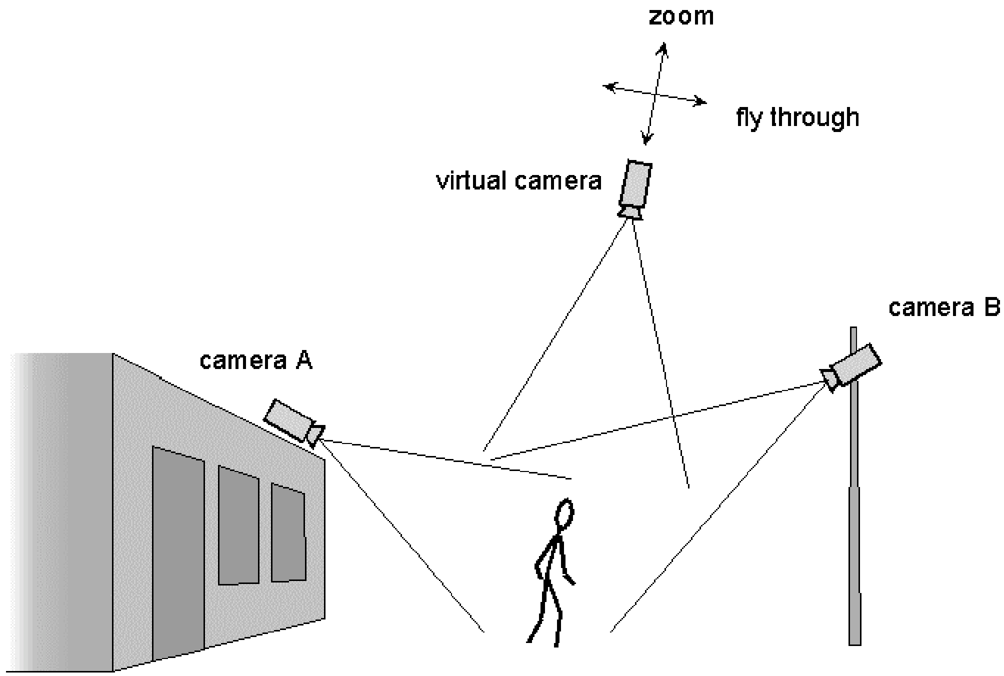
Geo-registration of video to site model

Re-projection of video after merging with model.

# Application : Model-based Video Visualization

# Immersive and Interactive Telepresence

## Total Facilities Visualization




Multiple cameras are merged to form a unified 3D scene representation.

Each observer views the scene with his own “virtual” camera.

# Distributed 2D Cameras

It is difficult to interpret activities viewed by multiple cameras

# Video Flashlights Concept

[EGWR'02]

## A tool for Global Visualization of Dynamic Environments

- **2D Video Flashlights:**
  - Project multiple 2D videos on a site model.
- **Moving Object Cued Video Flashlights:**
  - Project multiple 2D videos with automatically detected moving objects on a site model.
- **3D Video Flashlights:**
  - Project automatically extracted dynamic object models from multiple videos on a site model.

# Video Flashlights: Moving Target Cueing

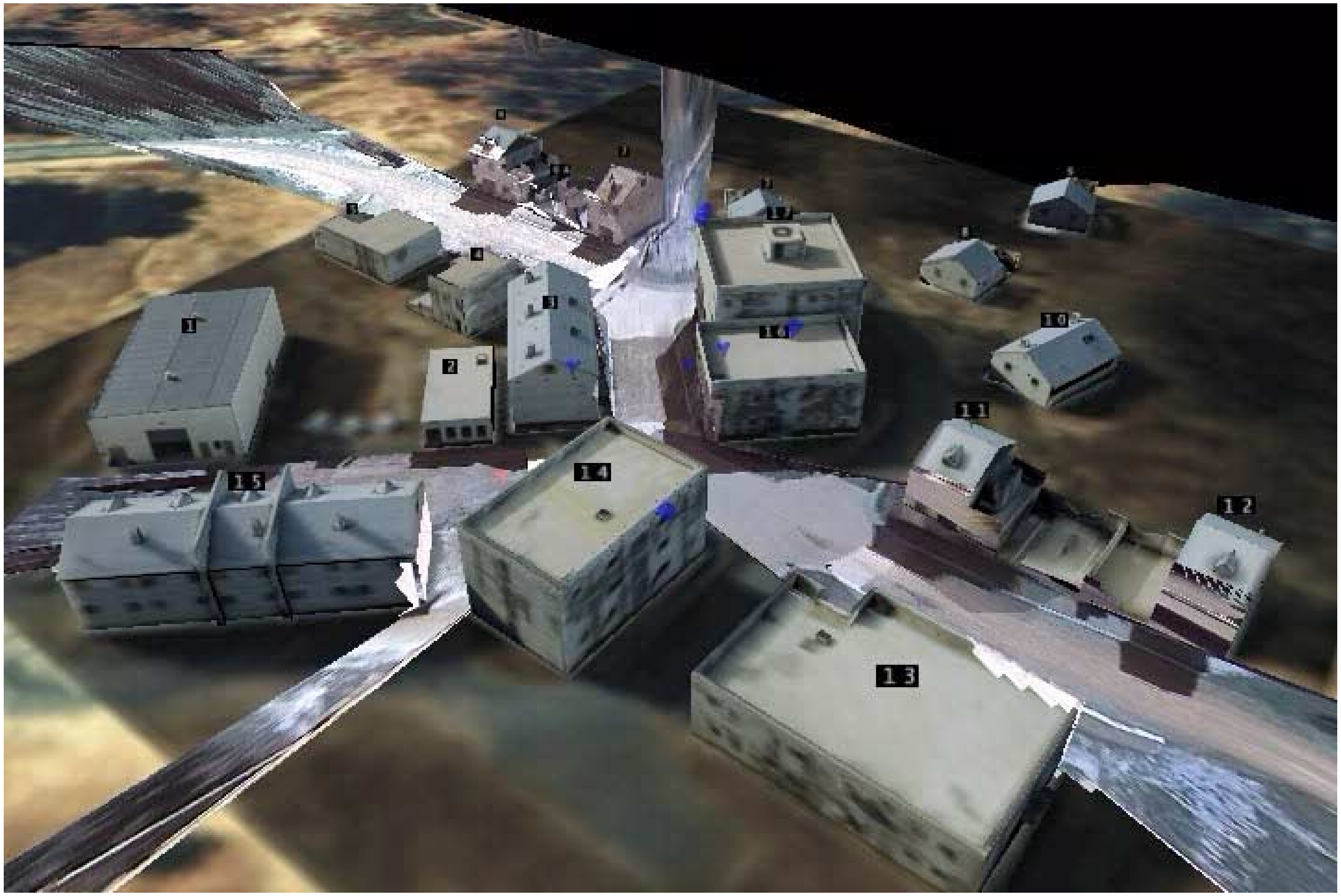
- Moving objects (humans & vehicles) are detected and segmented from live camera videos
- Shown as color coded dynamic visual cues from a bird's eye view
- Accurate dynamic positioning w.r.t. the model provides a global context for the action

[VIDEO]

**Moving Target Indication (MTI)**

# Video Flashlights

Live video streams are draped over a site model in real-time
Live videos are being viewed in the context of the model from a bird's eye view

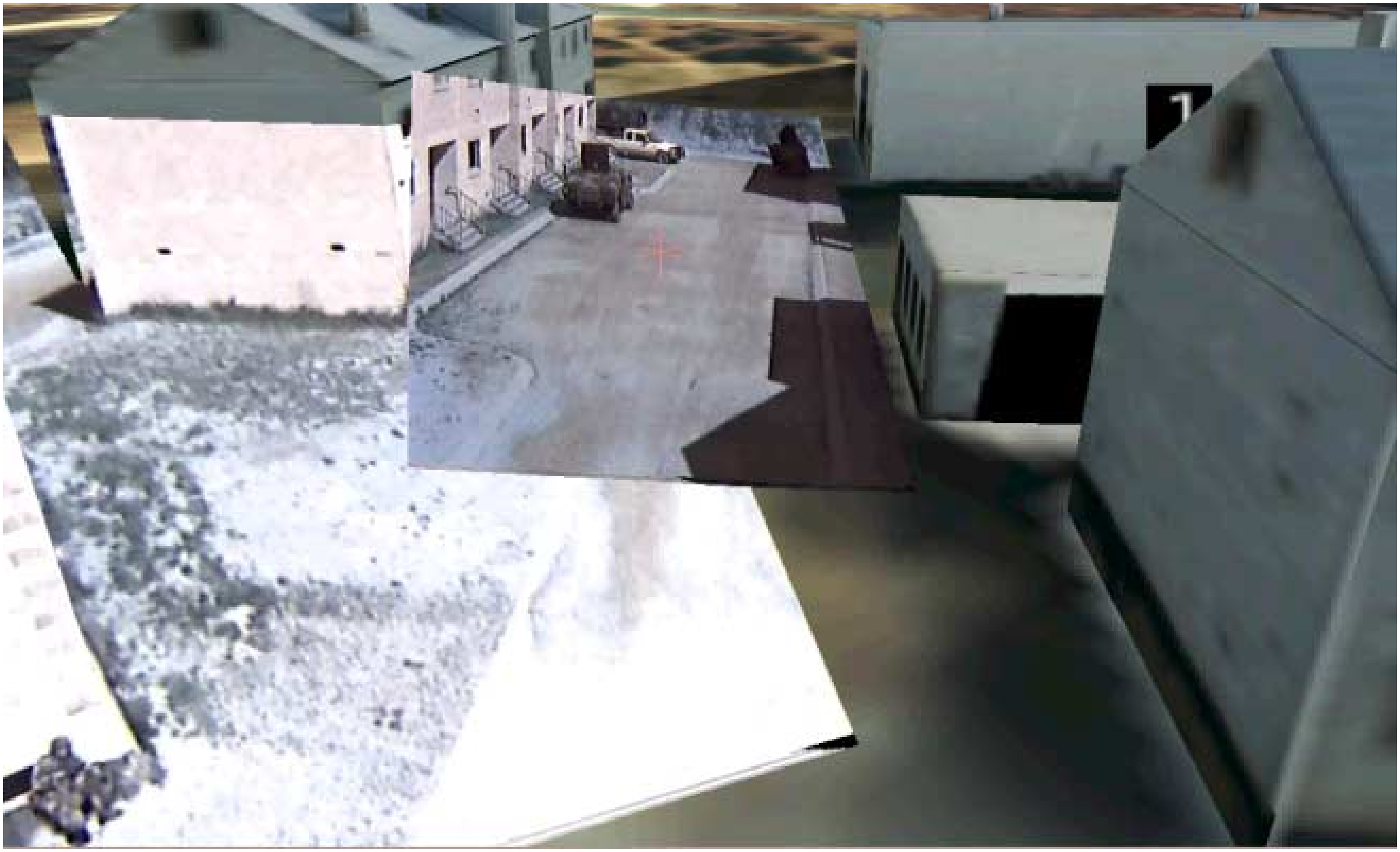



**Accurate Projection of multiple video streams onto the site model**

Enables interpretation of visual action in the global context of the model
Provides photo-realistic sky-to-street views at arbitrary scales and viewpoints

# Video Flashlights

Close up view of multiple video streams draped over the site model
Close up view allows zooming onto action that is happening over multiple video cameras



# Depth Computation for continuous video streams

# 2D and 3D MTI : Results

- 3D method can separate shadows from moving objects
- 3D method provides better delineation of moving objects

