3D Viewing

Thomas Funkhouser
Princeton University
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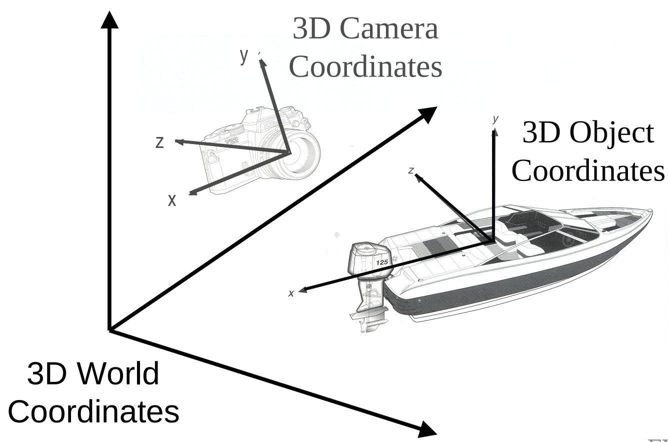
Overview

- Rendering pipeline
  - Camera analogy
- Camera placement
  - Center of projection, direction of projection
  - Change of coordinate systems
- Projections
  - Orthographic
  - Perspective

Coordinate Systems



FVFHP Figure 6.1

Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Camera Transformation

↓ 3D Camera Coordinates

Projection Transformation

↓ 2D Screen Coordinates

Window-to-Viewport Transformation

↓ 2D Image Coordinates

p'(x',y')

Transformations map points from one coordinate system to another

Viewing Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Camera Transformation

↓ 3D Camera Coordinates

Projection Transformation

↓ 2D Screen Coordinates

Window-to-Viewport Transformation

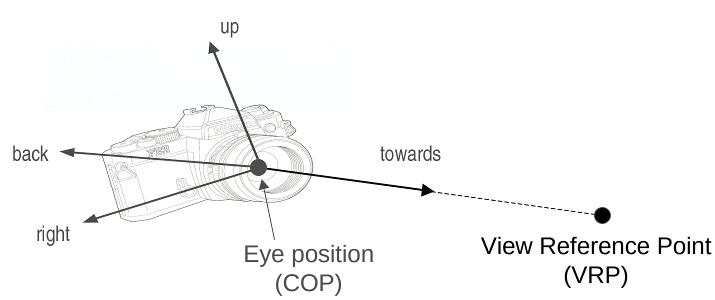
↓ 2D Image Coordinates

p'(x',y')

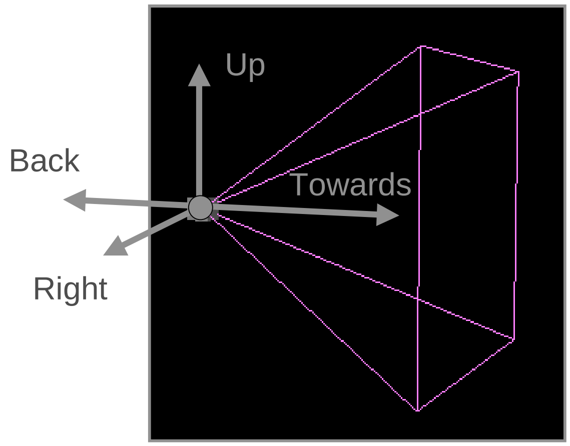
} Viewing Transformations

Camera Metaphor

- Camera parameters
  - Eye position (x, y, z)
  - View direction (towards vector, up vector)
  - Field of view (xfov, yfov)



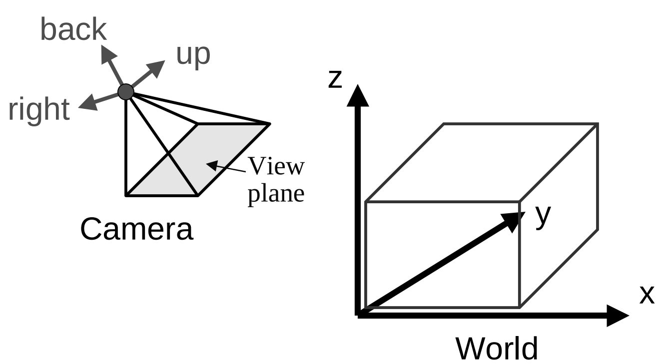
Demo



View Frustum

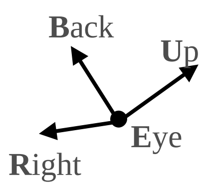
Camera Transformation

- Mapping from world to camera coordinates
  - Origin moves to eye position
  - Up vector maps to Y axis
  - Right vector maps to X axis

