Image Sampling and Reconstruction

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Overview

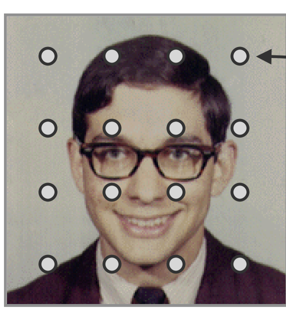
- Image sampling and reconstruction
  - Pixels are discrete samples of continuous function
  - Frequency analysis
- Image resampling
  - Aliasing
  - Filters and convolution

Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
  - Morphs
- Combining
  - Composite

Image Sampling

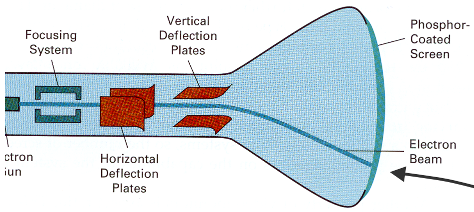
- An image is a 2D rectilinear array of samples
  - Quantization due to limited intensity resolution
  - Sampling due to limited spatial and temporal resolution



Pixels are
infinitely small
point samples

Image Reconstruction

- Re-create continuous image from samples
  - Example: cathode ray tube



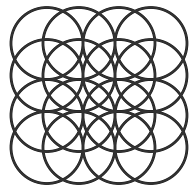


Image is reconstructed by displaying pixels with finite area (Gaussian)

Sampling and Reconstruction

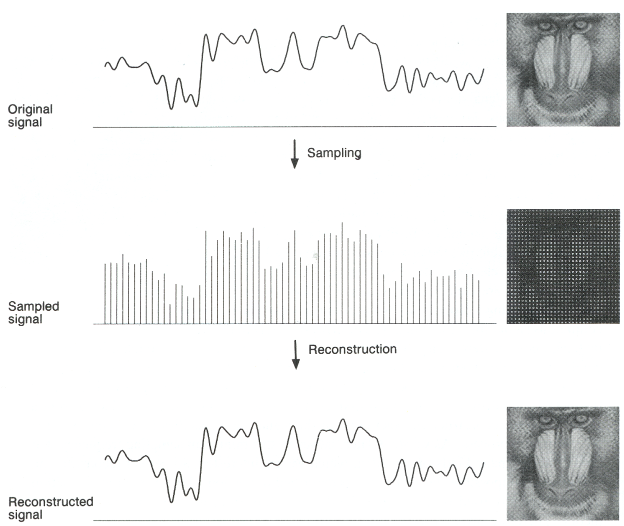
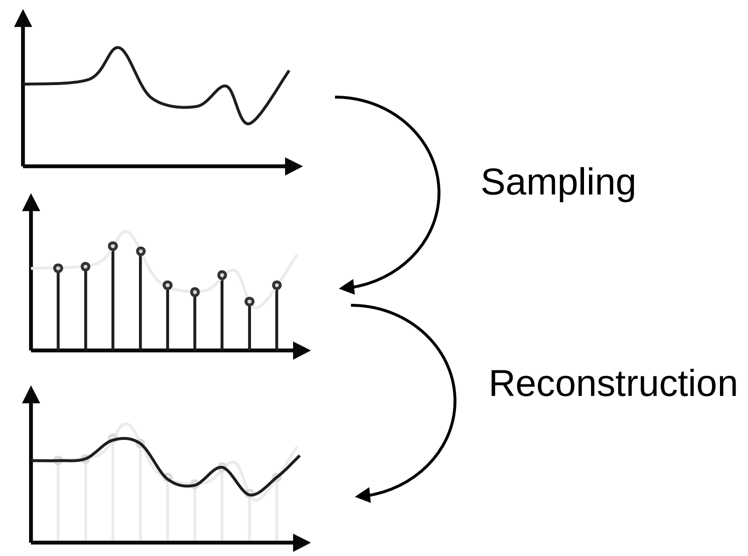


