Image Sampling and Reconstruction

Thomas Funkhouser
Princeton University
C0S 426, Fall 1999

Overview

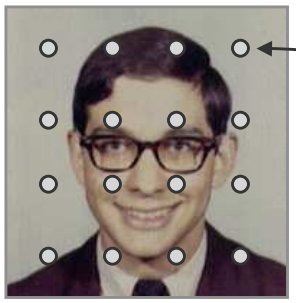
- Image sampling and reconstruction
  - Pixels are discrete samples of continuous function
  - Frequency analysis
- Image resampling
  - Aliasing
  - Filters and convolution

Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
  - Morphs
- Combining
  - Composite

Image Sampling

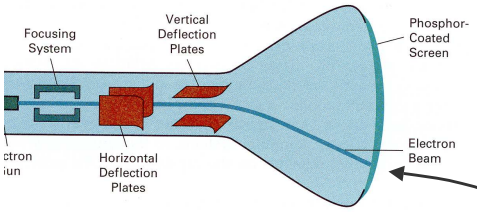
- An image is a 2D rectilinear array of samples
  - Quantization due to limited intensity resolution
  - Sampling due to limited spatial and temporal resolution



Pixels are infinitely small point samples

Image Reconstruction

- Re-create continuous image from samples
  - Example: cathode ray tube



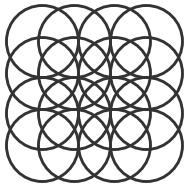


Image is reconstructed by displaying pixels with finite area (Gaussian)

Sampling and Reconstruction

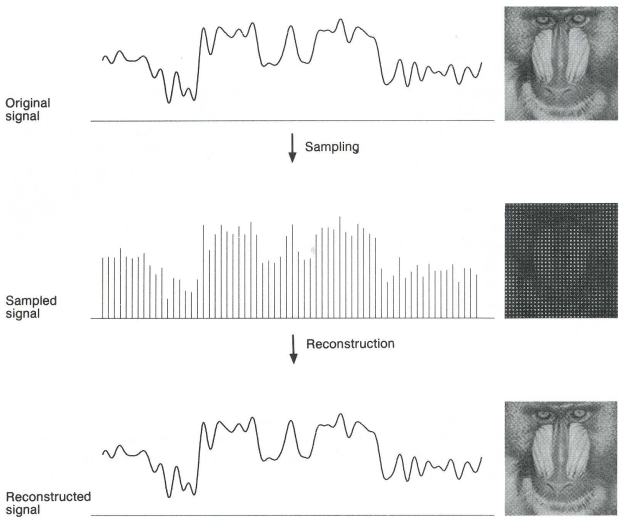
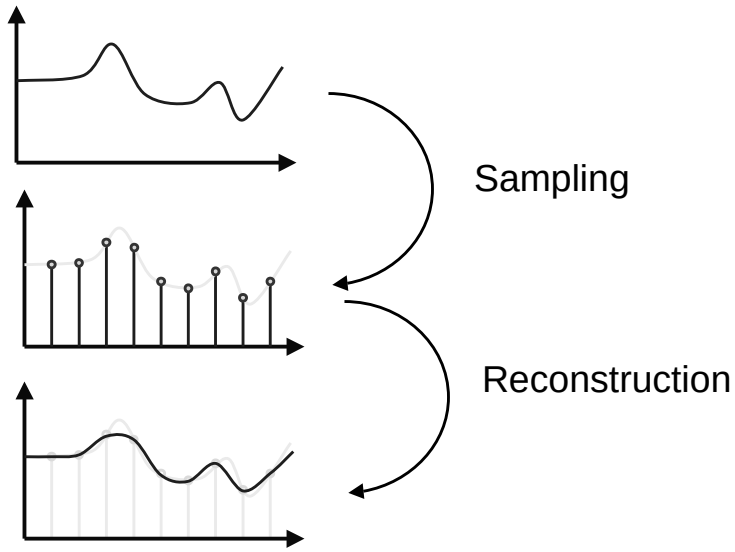