# Where are we headed ?

From Pixels
to
Intermediate Representations
for

Immersive Visualization

Immersive Communications

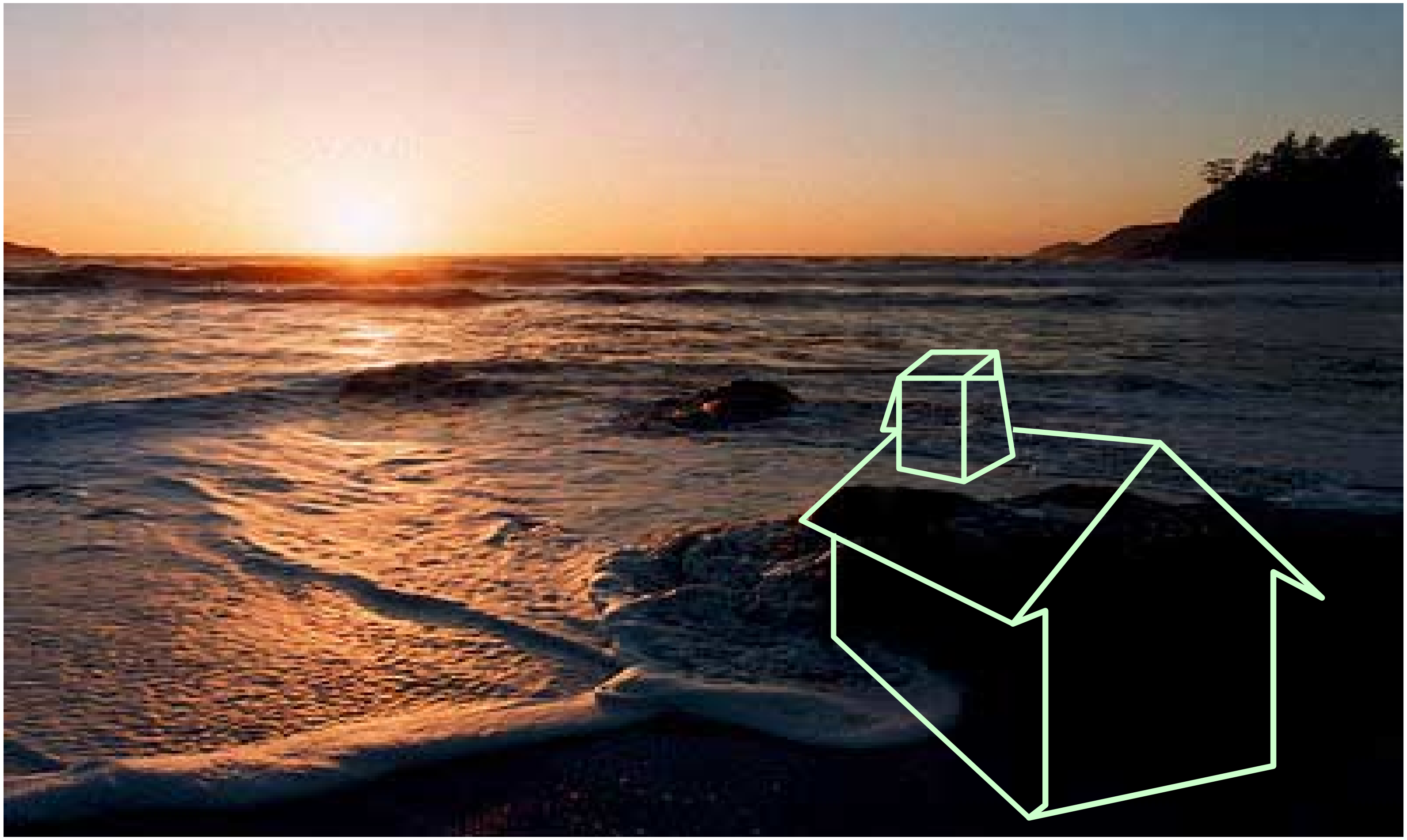
Object / Activity Recognition

Pattern Discovery

Perception

Cognition

# AN IMMERSIVE IBMR GRAND CHALLENGE



# AND IF WE DO IT RIGHT

# Handling Moving Objects in 2D Parametric Alignment & Mosaicing

# Multiple Motions : Robust Regression

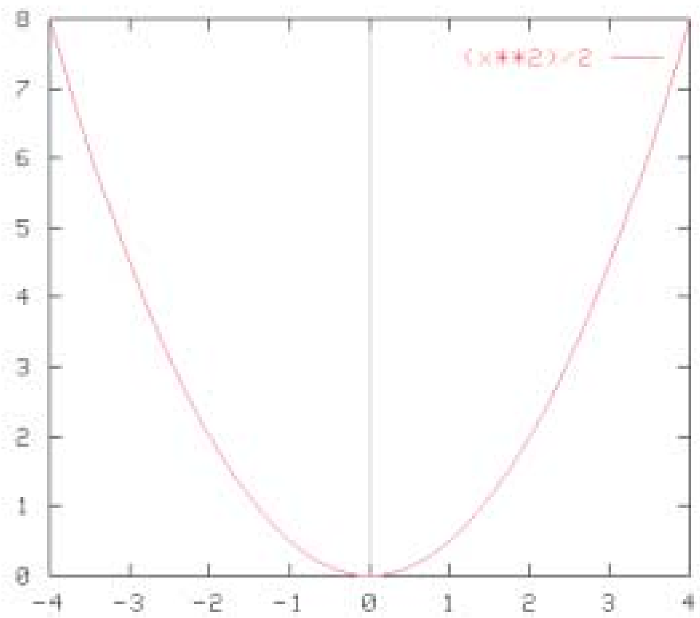
**Find the dominant motion while rejecting outliers**

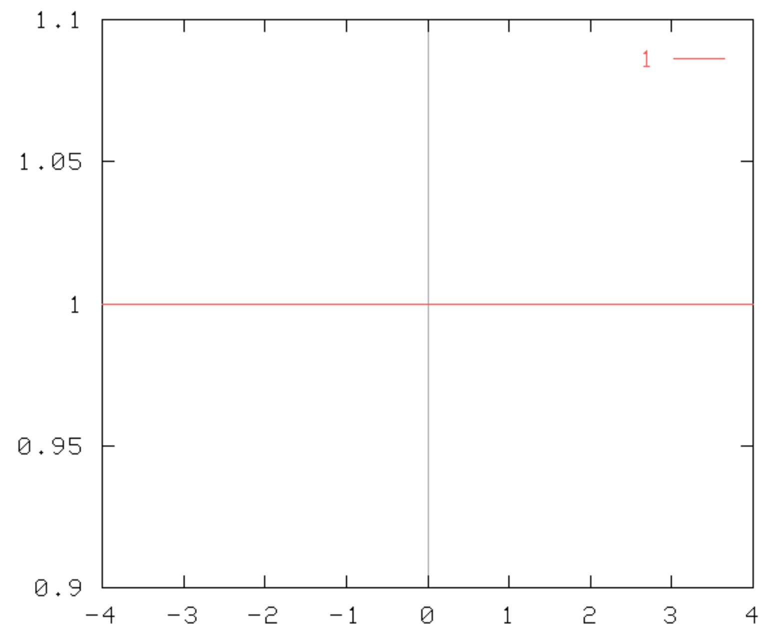
# Estimating the Mean

$$r_i = d_i - \mu \qquad \min_{\mu} \sum_i (\rho(r_i) = r_i^2) \qquad \mu = \frac{1}{N}\sum_i d_i$$

- Mean is the "least squares" estimate
- But each measurement is given the same weight
- Influence of an outlier: $\frac{\delta}{N}$ → Can be arbitrarily large

Influence is given by : $\frac{\rho(r)}{r}$



# Generalized M-Estimation

$$\min_{\Theta} \sum_i \rho(r_i;\sigma), \qquad r_i = I_2(p_i) - I_1(p_i - u(p_i;\Theta))$$

- Given a solution $\Theta^{(m)}$ at the $m$th iteration, find $\delta\Theta$ by solving :

$$\sum_l \sum_i \frac{\dot{\rho}(r_i)}{r_i} \frac{\partial r_i}{\partial \theta_k} \frac{\partial r_i}{\partial \theta_l} \partial \theta_l = -\sum_i \frac{\dot{\rho}(r_i)}{r_i} r_i \frac{\partial r_i}{\partial \theta_k} \quad \forall k$$

$w_i$

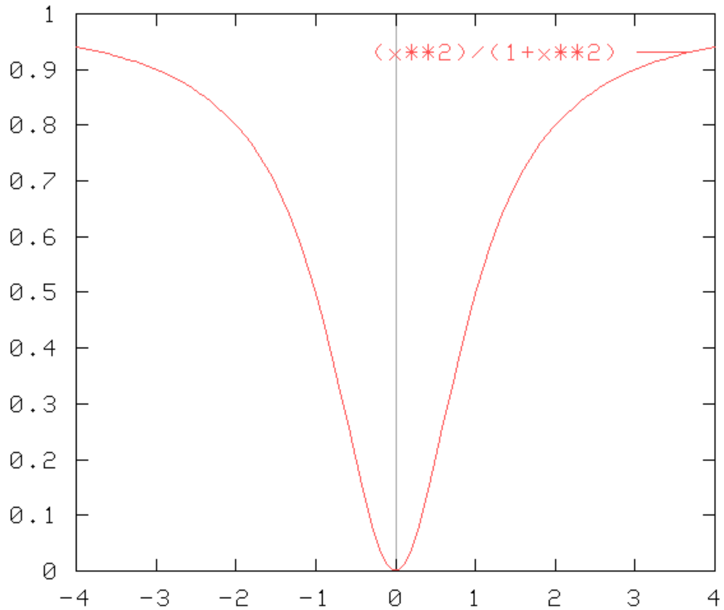
- $w_i$ is a weight associated with each measurement. Can be varied to provide robustness to outliers.

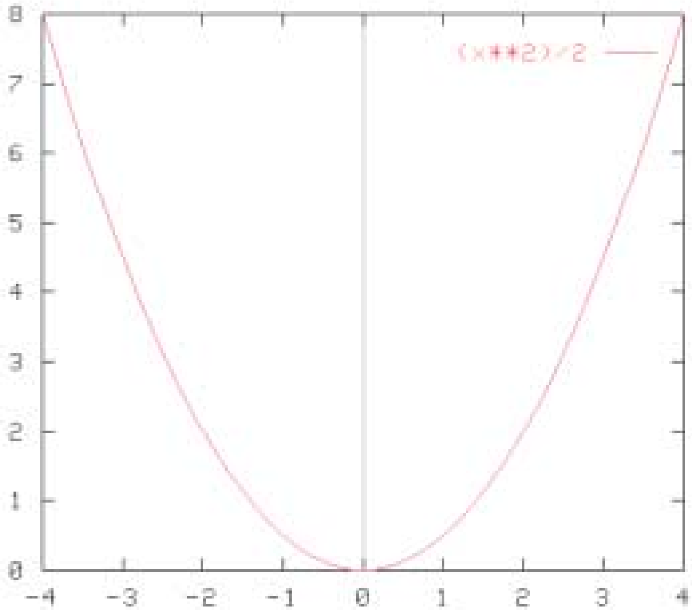
Choices of the $\rho(r_i;\sigma)$ function:

$$\rho_{SS} = \frac{r^2}{2\sigma^2} \qquad \rho_{GM} = \frac{r^2/\sigma^2}{1+r^2/\sigma^2}$$

$$\frac{\dot{\rho}_{SS}(r)}{r} = \frac{1}{\sigma^2} \qquad \frac{\dot{\rho}_{GM}(r)}{r} = \frac{2\sigma^2}{(\sigma^2+r^2)^2}$$

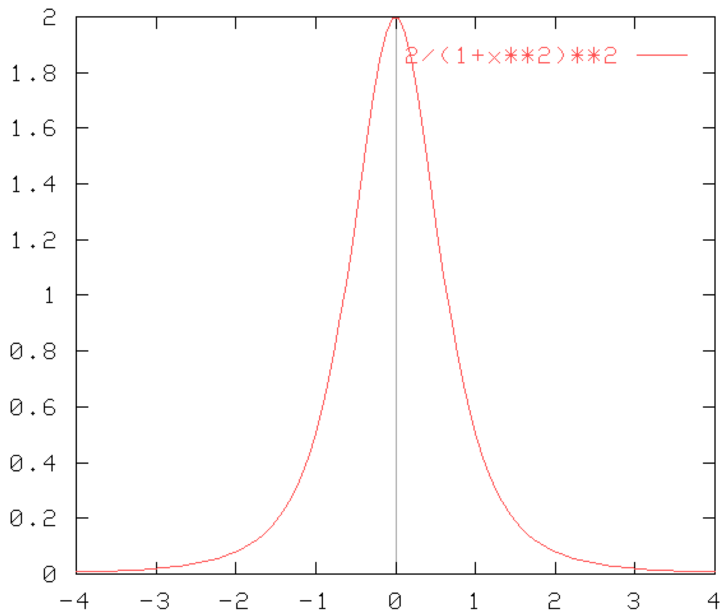
# Optimization Functions & their Corresponding Weight Plots

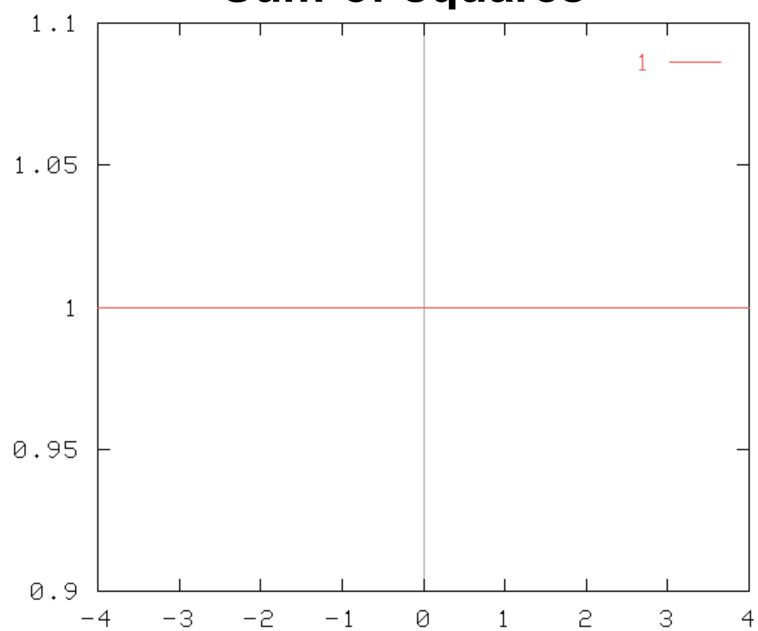


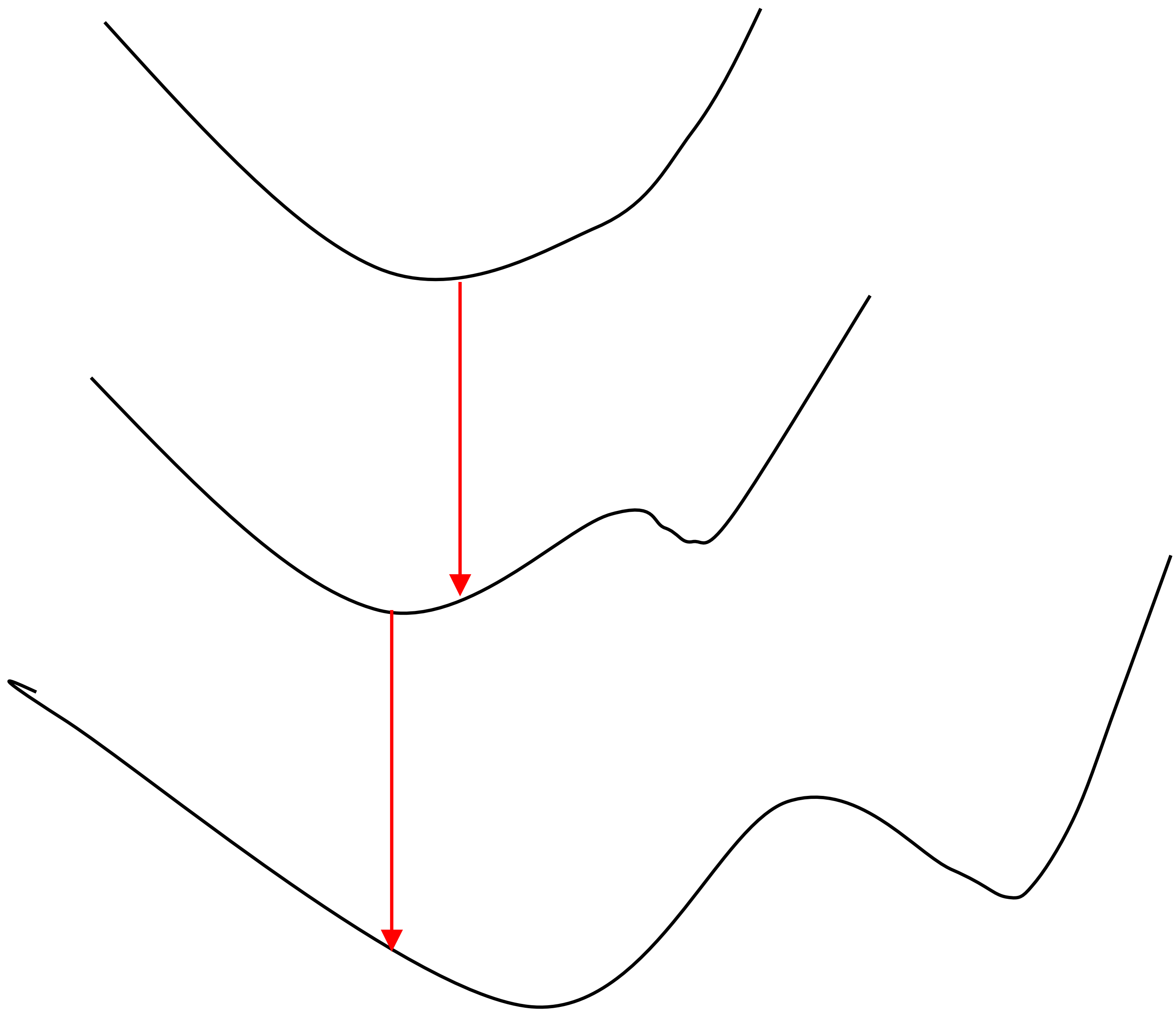

**Geman-Mclure**

**Sum-of-squares**



# Continuation Method: Coarse-to-fine



# With Robust Functions Direct Alignment Works for Non-dominant Moving Objects Too

Original two frames

Background Alignment

# Object Deletion with Layers

Original Video

Video Stream with
deleted moving object

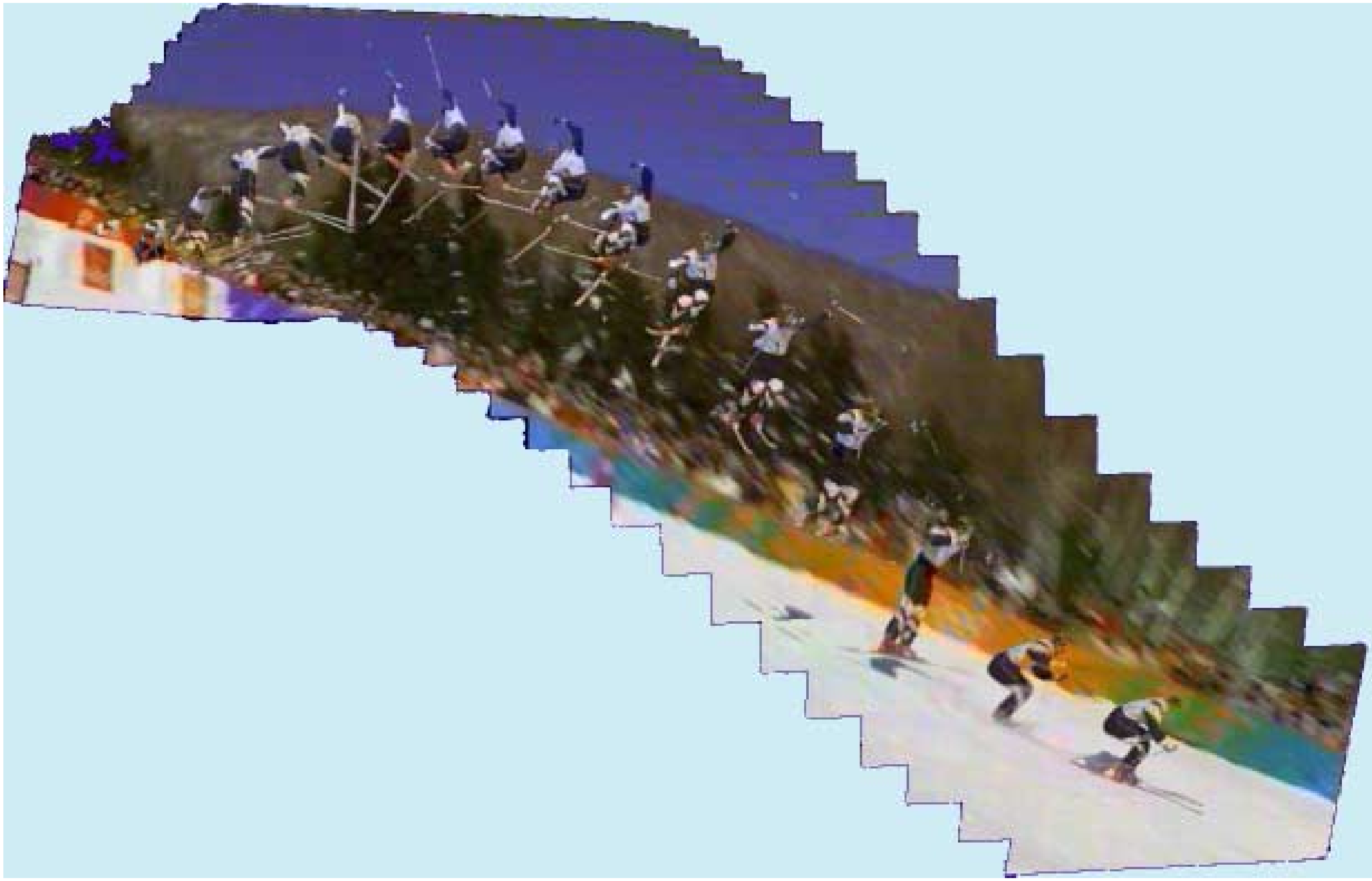
DYNAMIC MOSAICS

Original Video

**Video Stream with deleted moving object**

**Dynamic Mosaic Video**

# SYNOPISIS MOSAICS



# Problem

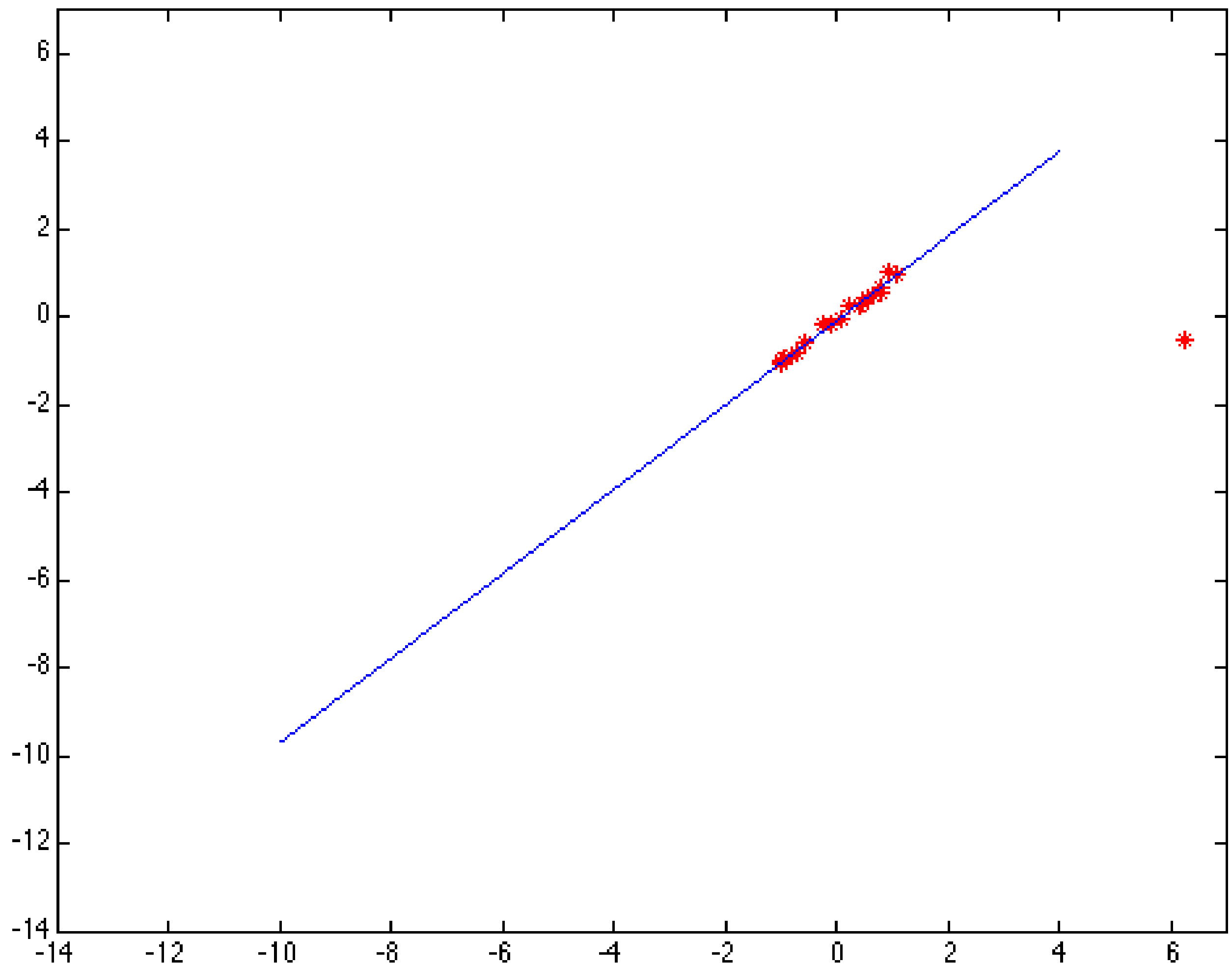
- Assumption:
  - Constraints that do not fit the dominant motion are treated as outliers : Extreme noise

- Problem:
  - But they are not noise
  - There indeed are multiple motions present in the scene

# Motivate Simultaneous Multiple Model Estimation

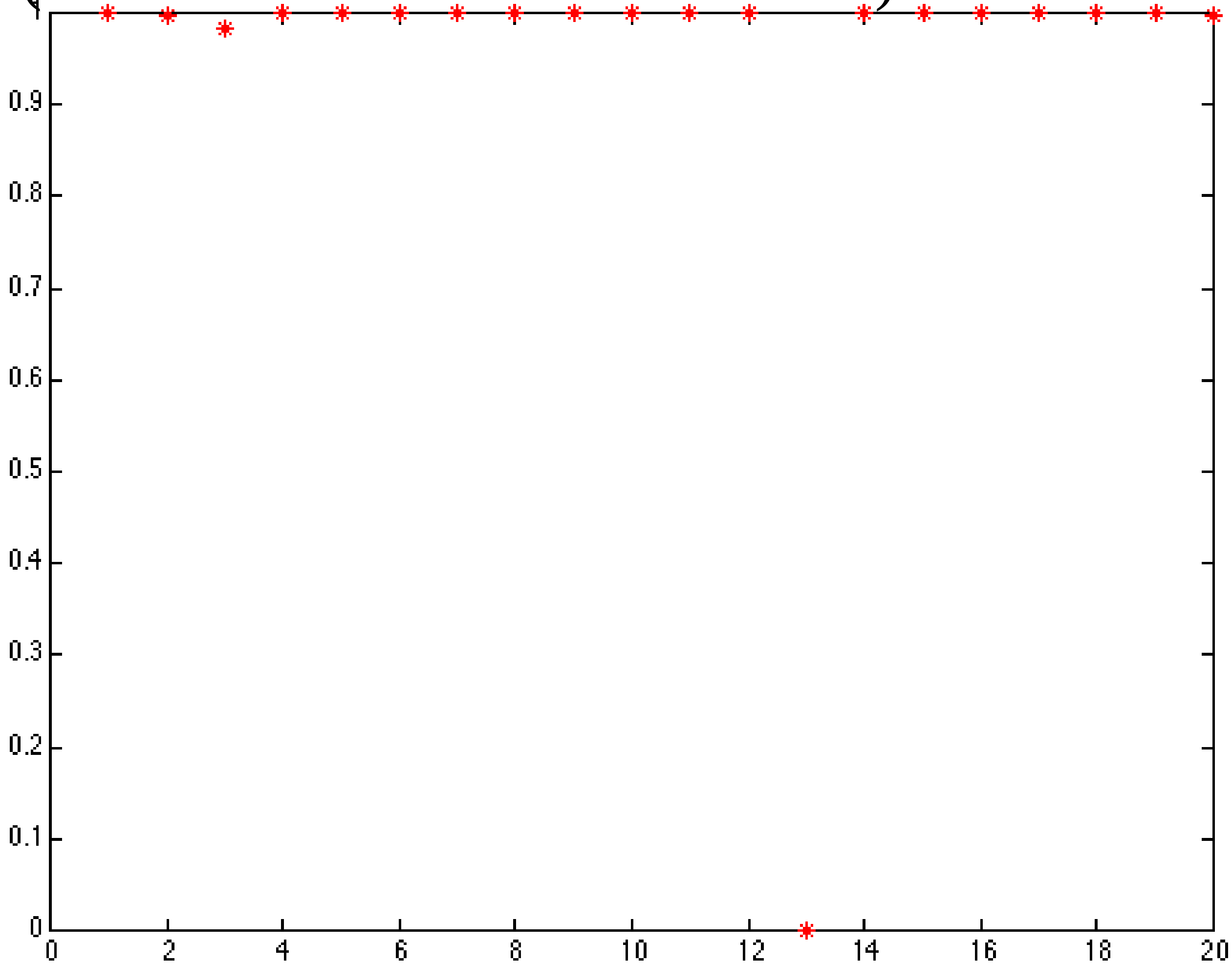
# Motivating Multiple Models

## Line Fitting

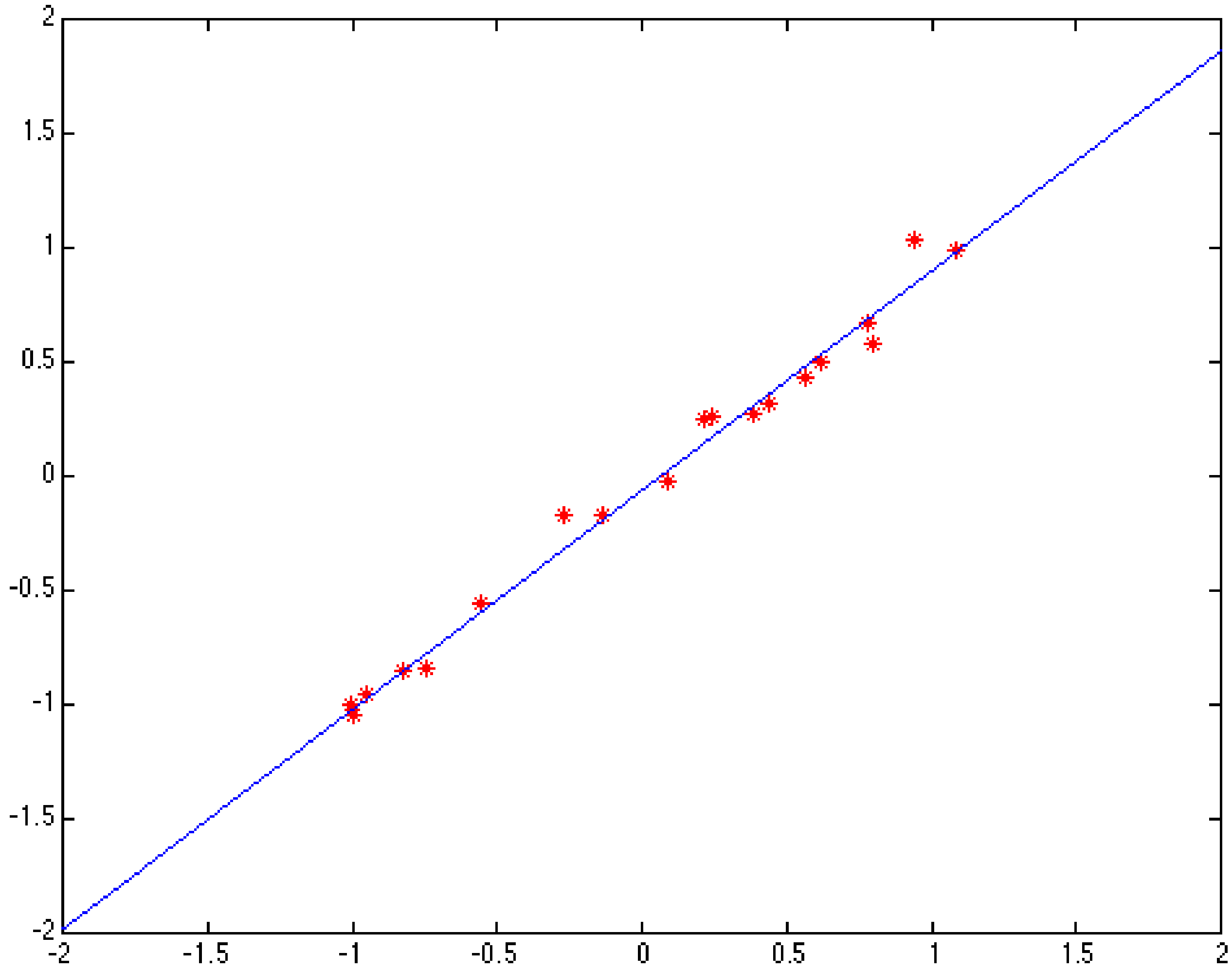


6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-14
-12
-10
-8
-6
-4
-2
0
2
4
6

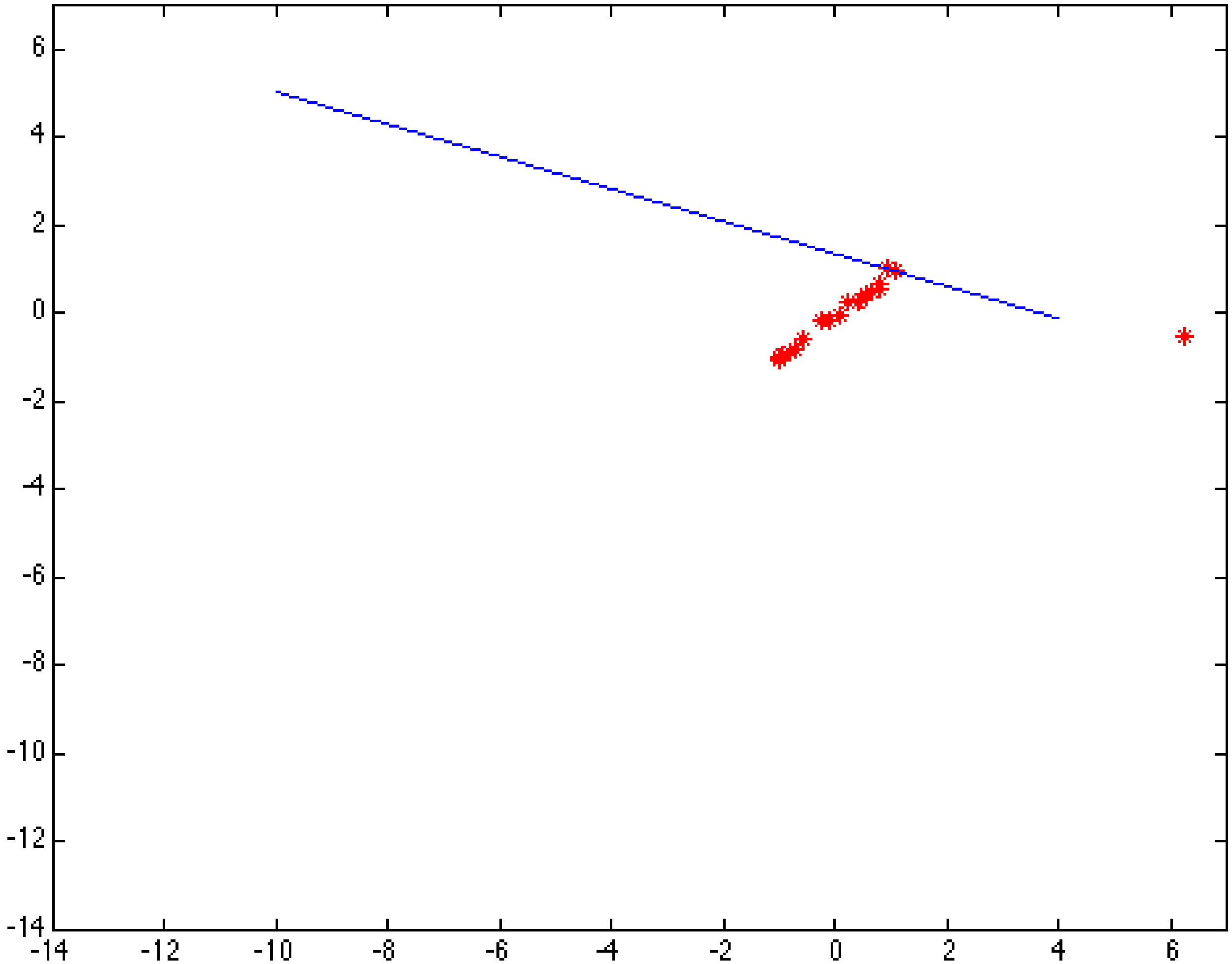
The expected values of the deltas at the maximum (notice the one value close to zero).