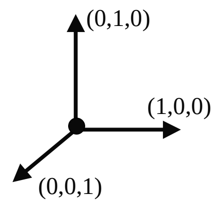


Camera Transformation

- Transformation matrix maps camera basis vectors to canonical vectors in camera coordinate system

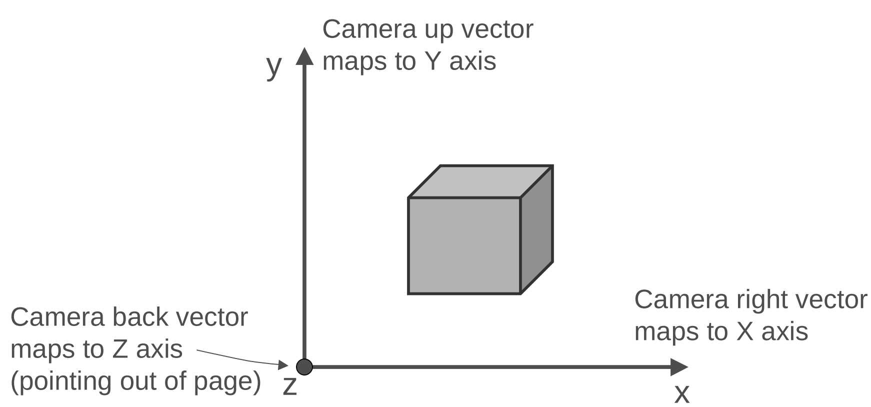


$$M = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix}^{-1}$$



Camera Coordinates

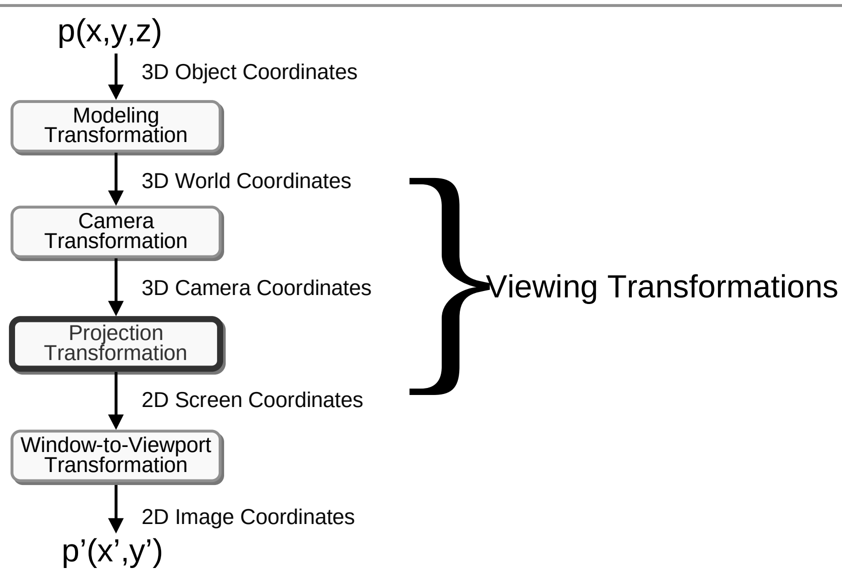
- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



Demo

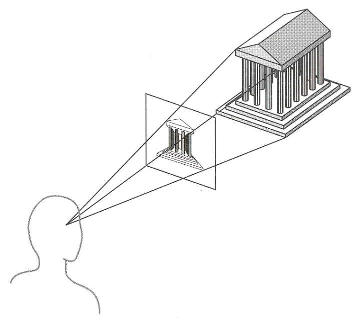
Patrick Min's
3D Viewing Applet

Viewing Transformations

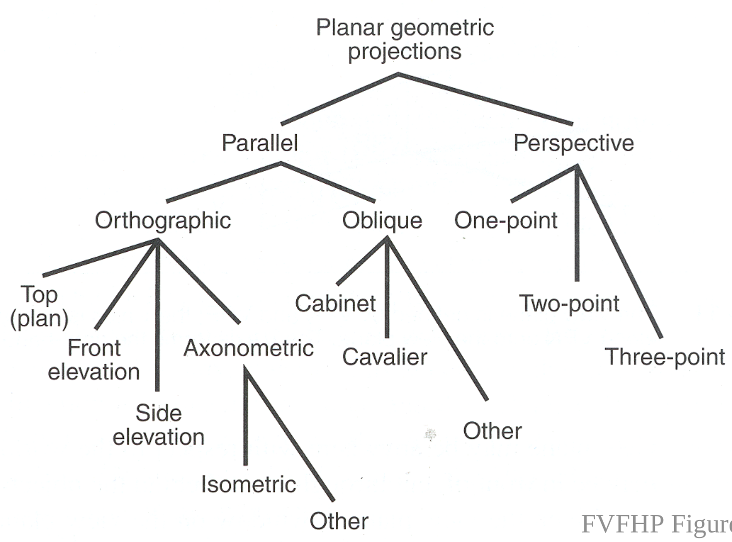


Projection

- General definition:
  - Transform points in *n*-space to *m*-space (*m*<*n*)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

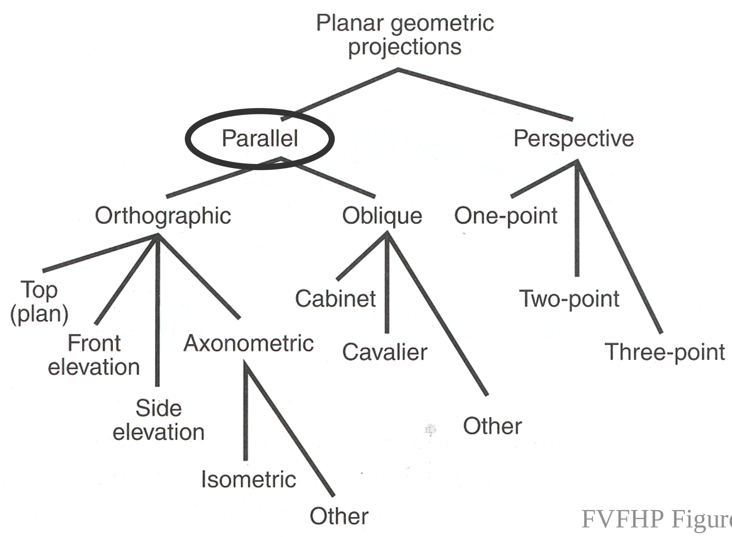


Taxonomy of Projections



FVFHP Figure 6.10

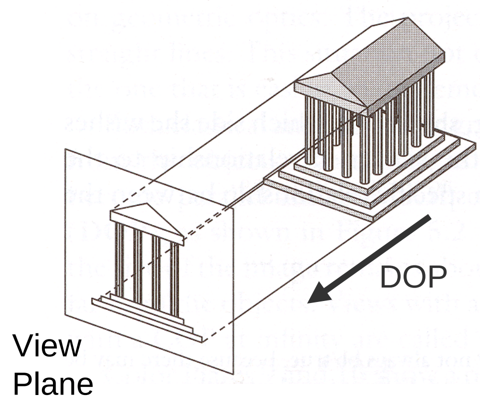
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

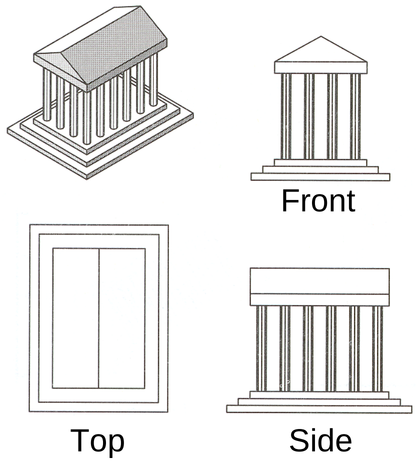
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Orthographic Projections