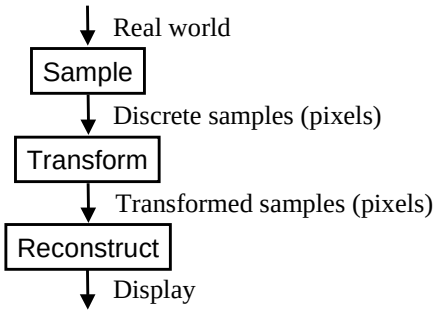
Figure 19.9 FvDFH

Sampling and Reconstruction



Sample Processing

- Apply function to samples
  - Linear functions
  - Linear reconstructions

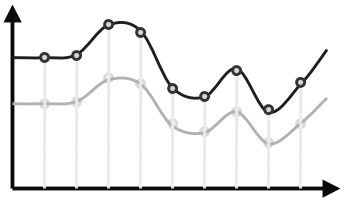


Adjusting Brightness

- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)

Original

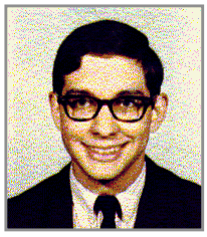
Brighter



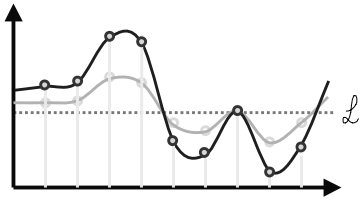
Adjusting Contrast

- Compute mean luminance $\mathscr{l}$ for all pixels
  - luminance = 0.30*r + 0.59*g + 0.11*b
- Scale deviation from $\mathscr{l}$ for each pixel component
  - Must clamp to range (e.g., 0 to 255)

Original

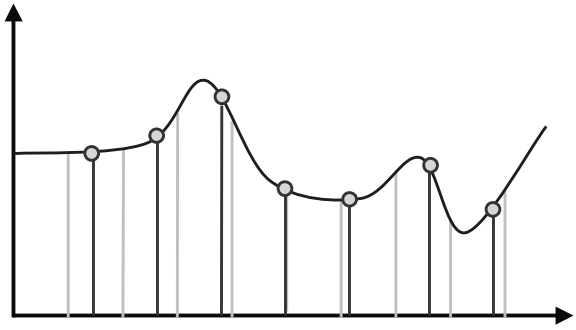


More Contrast



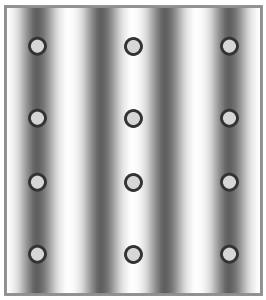
Resampling

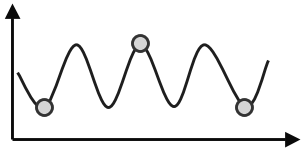
- What if samples are at new locations?
  - Image scaling, warping, etc.



Sampling Theory

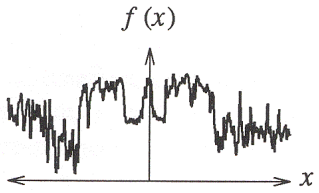
- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?

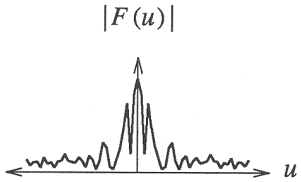




Spectral Analysis

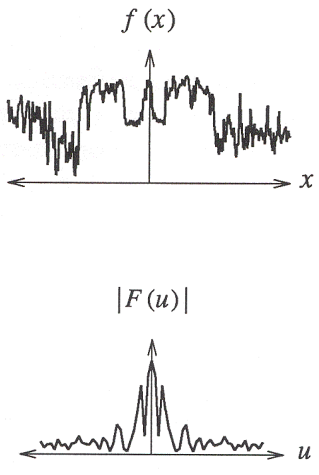
- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain:
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform



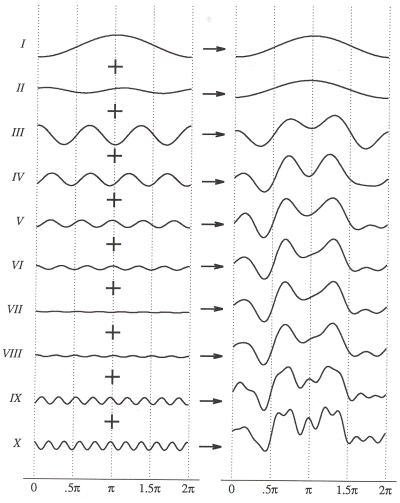
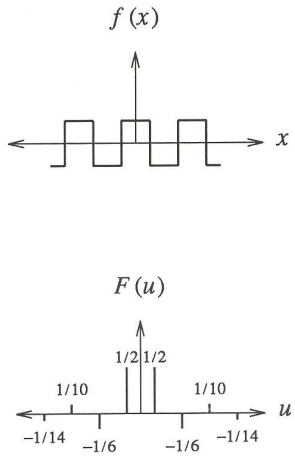


Figure 2.6 Wolberg

Fourier Transform



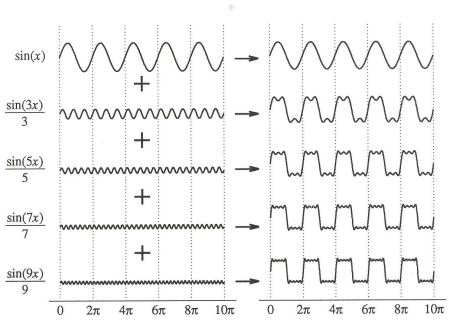


Figure 2.5 Wolberg

Fourier Transform

- Fourier transform:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi x} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi u} du$$

Aliasing

- Artifacts due to undersampling

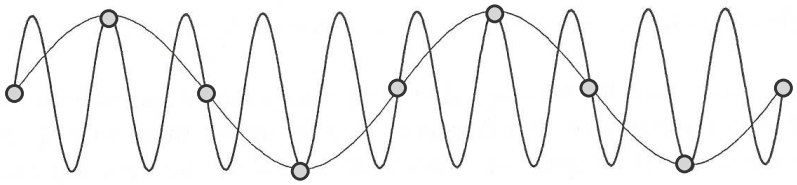
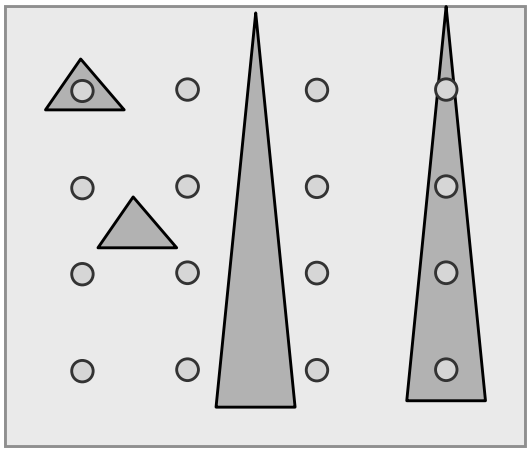