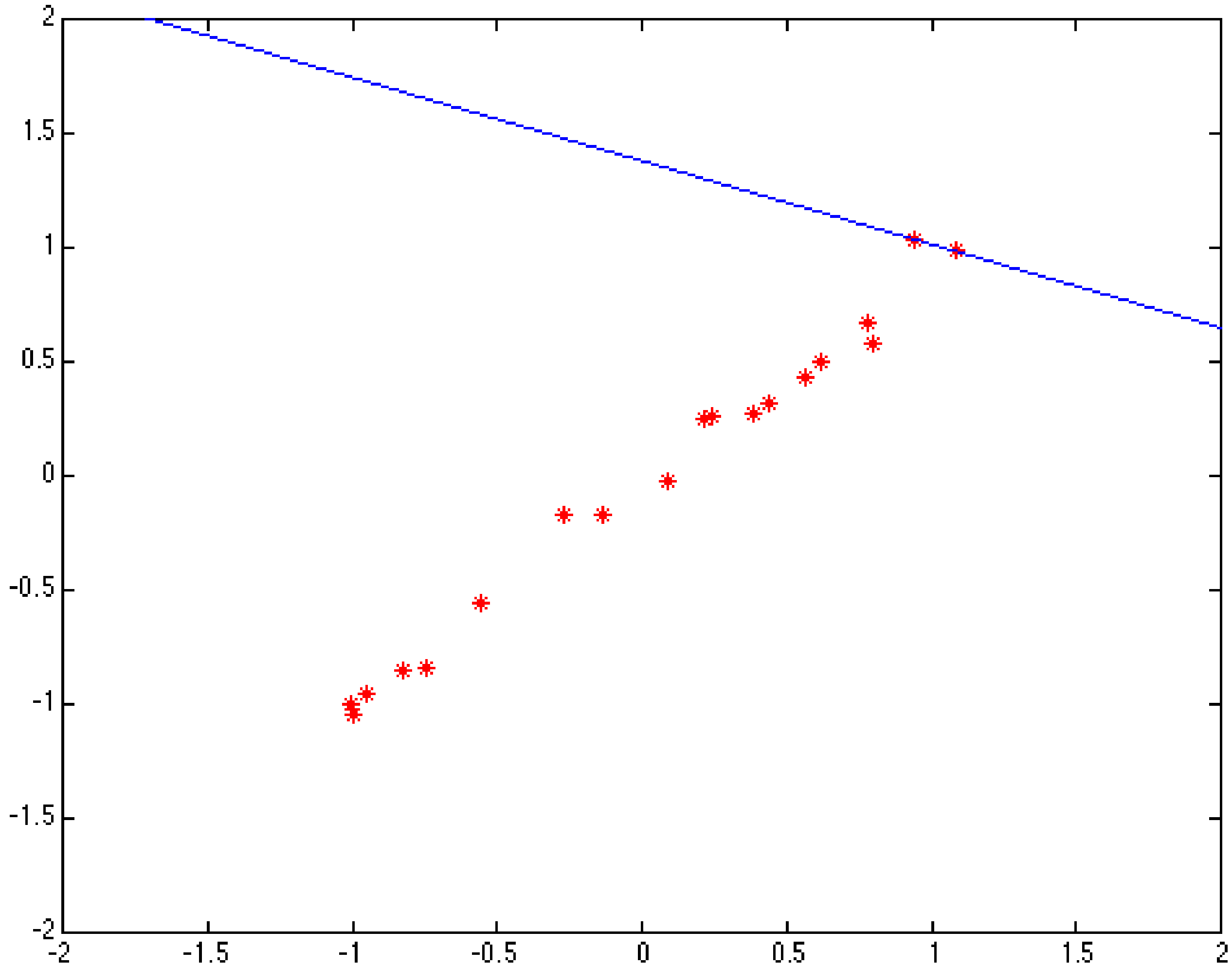
# Closeup of the fit



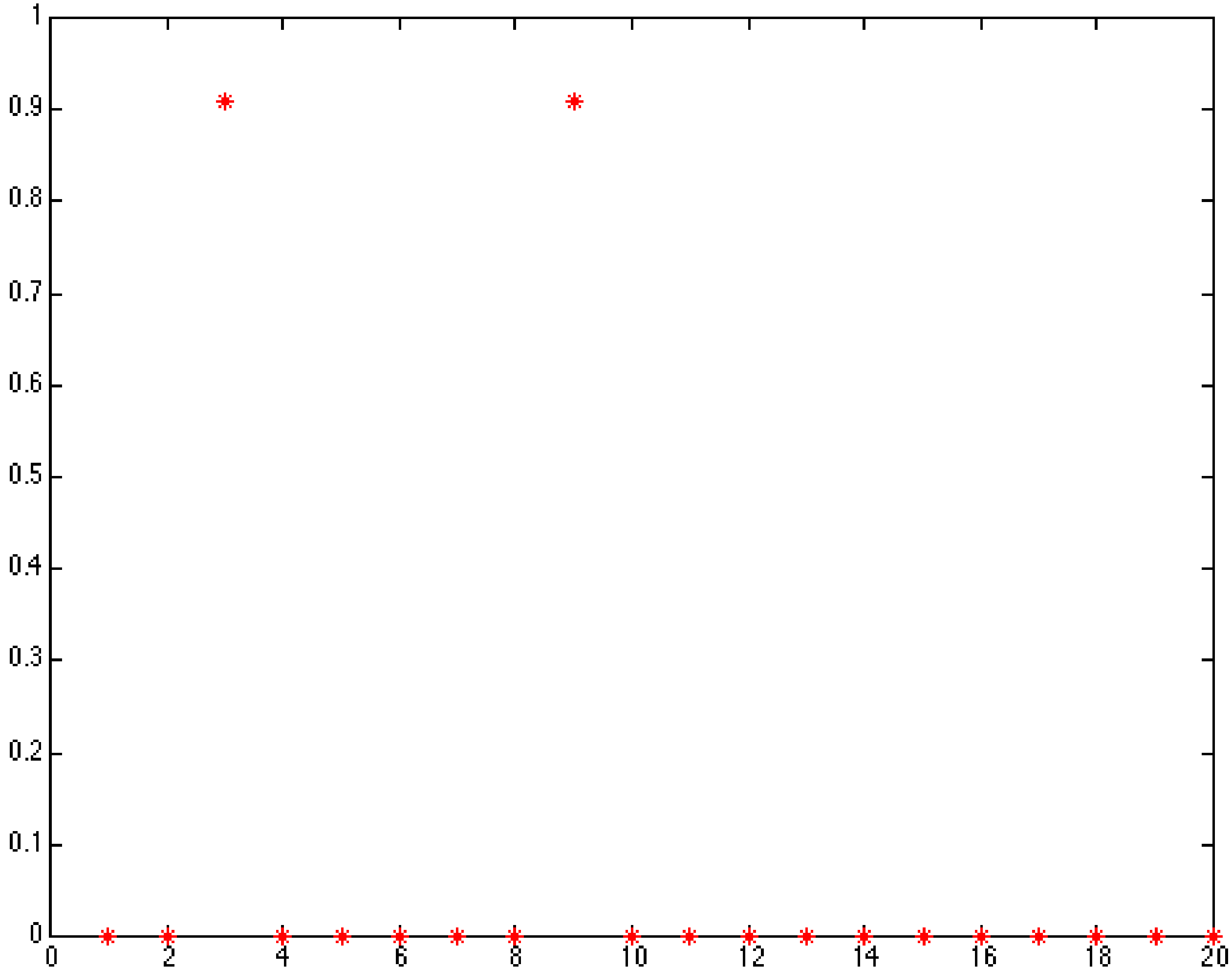
# Local maximum



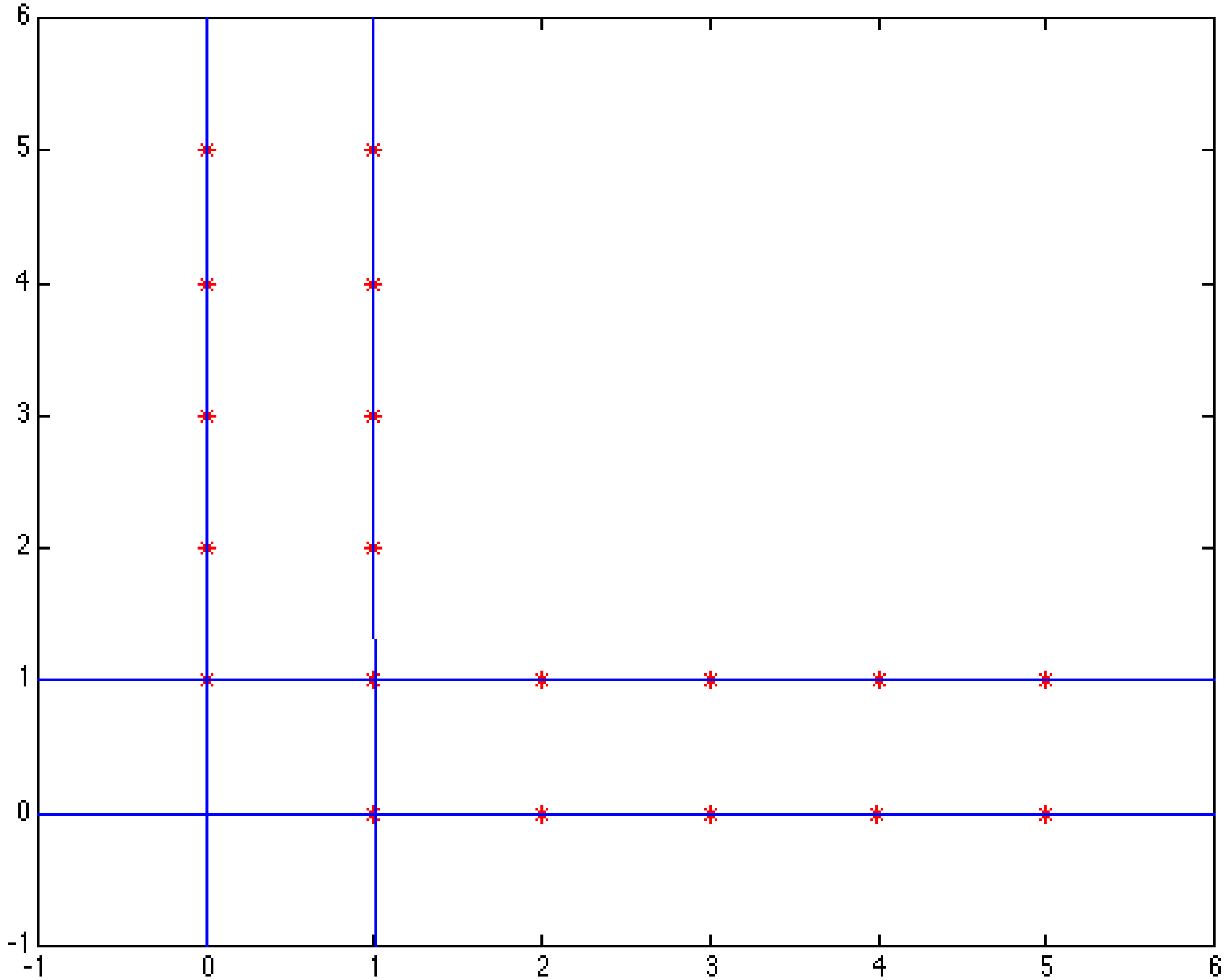
which is an excellent fit to some points