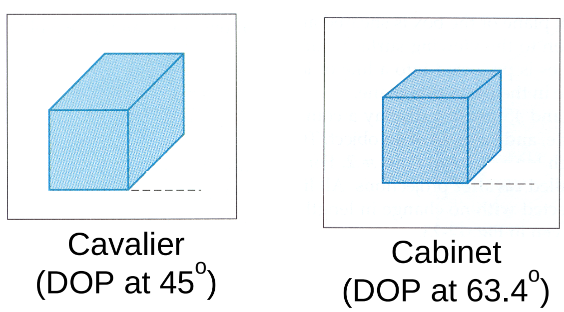
- DOP perpendicular to view plane



Angel Figure 5.5

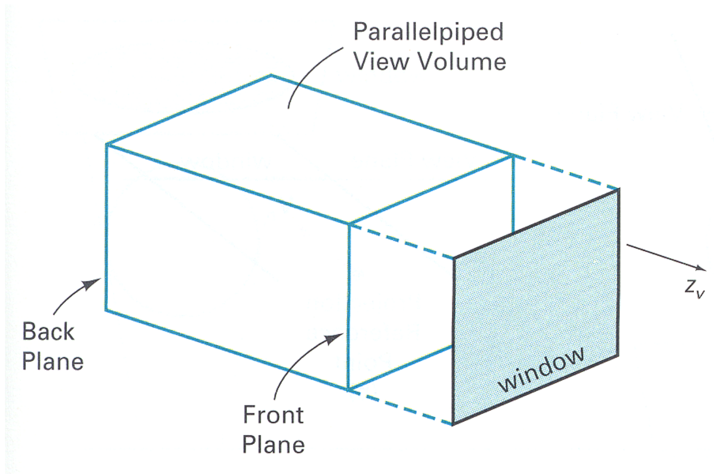
Oblique Projections

- DOP not perpendicular to view plane



H&B Figure 12.24

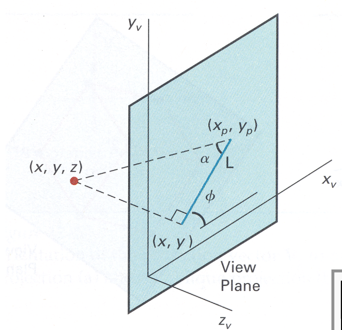
Parallel Projection View Volume



H&B Figure 12.30

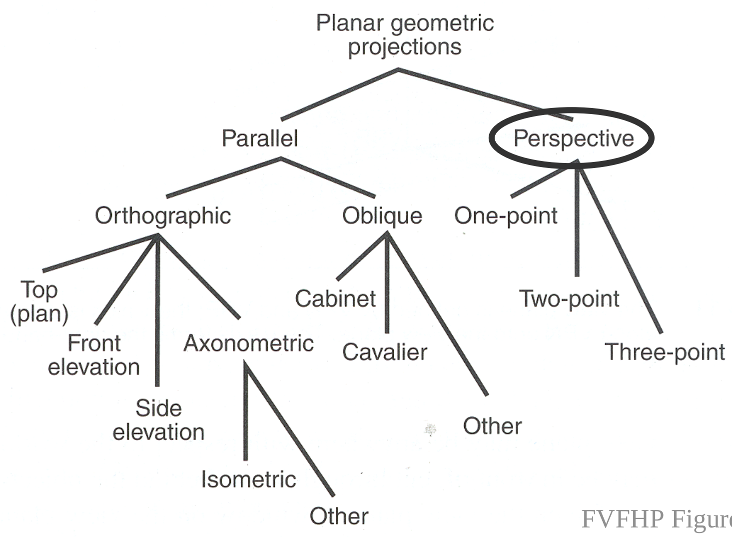
Parallel Projection Matrix

- General parallel projection transformation:



$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1 \cos\phi & 0 \\ 0 & 1 & L_1 \sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

