Problem Set 3

A road must be built. It will look horizontal on a map, ie, parallel to the x-axis. Where it should intersect the y-axis is the question we (or, rather, you) must answer. The road is an infinite line running parallel to the x-axis, and (amazingly enough) it costs nothing to build. What costs money are the n driveways leading into it. You see, there are n houses, each one modeled as a single point given by its (x, y) coordinates (these are very, very tiny houses). A driveway is the line segment connecting (x, y) to its nearest point on the road. So, obviously, driveways are parallel to the y-axis. The cost of a driveway is proportional to its length (same factor of proportionality for all n driveways). Design an $O(n)$ algorithm (ie, one that runs in time proportional to n) for finding the position of the road that minimizes the total cost (ie, the sum of the construction costs incurred by building the n driveways).

So, to summarize, the input consists of n points, ie, $2n$ numbers, and the output is a single number, ie, a y-coordinate. Justify your answers fully. This means: prove that your algorithm works, and that it runs in the right amount of time.

Due: at precept on Feb.28/29