

# Radiosity

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## Overview

- ♦ Motivation
- ♦ Radiosity equation
- ♦ Computing form factors
- ♦ Solution methods
- ♦ Combining radiosity and ray tracing

## Motivation

### ♦ Goal

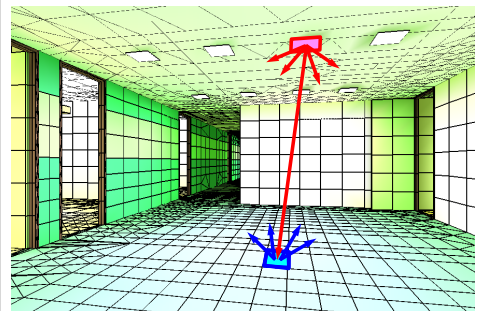
- Eliminate “ambient” term in illumination equation by more accurately modeling *diffuse* interobject reflection



## Radiosity

### ♦ Basic idea

- Treat every polygon as light source

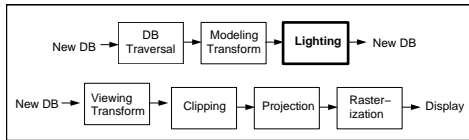
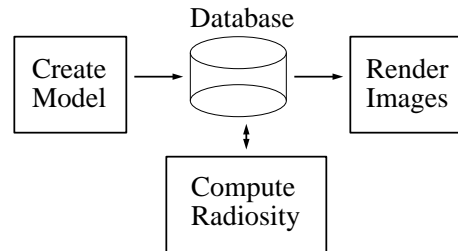


### ♦ Advantages

- Physically models shadows and indirect diffuse illumination
- Independent of any viewpoint

## Radiosity

### ◆ Two-Phase Rendering:



## Radiosity Equation

### ◆ Equation Formulation:

$$B_i = E_i + \rho_i \sum B_j F_{ij}$$

$B_i$  = Radiosity of patch  $i$

$E_i$  = Emission of patch  $i$

$\rho_i$  = Reflectivity of patch  $i$  ( $K_D$ )

$F_{ij}$  = Form-factor between patches  $i$  and  $j$

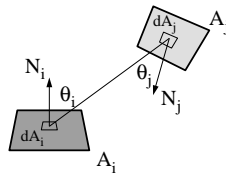
### ◆ Matrix Formulation:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \dots & -\rho_2 F_{2n} \\ -\rho_3 F_{31} & -\rho_3 F_{32} & \dots & -\rho_3 F_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \dots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_n \end{bmatrix}$$

## Form Factors

### ◆ Definition:

- Fraction of energy leaving patch  $j$  that arrives at patch  $i$



$$F_{di-j} = \frac{\cos \theta_i \cos \theta_j}{\pi r^2} H_{ij} dA_j$$

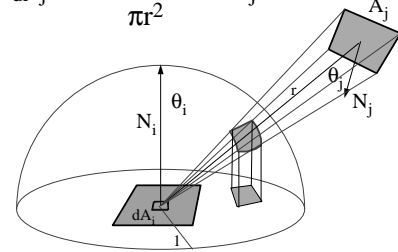
$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} H_{ij} dA_j dA_i$$

## Computing Form Factors

### ◆ Method:

- Project onto unit hemisphere ( $A_j \cos \theta_j / r^2$ )
- Project onto unit circle base ( $\cos \theta_i$ )
- Divide by area of circle ( $1/\pi$ )

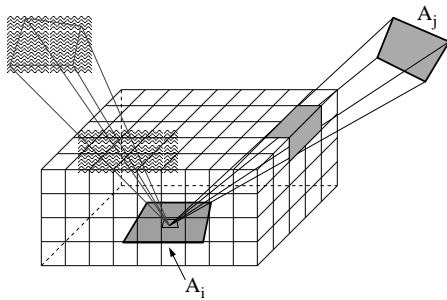
$$F_{di-j} = \frac{\cos \theta_i \cos \theta_j}{\pi r^2} A_j$$



## Hemi-Cube Form Factors

### ