Representations of Geometry for Computer Graphics

An Introduction

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Representations of Geometry

Reps provide the foundations for

- Computer Graphics, Computer-Aided Geometric Design, Visualization, Analysis, Robotics

They are languages for describing geometry

Semantics	Syntax
values	**data structures**
operations	**algorithms**

Data structures determine algorithms!

Semantic Equivalence of Representations

Thesis

- Each fundamental representation has enough expressive power to model the semantics of geometric sets
- Possible to perform all geometric operations with any fundamental representation!

Analogous to Turing-Equivalence

- But all computers today are turing-equivalent, but we still have many different processors

Computational Differences

Efficiency

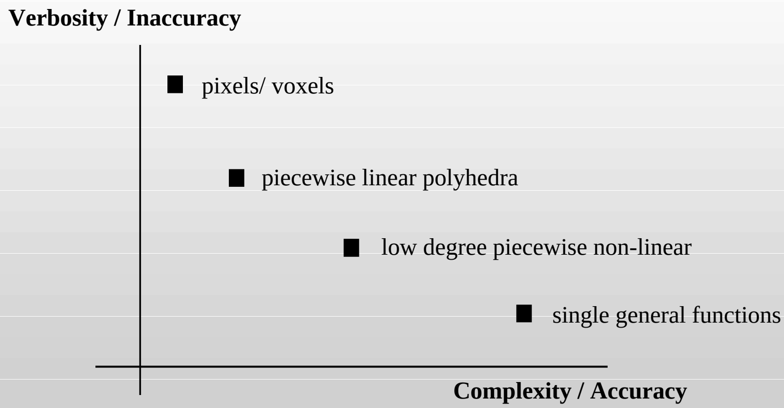
- combinatorial complexity (e.g. O(n log n))
- space/time trade-offs (e.g. z-buffer)
- numerical accuracy/stability (degree of polynomial)

Simplicity

- hardware acceleration
  - special purpose, multi-processor, cache hits
- software creation and maintenance

Useability

Complexity vs. Verbosity Tradeoff



Piecewise Constant Approximations

Pixels and Voxels

- The simplest and most verbose representation of geometry

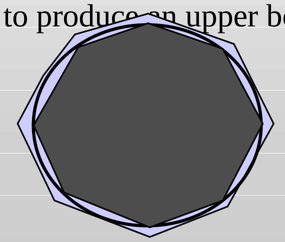
Ancient precursor: mosaics

- Sumarians created mosaics in third millennium B.C.
- Byzantine Roman Empire after the ascension of Christianity as the state religion.

Piecewise Linear Approximations

Archimedes

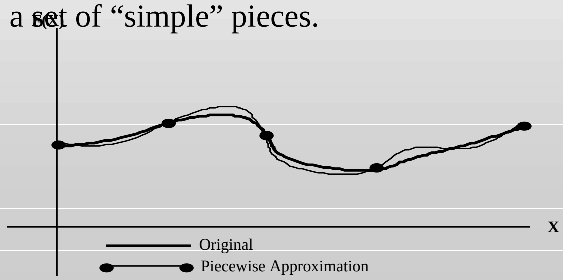
- Used polygonal approximations to approximate integration.
- Use the area of an inscribed n-gon to provide a lower bound on PI, and the area of a circumscribing n-gon to produce an upper bound.



Piecewise Polynomial Approxiations

Splines

- Finding the roots of high degree polynomials is infeasible.
- Splines approximated higher degree functions by a set of “simple” pieces.



Implications of Complexity / Verbosity Tradeoff

Central question

- When should we trade conciseness and accuracy to gain faster and simpler algorithms?

The Hardware Factor

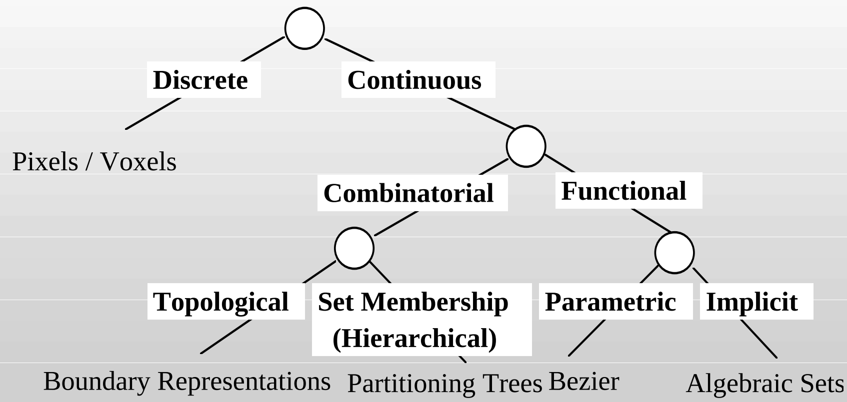
- Fast texture mapping of polygons -> voxel representations for all 3D geometry?
- Fast polygon rendering -> dispense with non-linear representations?