Figure 14.17 FvDFH

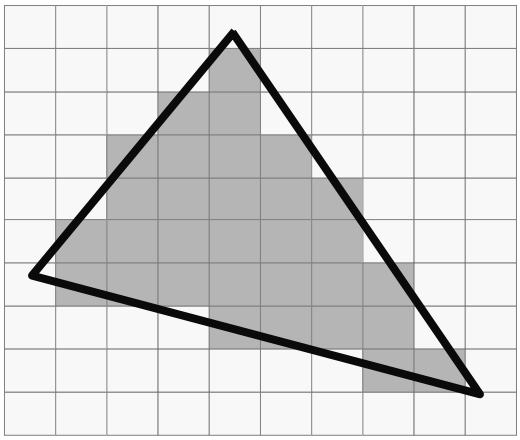
Spatial Aliasing

- Point sampling problems



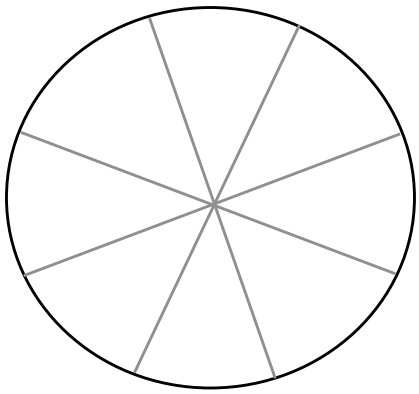
Spatial Aliasing

- Jaggies

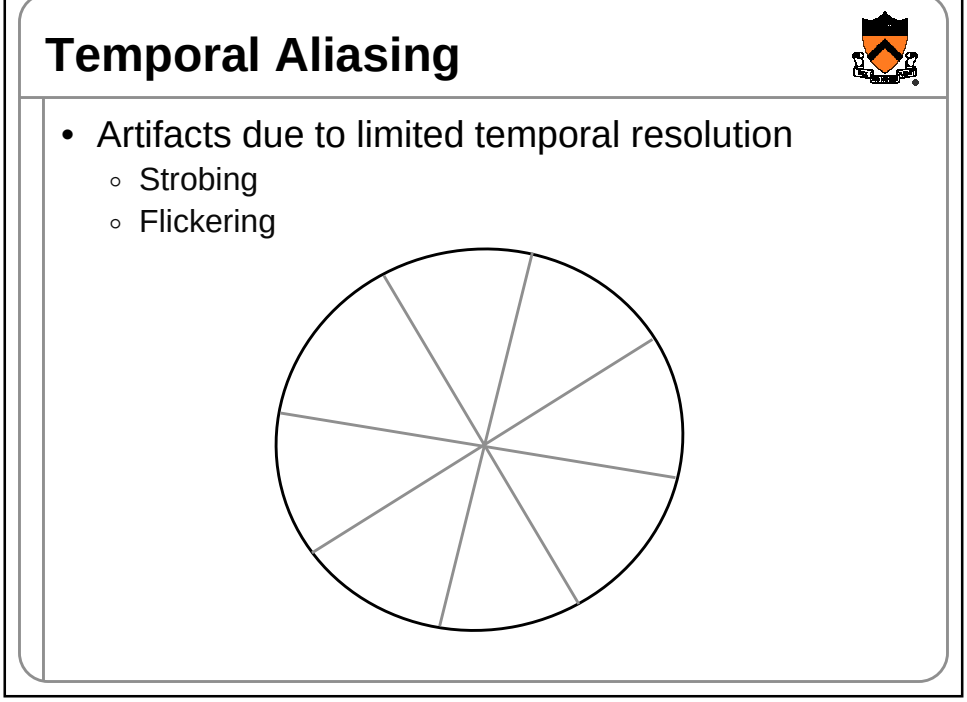


Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

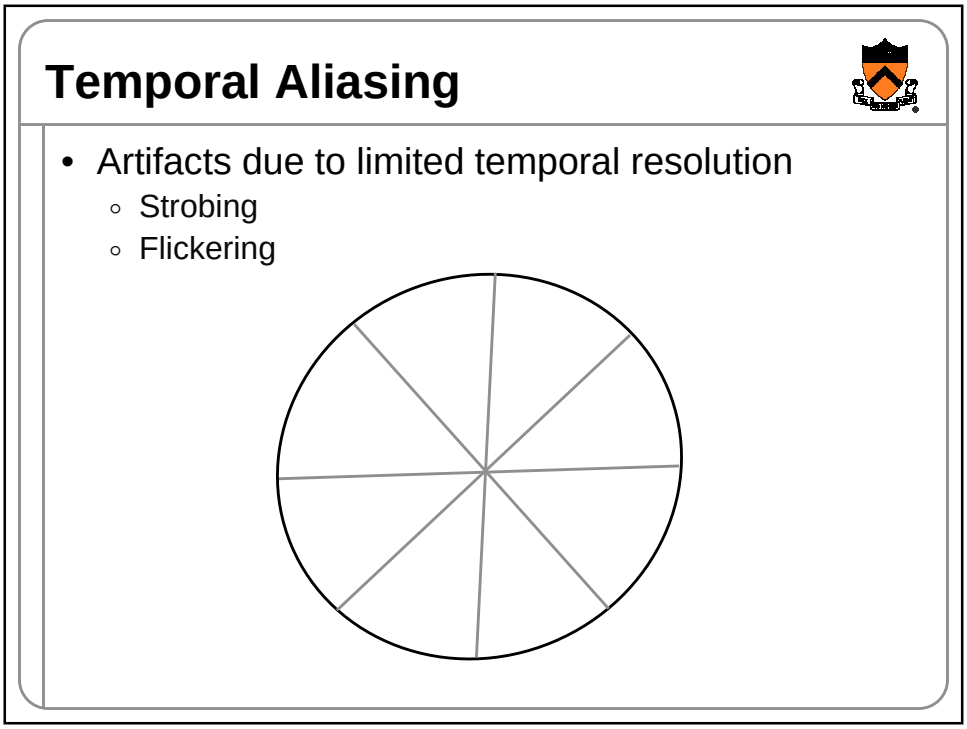


Temporal Aliasing



- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

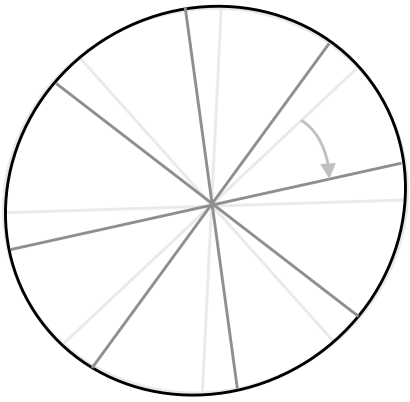
Temporal Aliasing



- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



Sampling Theorem

- A signal can be reconstructed from its samples, if the original signal has no frequencies above 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called “Nyquist rate”

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

Antialiasing

- Sample at higher rate
  - Not always possible
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)

Unfortunately,
real image signals
are rarely bandlimited

Convolution

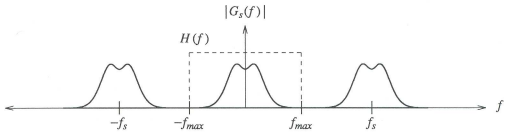
- Convolution of two functions (= filtering):

$$g(x) = f(x) \otimes h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x - \lambda) d\lambda$$

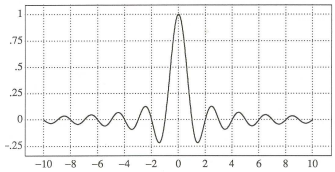
- Convolution theorem
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