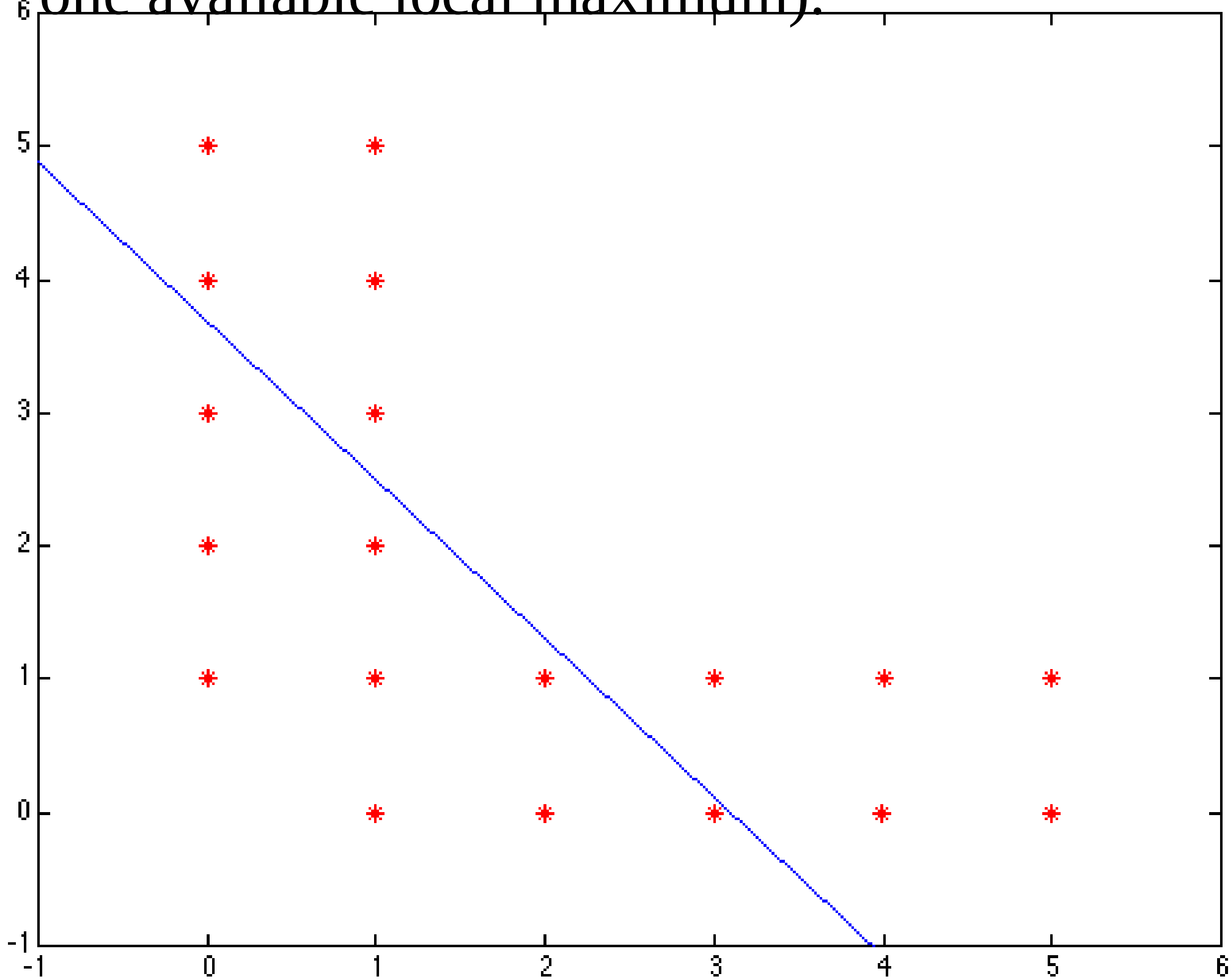
and the deltas for this maximum



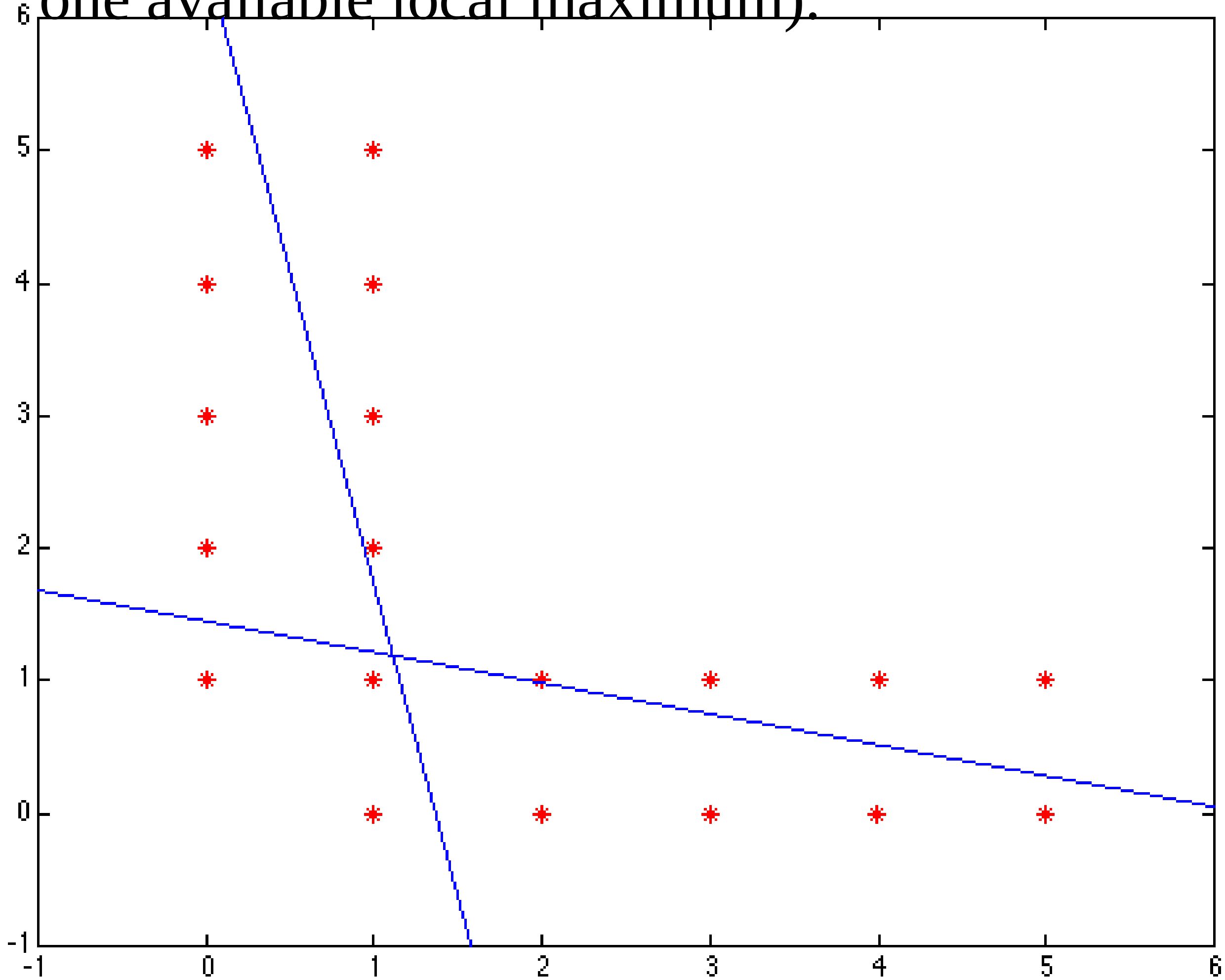
# A dataset that is well fitted by four lines



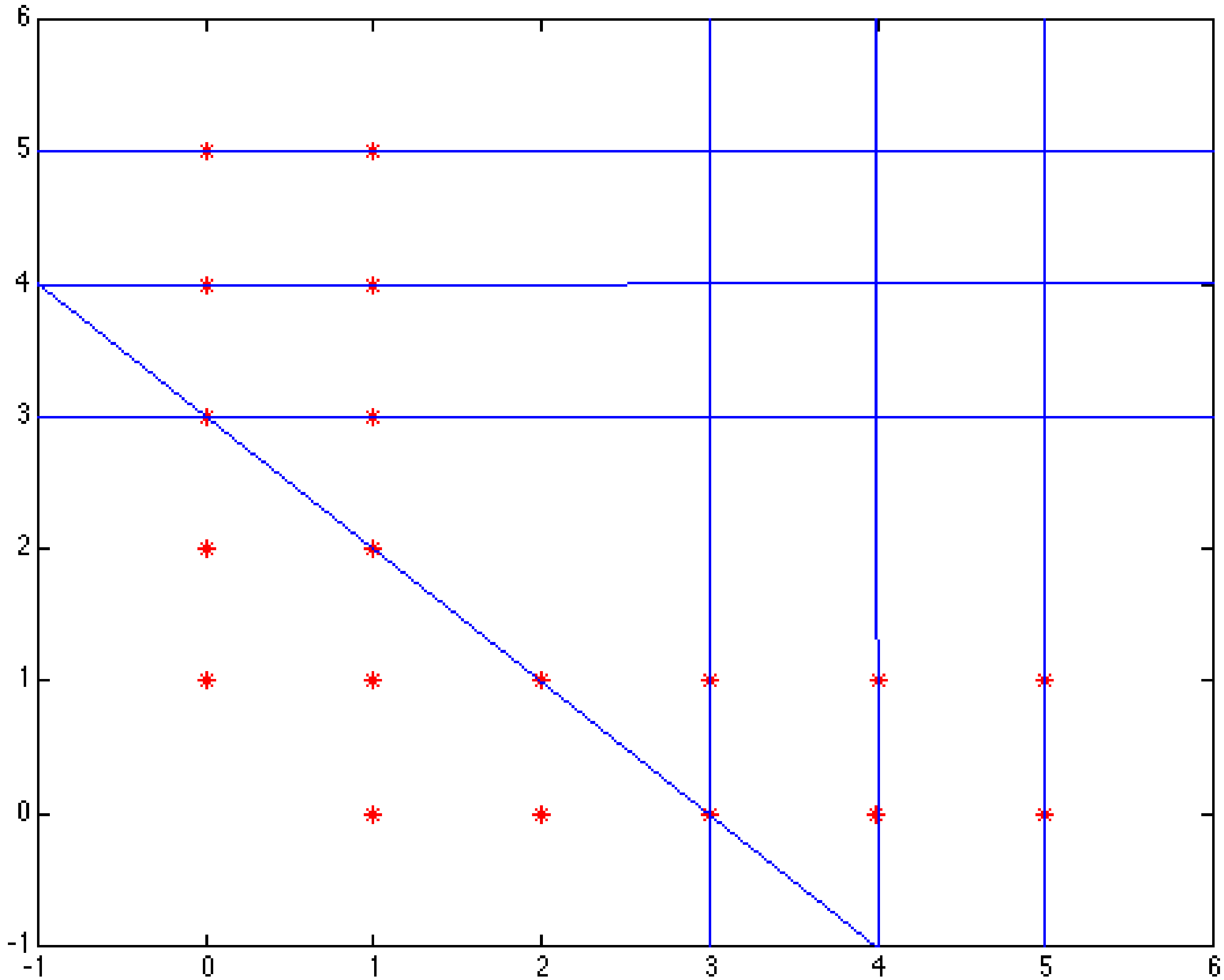
Result of EM fitting, with one line (or at least, one available local maximum).



Result of EM fitting, with two lines (or at least, one available local maximum).



Seven lines can produce a rather logical answer



# Motivating Multiple Models

## Multiple Motions

# Independent Object Motion

**Objects are the Focus**
**Camera is more or less steady**

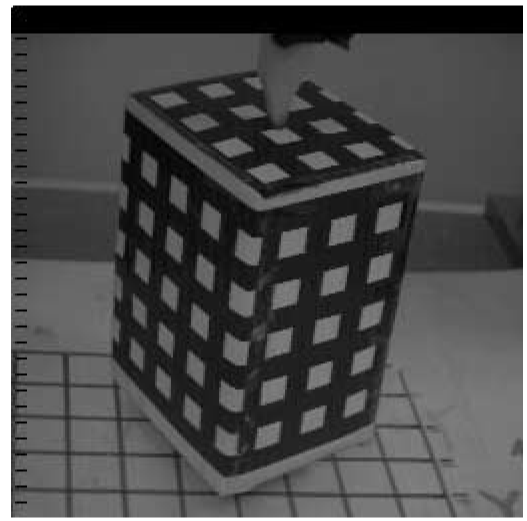
# Independent Object Motion with Camera Pan

**Most common scenario for capturing performances**

## Multiple Motions may not be due only to independent object motions but due to different surfaces

## Or

## “ Motion Layers “



# Multiple Motions as a Segmentation & Estimation Problem

- If we know which pixels go with what motion, can apply the now well-known methods of motion estimation to compute the motions

- Alternatively, given the motion parameters, potentially can label pixels corresponding to each of the motions.

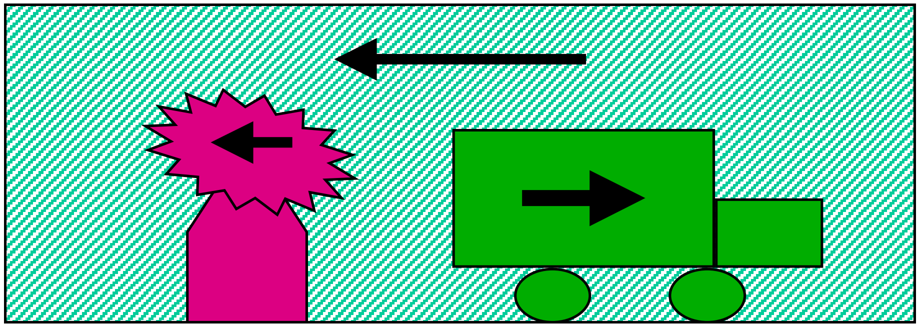
# Represent Multiple Motions as Layers

Layers

Input Sequence

# Compact Video Representation

***...motion and scene structure analysis...***



Separate coherent & significant motion & structure components

- Coherence : Align images using 2D/3D models of motion and structure

  Separate backgrounds and moving objects with layers

- Significance : Regions of support for various motion & structure components

# MULTIPLE 2D PARAMETRIC MODEL ESTIMATION

***… layered scene representation …***

## THREE ISSUES

- How many models ?
- What are the model parameters ?
- What is the spatial support layer for each model ?

# Competitive Multiple Model Estimation

[Ayer,Sawhney '95 '96]

- Model image motion in terms of a mixture of Gaussian models
- Layers of support represented as ownership probabilities
- Robust Maximum-Likelihood estimation of mixture and layer parameters using the Expectation-Maximization algorithm
- Minimum Description Length (MDL) encoding to select adequate number of models

# Automatic Layer Extraction : Intuition

**Input Sequence**

**Layer**

**Motion**

Assume that the segmentation of pixels into layers is known,

then estimating the motion is easy.

$$\mathbf{E}_{\mathbf{SSD}}(\mathbf{u}; \mathbf{A}_i) = \sum_{\mathbf{p} \in \mathbf{R}} \mathbf{w}_i(\mathbf{p})(\nabla \mathbf{I}_1^T \mathbf{u}(\mathbf{p}; \mathbf{A}_i) + \delta \mathbf{I}(\mathbf{p}))^2$$

# Where do we get the weights from ?

**Input Sequence**

**Layer**

**Motion**

Model each pixel as potentially belonging to N layers each with its own motion model.