Figure 19.9 FvDFH

Sampling and Reconstruction



Sample Processing

- Apply function to samples
  - Linear functions
  - Linear reconstructions

↓ Real world

Sample

↓ Discrete samples (pixels)

Transform

↓ Transformed samples (pixels)

Reconstruct

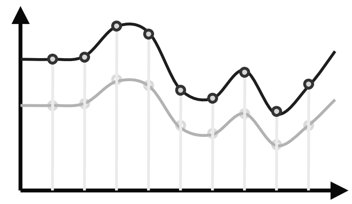
↓ Display

Adjusting Brightness

- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)

Original

Brighter

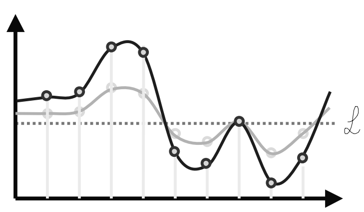


Adjusting Contrast

- Compute mean luminance $\mathcal{L}$ for all pixels
  - luminance = 0.30*r + 0.59*g + 0.11*b
- Scale deviation from $\mathcal{L}$ for each pixel component
  - Must clamp to range (e.g., 0 to 255)

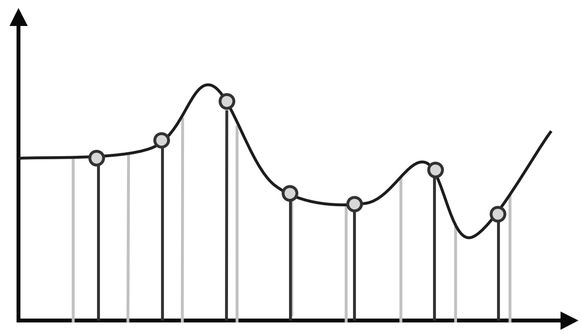
Original

More Contrast



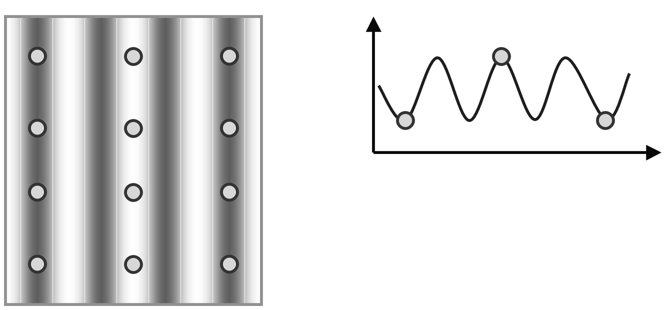
Resampling

- What if samples are at new locations?
  - Image scaling, warping, etc.



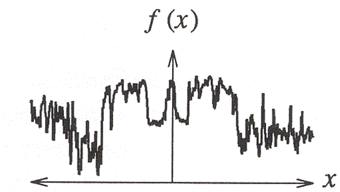
Sampling Theory

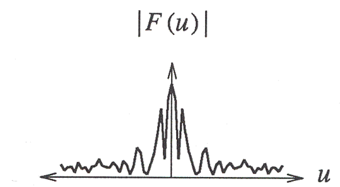
- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?



Spectral Analysis

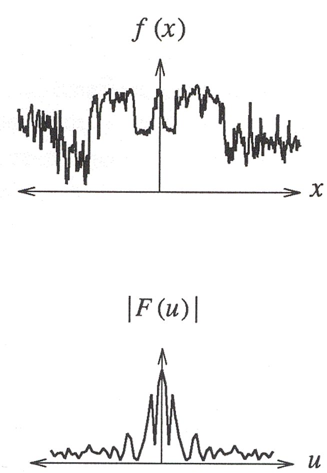
- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain:
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform



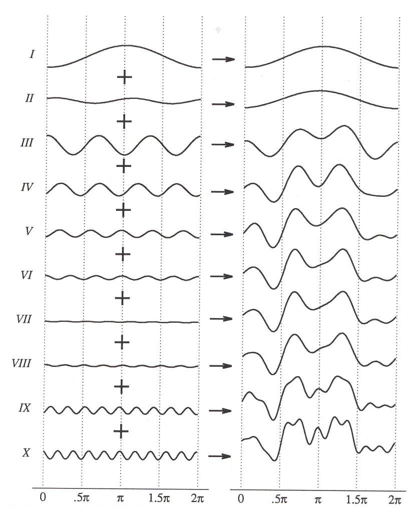


Figure 2.6 Wolberg

Fourier Transform

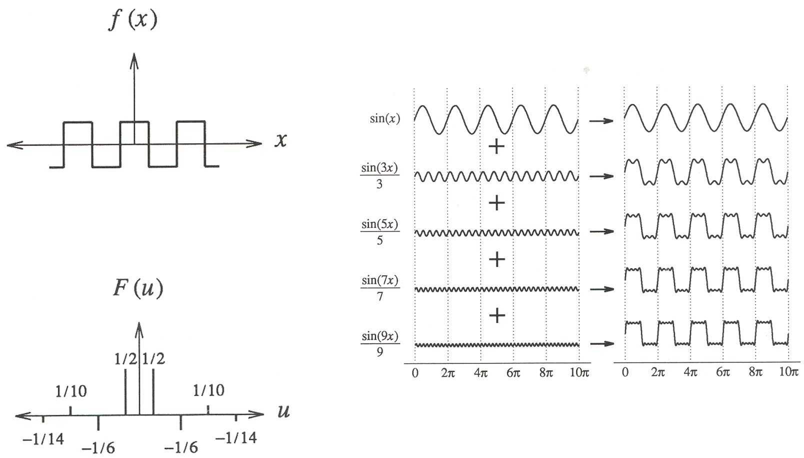


Figure 2.5 Wolberg

Fourier Transform

- Fourier transform:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi x} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi u} du$$

Aliasing

- Artifacts due to undersampling

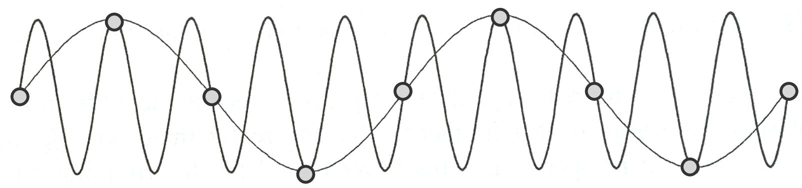
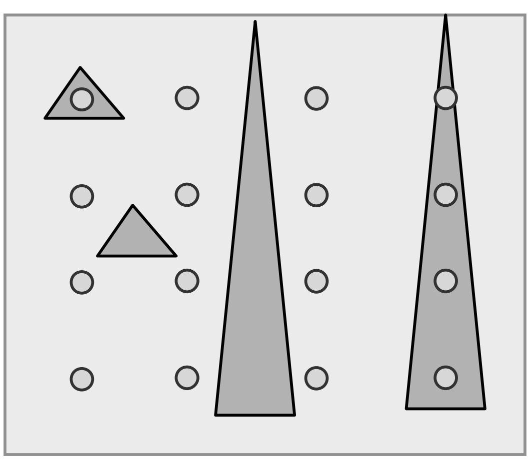