Ideal Low-Pass Filter

- Frequency domain



- Spatial domain



$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

Image Processing

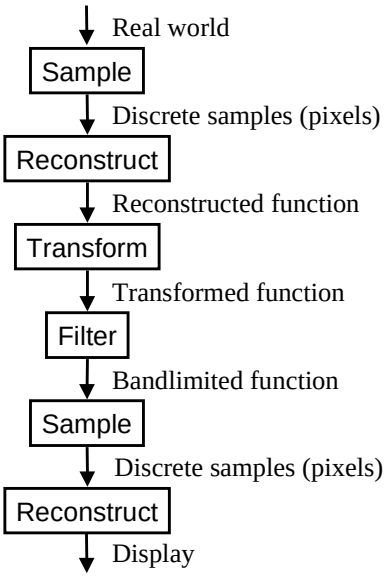


Image Processing

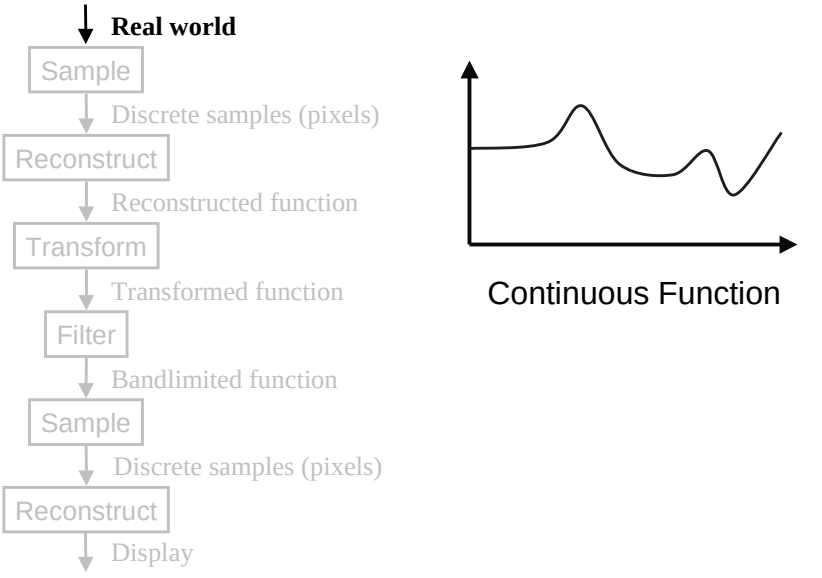


Image Processing