Assume that we know the motion model, but not the pixel ownership to the model

$$\mathbf{E}_{SSD}(\mathbf{u};\mathbf{A}_i) = \sum_{p \in R} \mathbf{w}_i(\mathbf{p})(\nabla \mathbf{I}_1^T \mathbf{u}(\mathbf{p};\mathbf{A}_i) + \delta \mathbf{I}(\mathbf{p}))^2$$

# Where do we get the weights from ?

Input Sequence

Layer

Motion

Each pixel has a likelihood associated with a motion model and the two images

$$L(I_2(p) \mid I_1(p), A_i) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(\nabla I_1^T u(p; A_i) + \delta I(p))^2}{2\sigma^2}\right)$$

$$w_i(p) = \frac{L(I_2(p) \mid I_1(p), A_i)}{\sum_i L(I_2(p) \mid I_1(p), A_i)}$$

# Multiple Models : Mixture Models

- Model an image as a density function created using a mixture of Gaussian models conditioned on the adjacent images :

No. of models

$$f(I(\mathbf{x},t) \mid I(\mathbf{x},t-1),\Phi) = \sum_{i=1}^{g} \pi_i \, p(I(\mathbf{x},t) \mid I(\mathbf{x}-\mathbf{u}(\mathbf{x},\Theta_i),t-1),\sigma_i)$$

The mixture model is parameterized by

$$\Phi = [\Pi, \Sigma, \Theta]$$

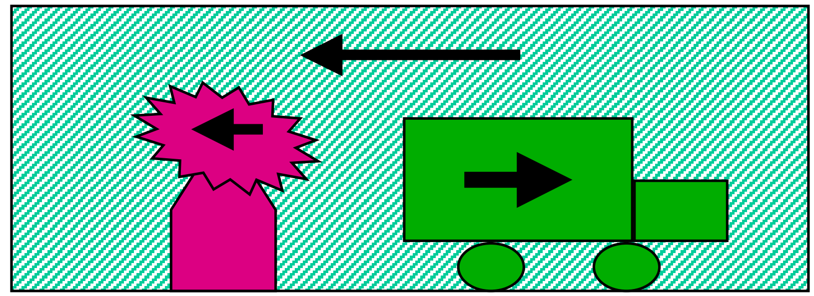
$\Theta$ : Ensemble of motion parameters for the g models

$\Sigma$ : Ensemble of the Gaussian distribution parameters for the models

$\Pi$ : Ensemble of the ownership layers for the models

# Mixture Models

*... represent layer ownerships as binary hidden variables ...*



- $Z = \{z_i(p_j), i = 1..g, j = 1..N\}$

  is a set of binary indicator variables representing the model labels.

- The stochastic model for the complete data, measurements and hidden variables is:

$$f(I, Z \mid \Phi) = \underbrace{f(I \mid Z, \Phi)}_{\text{Observation Likelihood}} \underbrace{p(Z \mid \Phi)}_{\text{Prior on the labels}}$$

# Mixture Model Estimation

***The Expectation-Maximization (EM) Algorithm***

- Maximize the negative log-likelihood of the parameters given the observations :

$$L(\Phi \mid I, Z) = -\log(f(I, Z \mid \Phi))$$

- Define the expectation of the likelihood :

$$Q(\Phi \mid \hat{\Phi}^{(k)}) = E\left[L(\Phi \mid I, Z) \mid I, \hat{\Phi}^{(k)}\right]$$

Mixing Proportions

Ownership

$$Q(\Phi \mid \hat{\Phi}^{(k)}) = \sum_{i=1}^{g} \sum_{x} \log(\pi_i)\, p\left(i \mid x, \hat{\Phi}^{(k)}\right) +$$

Data Likelihood

$$\sum_{i=1}^{g} \sum_{x} \log\left(p_l(I(x) \mid \hat{\Phi}^{(k)})\right) p\left(i \mid x, \hat{\Phi}^{(k)}\right)$$

# The EM Algorithm

*... iterate between layer and motion estimation ...*

Starting with an initial estimate $\hat{\Phi}^{(0)}$ repeat :

- **E-step :** Compute the function $Q(\Phi \mid \hat{\Phi}^{(k)})$

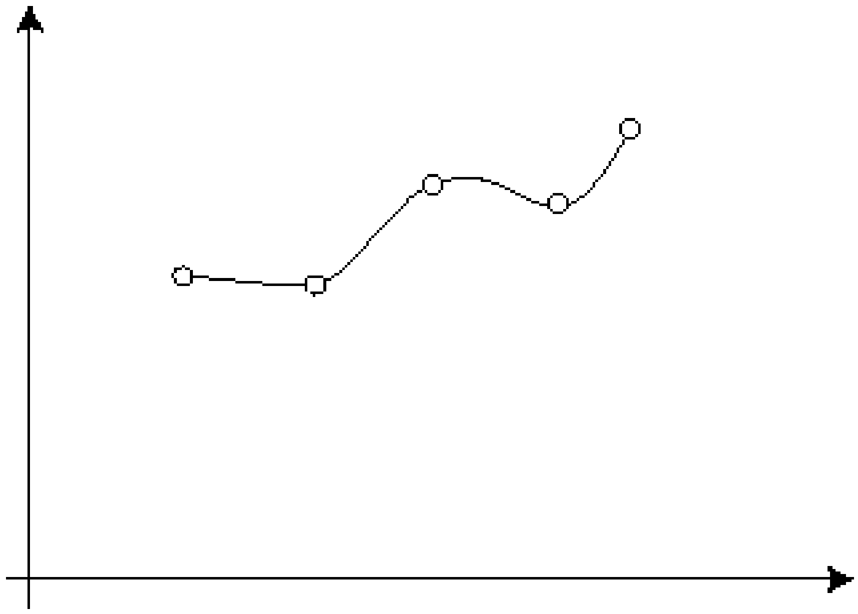
  Given the current estimate of the alignment parameters, compute the layer ownerships.

- **M-step :** Compute $\hat{\Phi}^{(k+1)} = \arg\max_{\Phi} Q(\Phi \mid \hat{\Phi}^{(k)})$

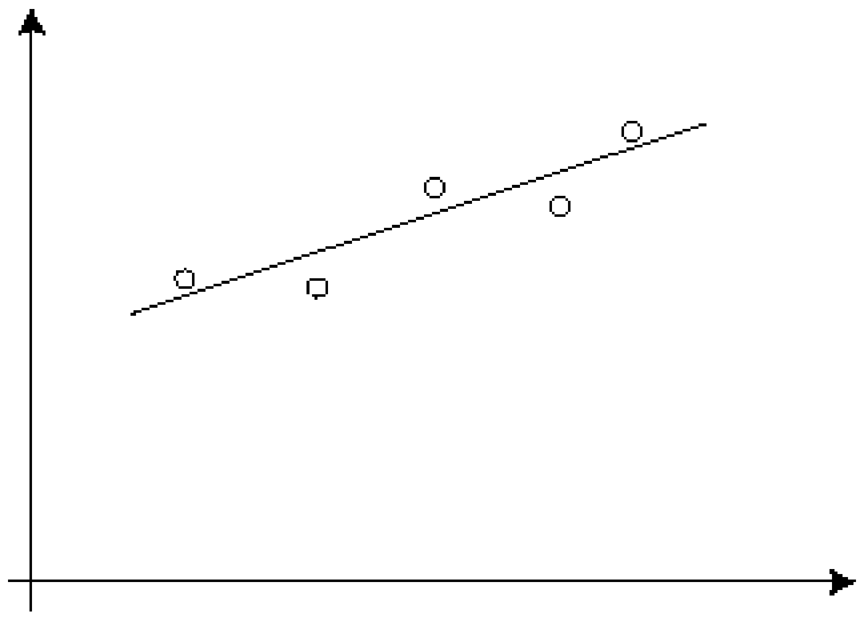
  Given the layer ownerships compute the alignment parameters.