Figure 14.17 FvDFH

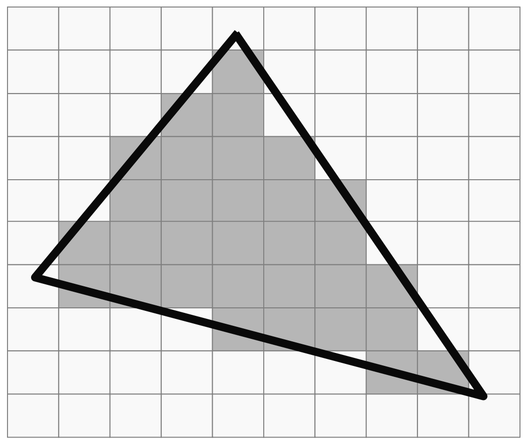
Spatial Aliasing

- Point sampling problems



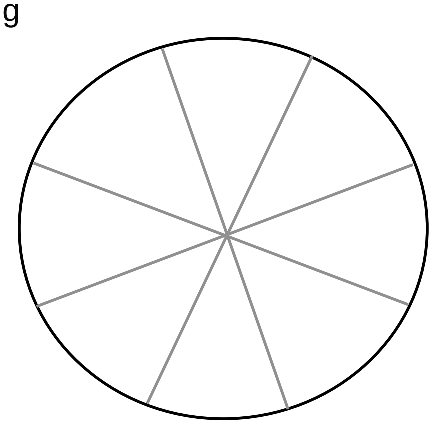
Spatial Aliasing

- Jaggies



Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



Temporal Aliasing

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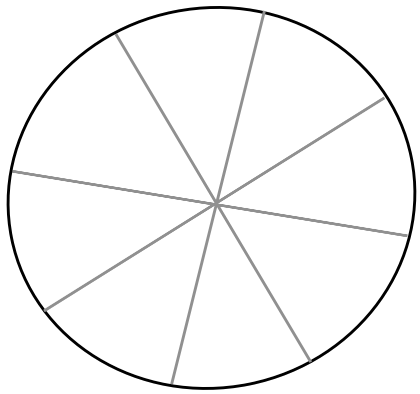

Temporal Aliasing

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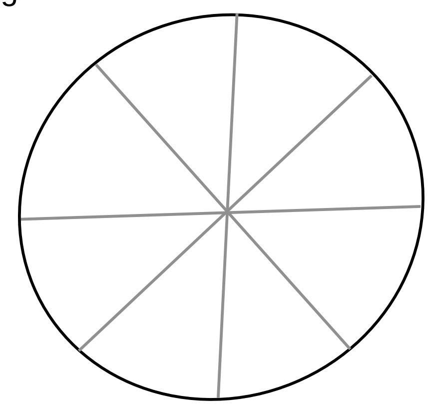

Temporal Aliasing

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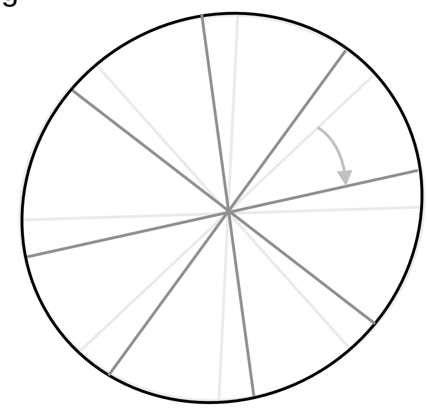


Sampling Theorem

- A signal can be reconstructed from its samples, if the original signal has no frequencies above 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called "Nyquist rate"

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

Antialiasing

- Sample at higher rate
  - Not always possible
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)

Unfortunately,
real image signals
are rarely bandlimited

Convolution

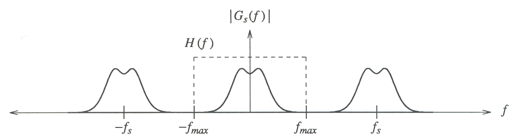
- Convolution of two functions (= filtering):

$$g(x) = f(x) \otimes h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda) d\lambda$$

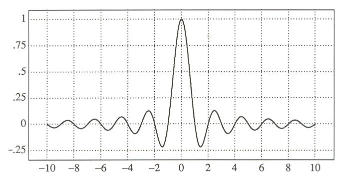
- Convolution theorem
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

Ideal Low-Pass Filter

- Frequency domain



- Spatial domain



$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

Image Processing

↓ Real world

Sample

↓ Discrete samples (pixels)

Reconstruct

↓ Reconstructed function

Transform

↓ Transformed function

Filter

↓ Bandlimited function

Sample

↓ Discrete samples (pixels)

Reconstruct

↓ Display

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

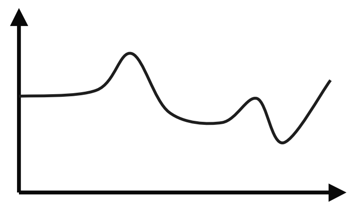
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Continuous Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

