COS 522, February 29, 2000
Due: March 21, 2000

Homework Set 2

Do Exercises 5.13, 5.15, and 5.21 in the book by Du and Ko, plus the following special problems.

Problem 1 Let $A = \{K_1, K_2, K_3, K_4, K_5\}$ be a family of affine subspaces in R^2, where K_1, K_2, K_3, K_4 and K_5 are respectively the lines $x = 0$, $y = 0$, $x + y = 0$, $x + y = 1$ and $x + y = -1$.
(a) Let $V_A = \cup_{1 \leq i \leq 5} K_i$, regarded as a set in the compactified R^2. What is $\beta_0(V_A)$? What is $\beta_1(V_A)$?
(b) Draw the partially ordered set L_A. Calculate $\sum_{\vec{0} < x \in L_A} |\mu(\vec{0}, x)|$.

Problem 2 Let P be the property of an n-vertex graph being Hamiltonian. Use the following adversary strategy to prove $C(P) \geq \Omega(n^2)$. An *n-cycle* is a graph consisting exactly of a cycle of length n. Clearly, there are $(n-1)!$ such n-cycles. Note that any n-cycle is Hamiltonian. At any time, the adversary keeps a count of the number of n-cycles consistent with the revealed information about the graph so far. When a query "$x_{i,j} = 0$?" is asked, the adversary picks the answer to maximize this count consistent with this answer.

Problem 3 Let $p > 1$ be a prime number and $n \geq 2p$. Let P be the n-vertex graph property such that $P(G) = 1$ if and only at least one of the following holds: (a) G is connected, (b) G contains a clique of size p. Prove that property P is elusive.

Hint: Use an extension of the Kahn-Saks-Sturtevant method.