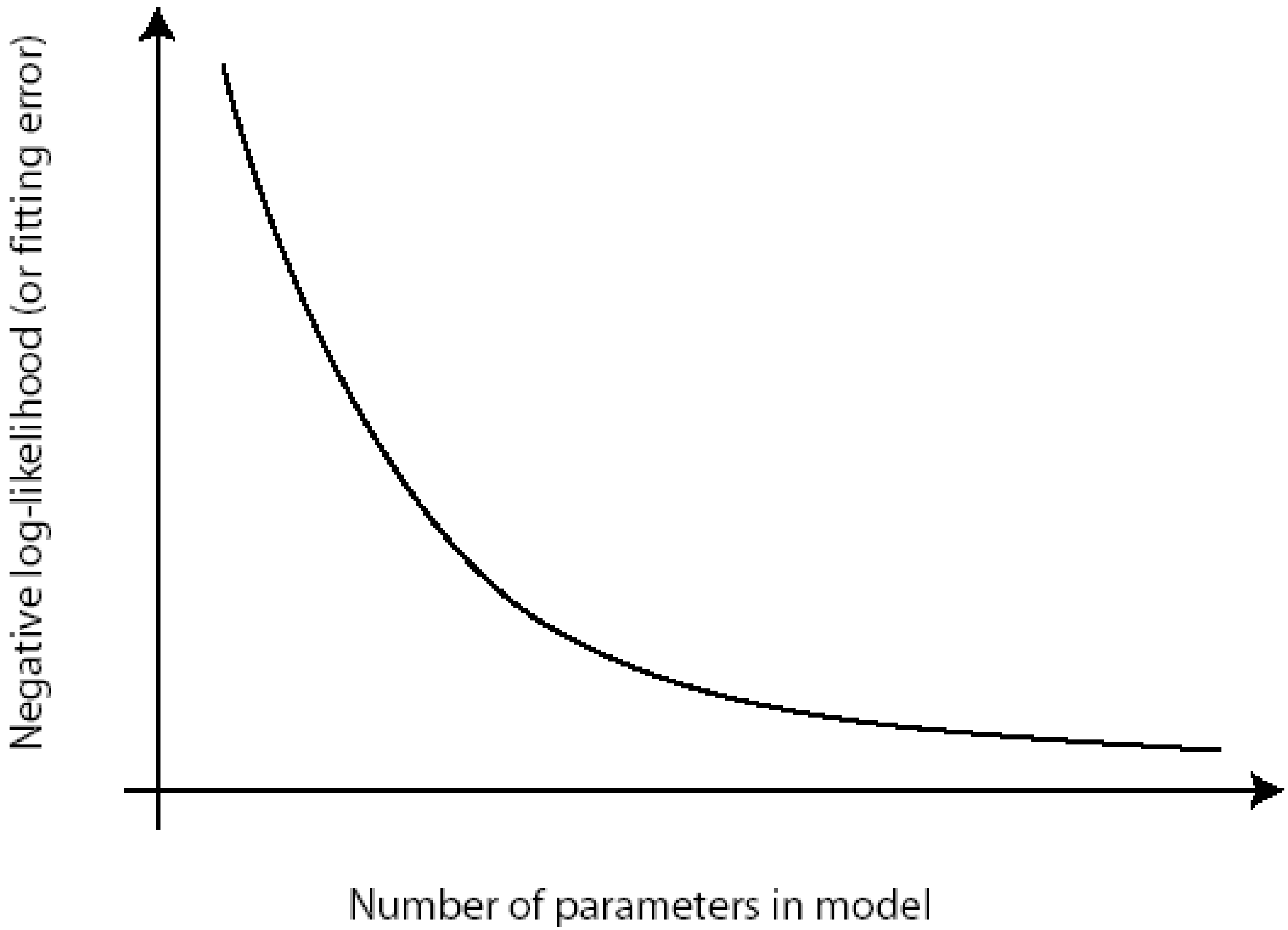
# Model Selection

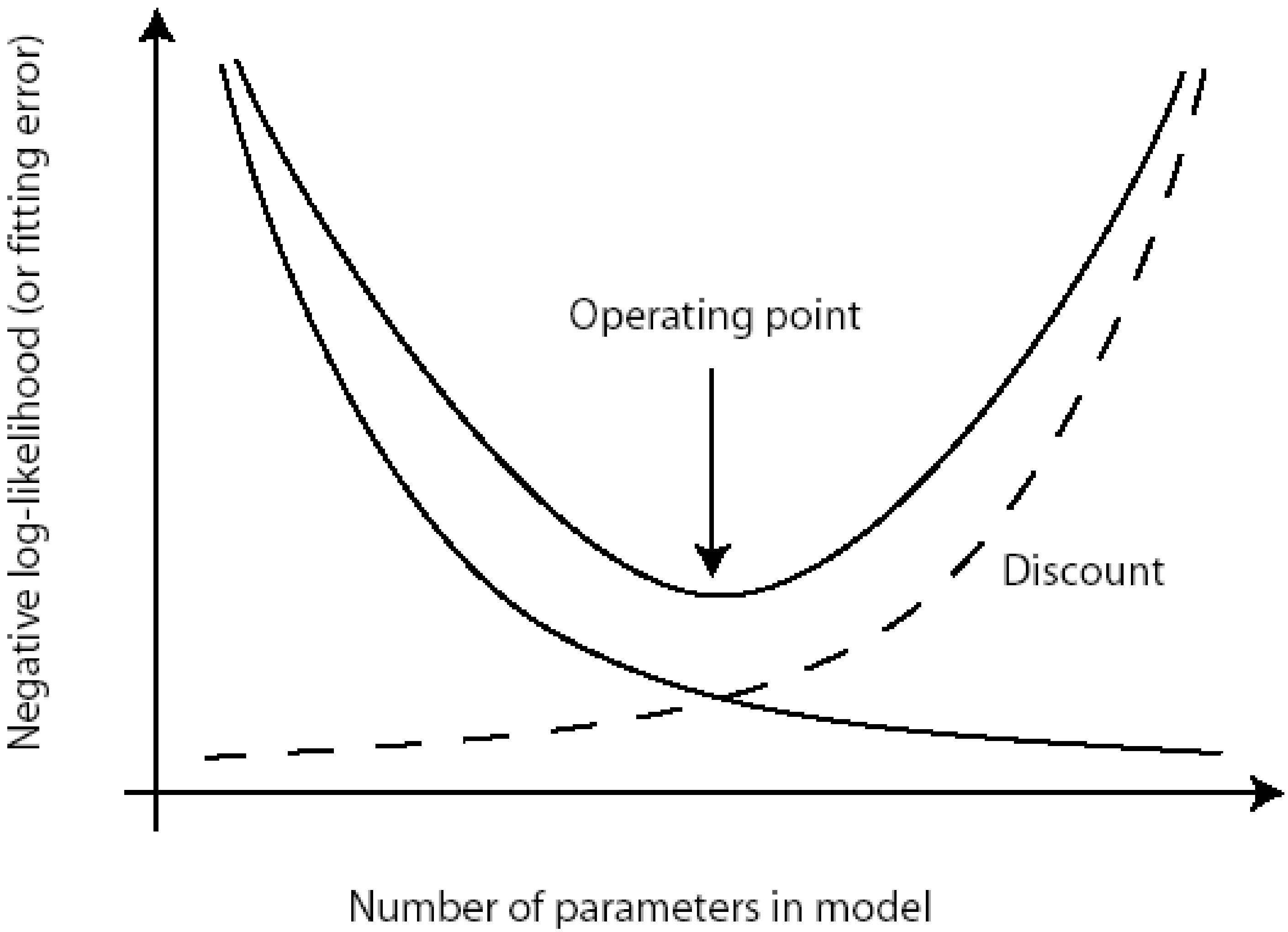
- We wish to choose a model to fit to data
  - e.g. is it a line or a circle?
  - e.g is this a perspective or orthographic camera?
  - e.g. is there an aeroplane there or is it noise?
- Issue
  - In general, models with more parameters will fit a dataset better, but are poorer at prediction
  - This means we can't simply look at the negative log-likelihood (or fitting error)



Top is not necessarily a better fit than bottom
(actually, almost always worse)




Negative log-likelihood (or fitting error)
Number of parameters in model




We can discount the fitting error with some term in the number of parameters in the model.

# How Many Models Are Adequate ?

*Minimum Description Length (MDL) encoding*
*for*
*Optimizing Modeling Complexity*

- Define model complexity as the total number of bits needed to encode the data and the models:

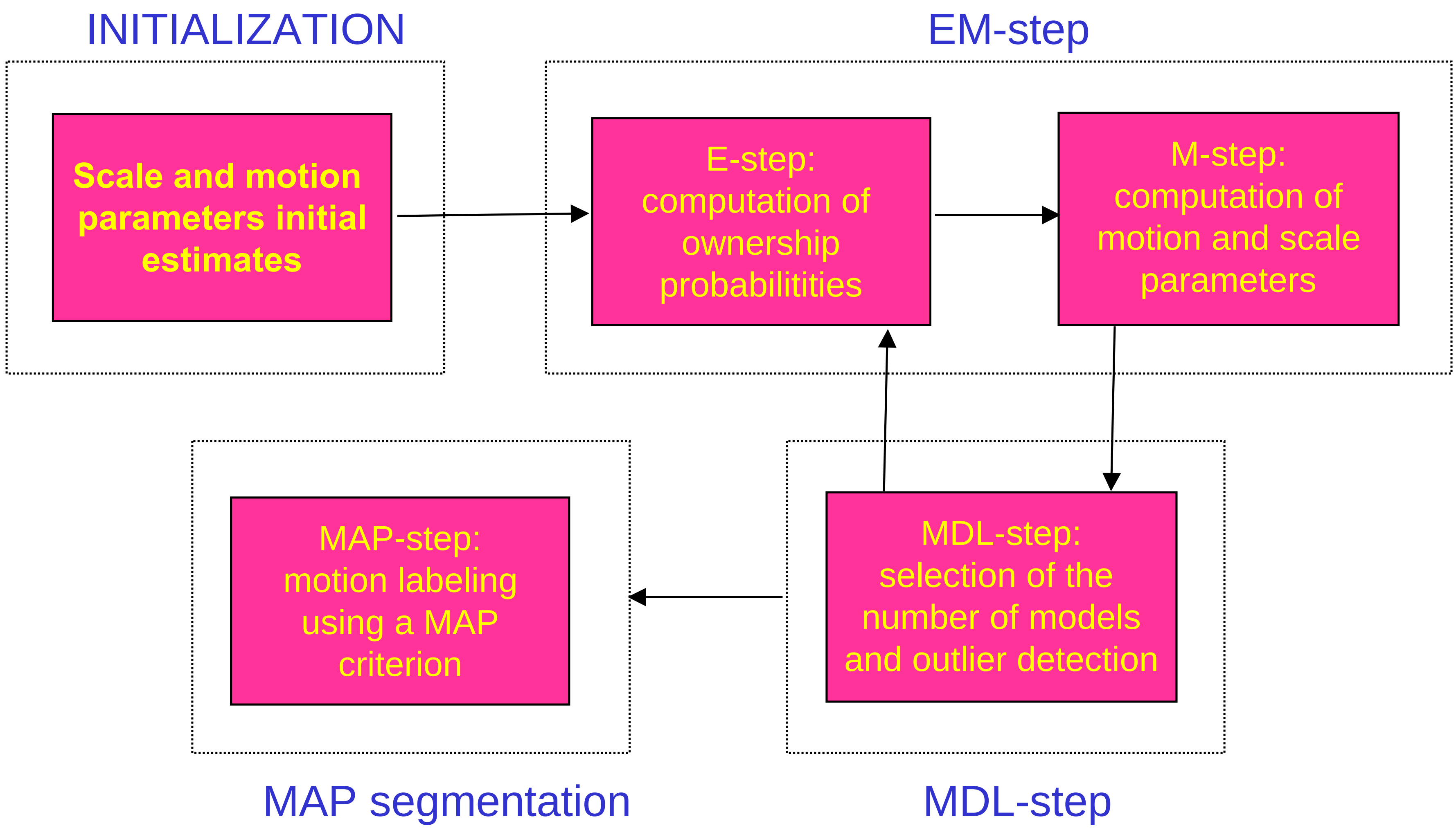
$$L(\{I(p_j), \Phi\}) = \underbrace{L_M(\Phi)}_{\text{Model Encoding Length (including pixel ownerships)}} + \underbrace{L_D(\{I(p_j)\} \mid \Phi)}_{\text{Data Encoding Length (residuals after alignment)}}$$

Model Encoding Length
(including pixel ownerships)

Data Encoding Length
(residuals after alignment)

- Find the optimum number of models and the model parameters that minimize the total encoding complexity.

# The Complete Algorithm

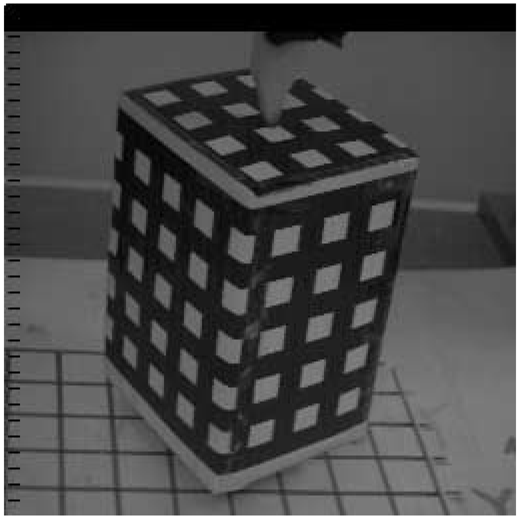


# Automatic Extraction of 2D Layers

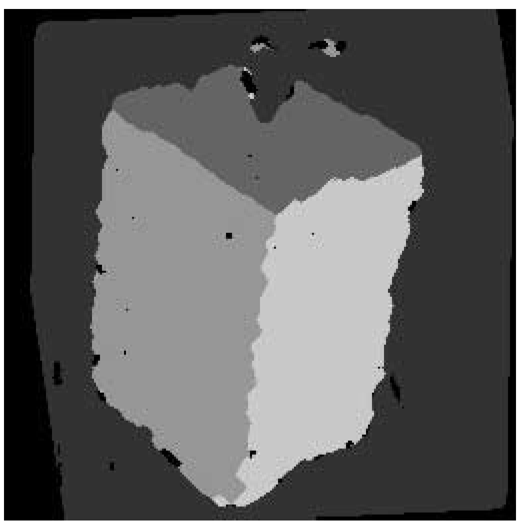
Input Sequence

Layers

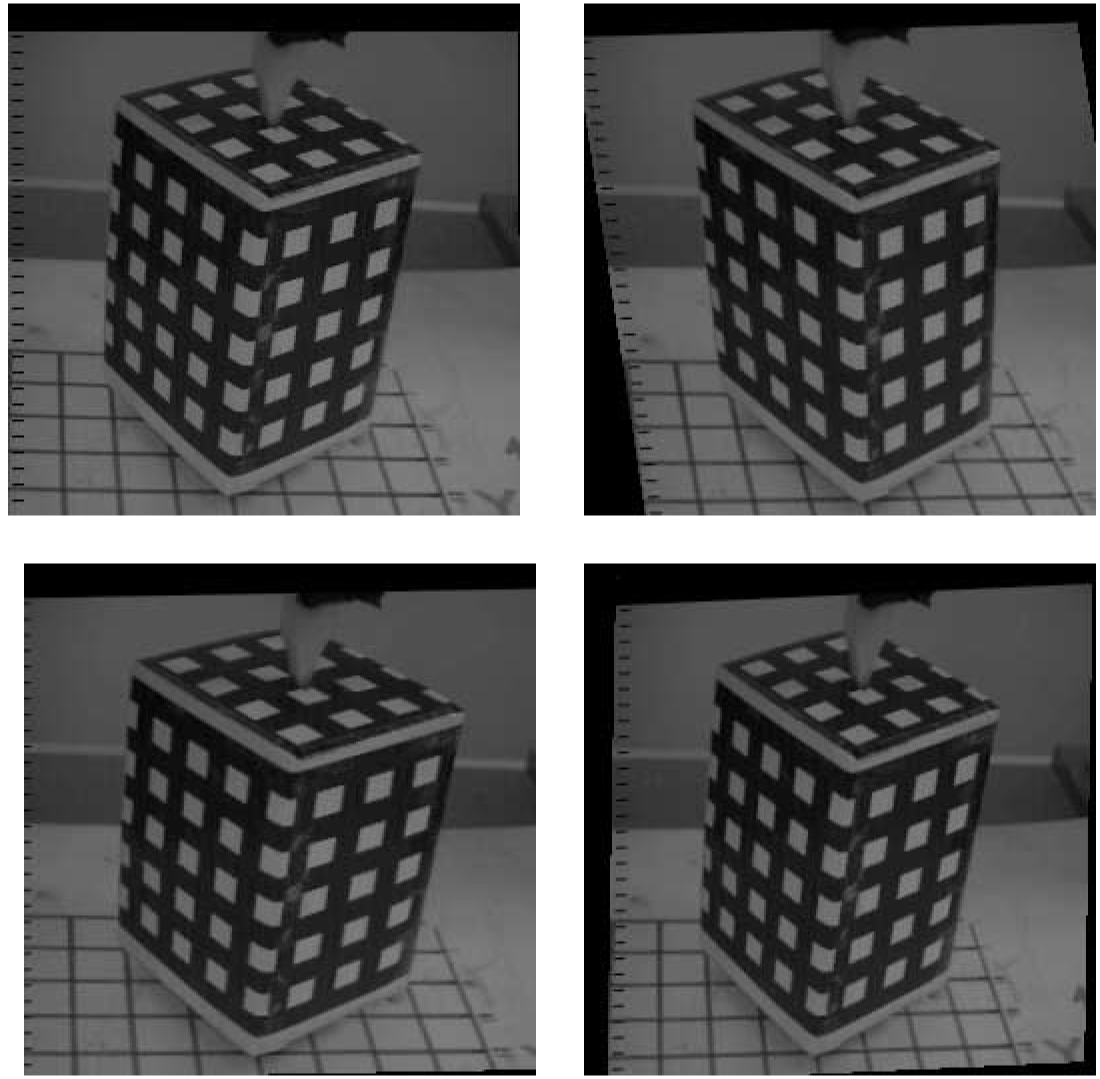
# Automatic Extraction of 2D Layers



Input Sequence



Layers



# The End