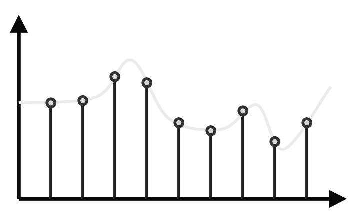
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Discrete Samples

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

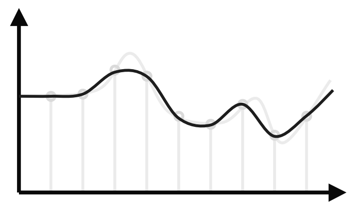
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Reconstructed Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

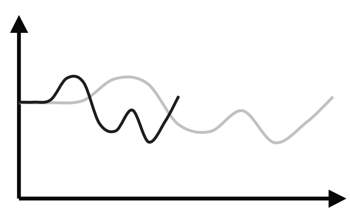
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

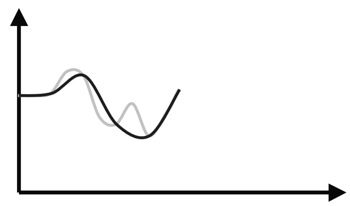
Display



Transformed Function

Image Processing

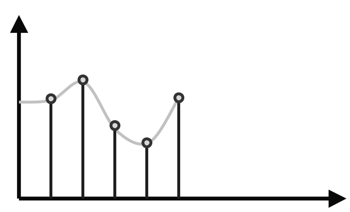
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Bandlimited Function

Image Processing

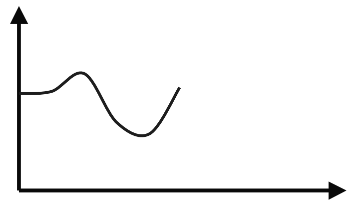
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Discrete samples

Image Processing

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Display

Practical Image Processing

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Triangle Filter

- Convolution with triangle filter

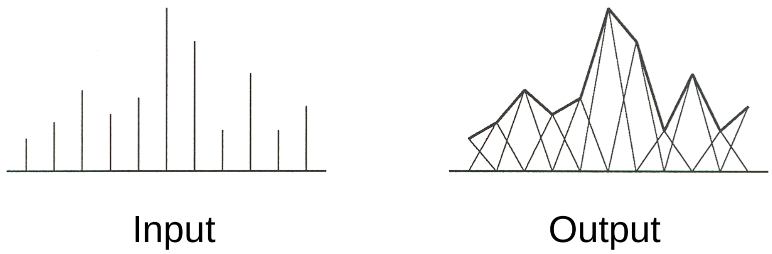


Figure 2.4 Wolberg

Gaussian Filter

- Convolution with Gaussian filter

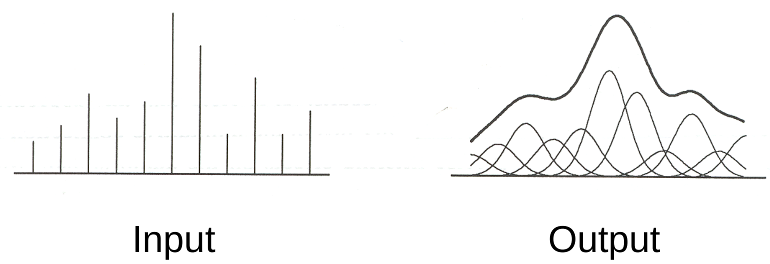
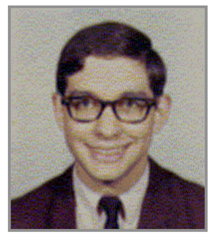


Figure 2.4 Wolberg

Adjust Blurriness

- Convolve with filter whose support covers more than one pixel


Original


Blur

Edge Detection

- Convolve with a filter that finds differences between neighbor pixels


Original


Detect edges

Summary

- Image processing is a resampling problem
  - Aliasing
  - Filtering