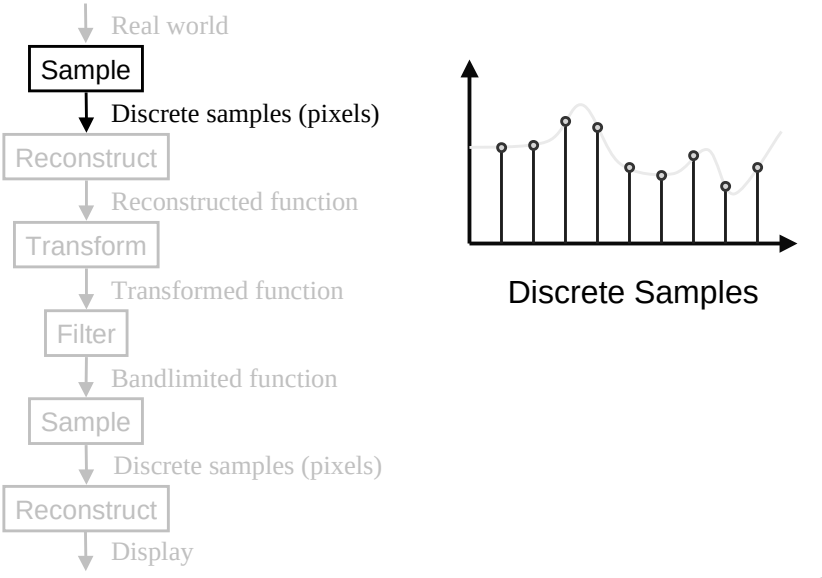


Image Processing

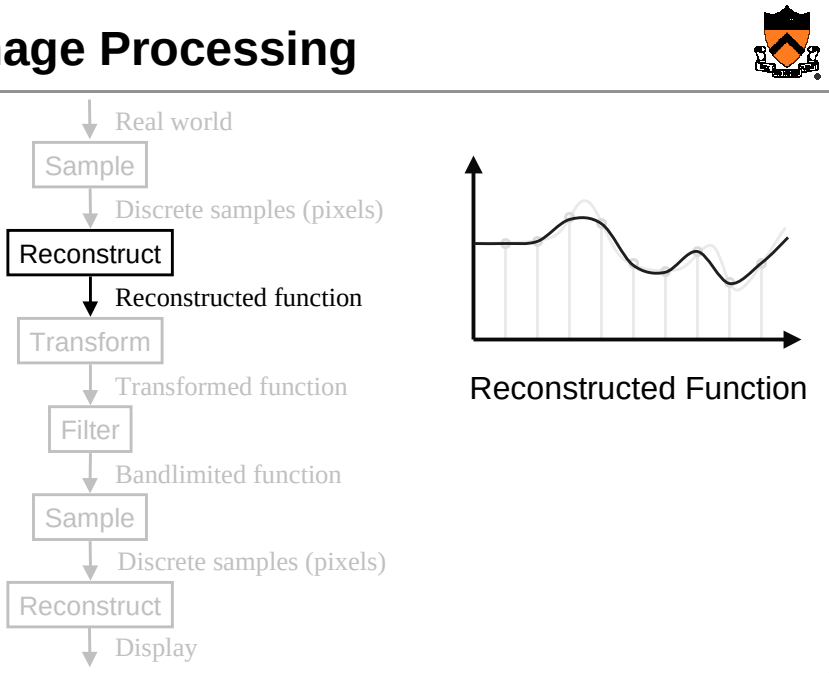


Image Processing

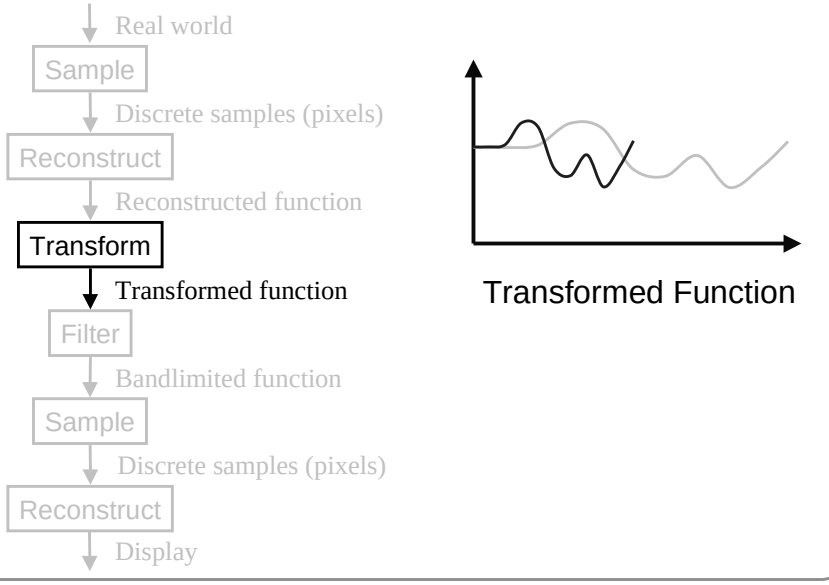
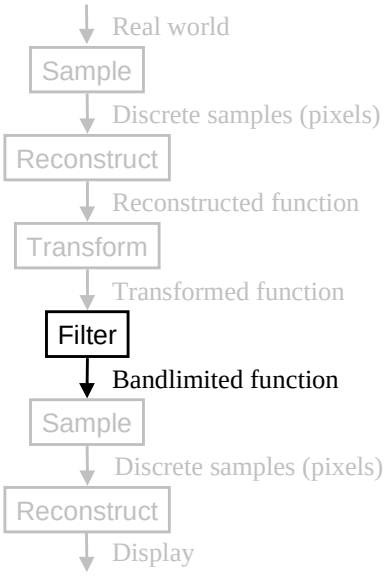
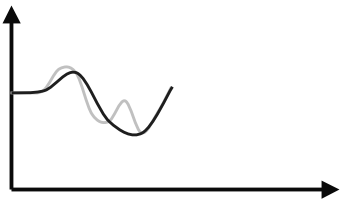