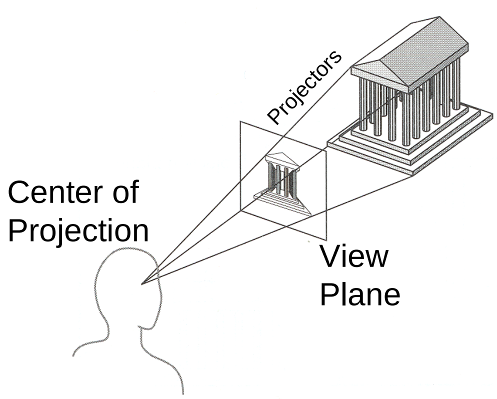
Taxonomy of Projections



FVFHP Figure 6.10

Perspective Projection

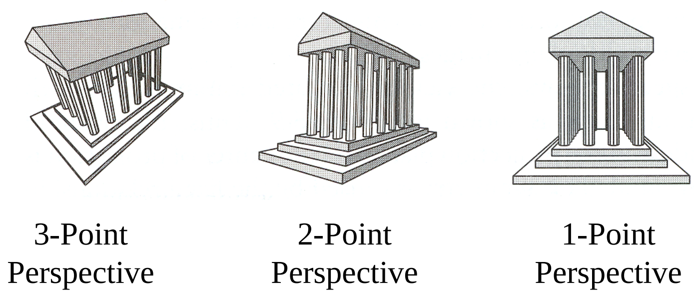
- Map points onto "view plane" along "projectors" eminating from "center of projection" (COP)



Angel Figure 5.9

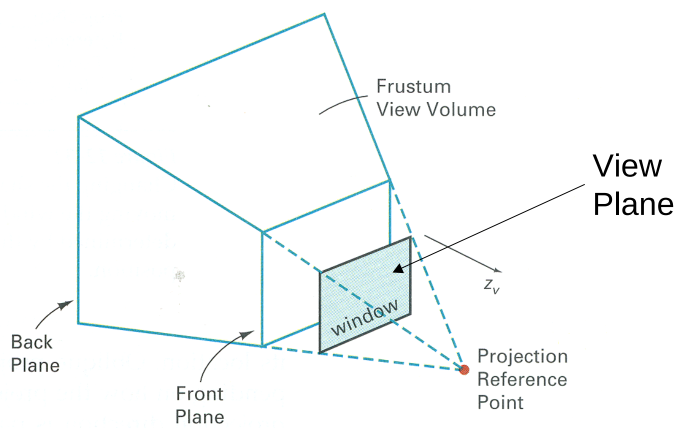
Perspective Projection

- How many vanishing points?



Angel Figure 5.10

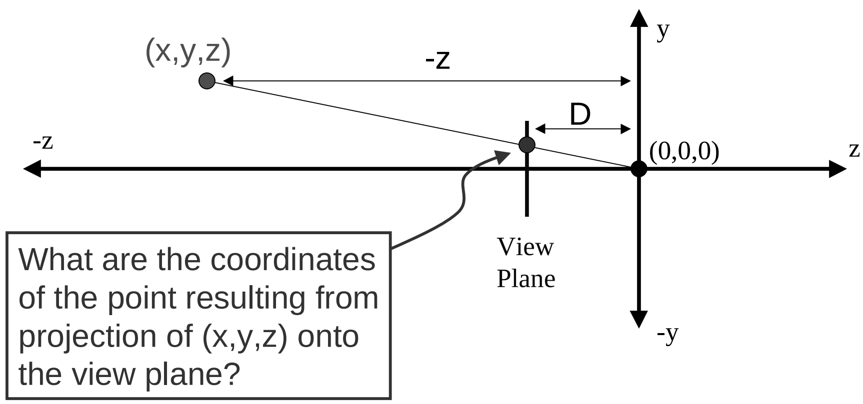
Perspective Projection View Volume



H&B Figure 12.30

Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

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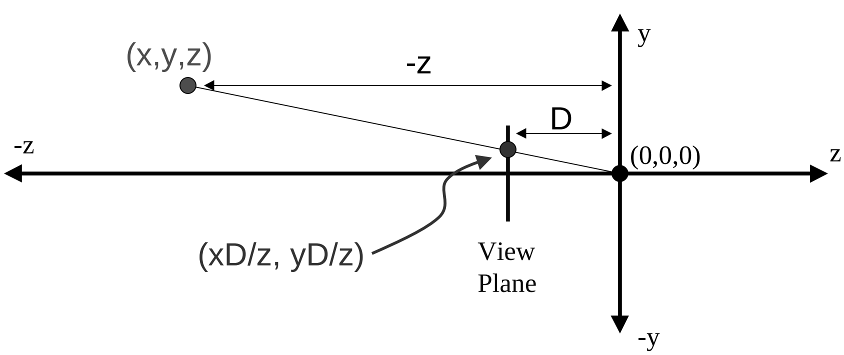


Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

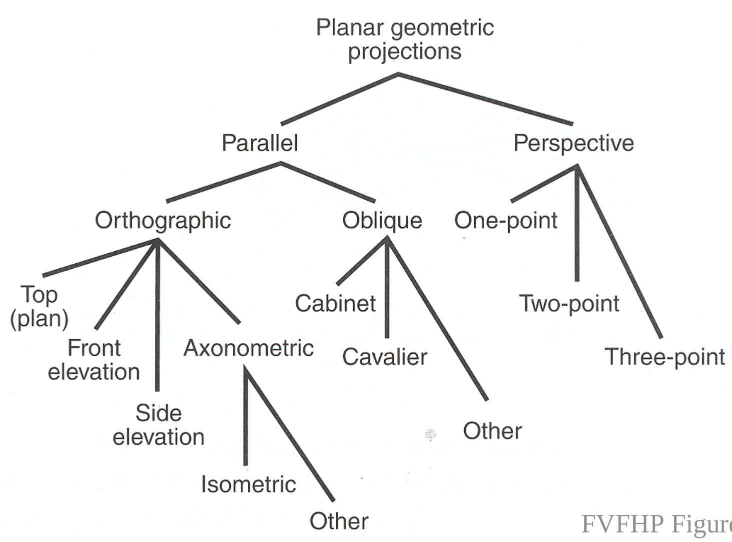
- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$x' = x_c$$
$$y' = y_c$$
$$z' = z_c$$
$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

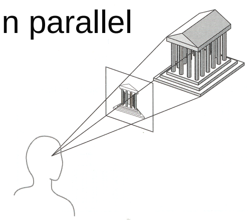
Taxonomy of Projections



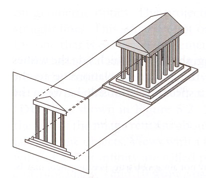
FVFHP Figure 6.10

Perspective vs. Parallel

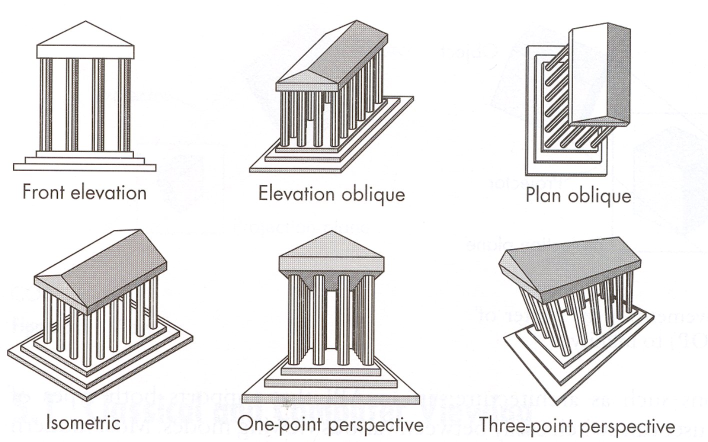
- Perspective projection
  + Size varies inversely with distance - looks realistic
  – Distance and angles are not (in general) preserved
  – Parallel lines do not (in general) remain parallel



- Parallel projection
  + Good for exact measurements
  + Parallel lines remain parallel
  – Angles are not (in general) preserved
  – Less realistic looking



Classical Projections



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Summary

- Camera transformation
  - Map 3D world coordinates to 3D camera coordinates
  - Matrix has camera vectors as rows
- Projection transformation
  - Map 3D camera coordinates to 2D screen coordinates
  - Two types of projections:
    - » Parallel
    - » Perspective