Implications of Complexity / Verbosity Tradeoff

Central answer

- Each principal representation will have its its niche if and only if it describes some essential aspect of geometry.
- Need to recognize niches, and understand how the representations will be related to one another in a complete system.

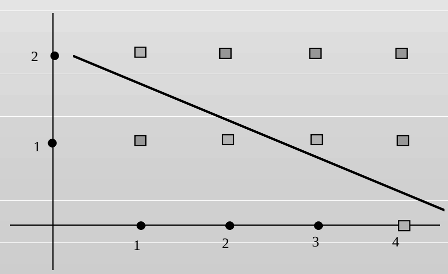
Taxonomy for Representations of Geometry



Discrete vs. Continuous

Same distinction as Integers vs. Reals

- Represented computationally as a multi-dimensional array.
- Can be as accurate as desired.
- Simplicity can be great advantage when designing special purpose hardware.



Combinatorial

Combinatorial representations

- Adresses the problem of representing "the many" with finite combinatorial structures.
- Describes how pieces are connected to one another or how to organize hierarchically.
- Can be used to introduce discontinuities absent from continuous functions.

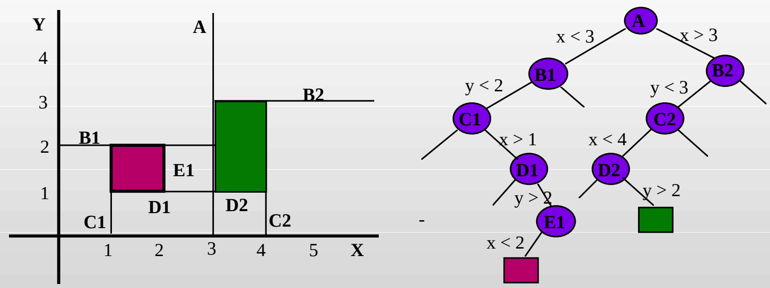
Topological Representations

Topological Structure

- Encodes the incidence relations between geometrically continuous pieces (quilting).
- Ideal for use in specifying topological deformations.
- Most often used with parametric representations to define boundaries

This is closer to how our brains perceive the world.

Set Membership Hierarchy



Set Membership Hierarchy

Set Membership Relation

- Represents hierarchy of bounding volumes with a tree that encodes <u>containment</u>.
- Octrees, Binary Space Partitioning Trees, Quadric bounding volume hierarchies

Targeted at computational efficiency

- Reduces $O(n^2)$ for <u>intersection</u> and/or <u>visibility</u> calculations using transitivity of set membership.

Functional

**Functional + Combinatoral ->
representation of geometry**

Functional representations

- Uses C^∞ functions as a finite rep. of an uncountable number of points to define curves and surfaces.
- Poorly suited for expressing arbitrary discontinuites.
  - Surfaces are discontinuities in function from points in 3-space to some set of attributes.

Parametrics

Geometry in the range of a function

$x = x0\ t + x1\ (1-t)$

$y= y0\ t + y1\ (1-t)$

Definable by weighted sum of points

- Good for interactive design and deformations.

Provides an enumeration function for points

- Generate samples for polygonal meshes and for scan-conversion.

Implicits

Geometry in domain of a function

Ax + By + Cz + D <= 0

Provides a membership function

- needed for computing intersections.
  - CSG, collision detection, ray-tracing, radioisty, visibility, etc.
- may be paired with a parametric to compute intersections

Implications for Designing Geometry Systems

Expression vs. Efficiency

- All fundemental representations can describe, approximately, any geometry.
- No single representation is the most efficient for every operation.

Multiple Representations

- Support multiple representations and initiate data-type conversions as needed.
- Can be hidden from the user by using the class "Geometric Set".
- Costs are: conversion time and software complexity