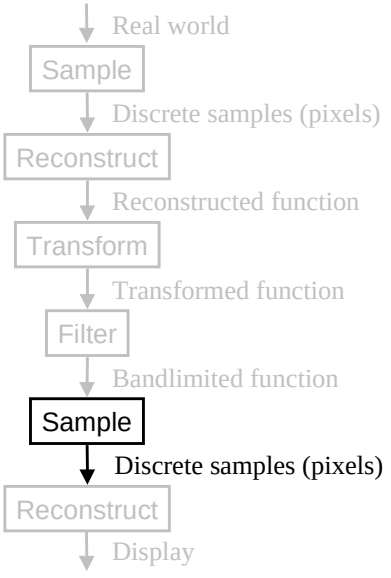
Image Processing

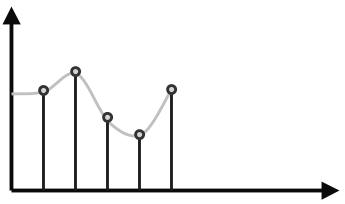




Bandlimited Function

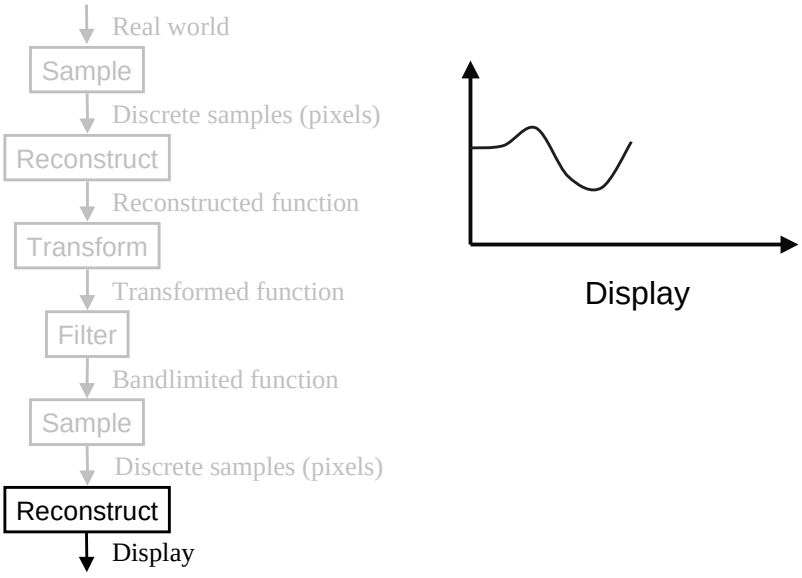
Image Processing





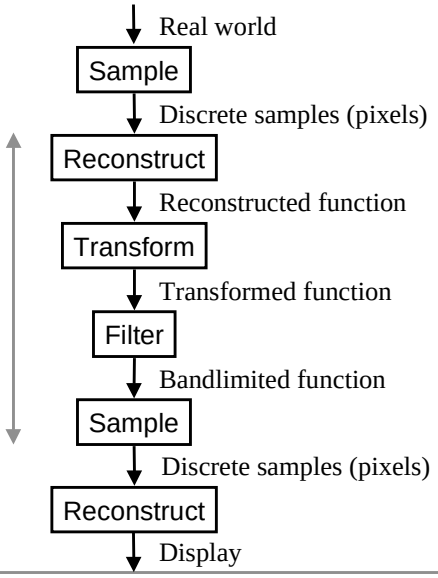
Discrete samples

Image Processing



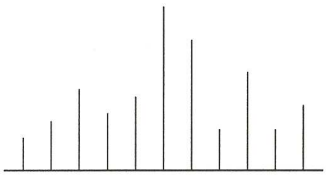
Practical Image Processing

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter

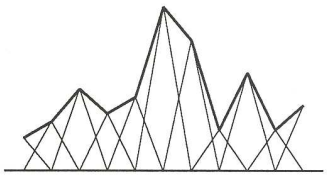


Triangle Filter

- Convolution with triangle filter



Input

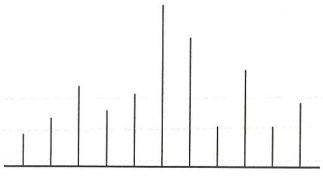


Output

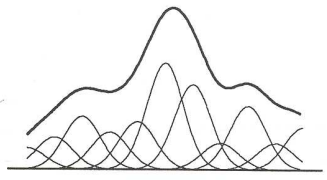
Figure 2.4 Wolberg

Gaussian Filter

- Convolution with Gaussian filter



Input



Output

Figure 2.4 Wolberg

Adjust Blurriness

- Convolve with filter whose support covers more than one pixel

Original

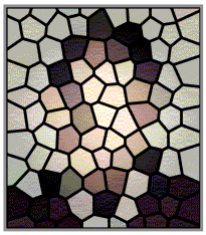
Blur

Edge Detection

- Convolve with a filter that finds differences between neighbor pixels

Original

Detect edges

Summary

- Image processing is a resampling problem
  - Aliasing
  - Filtering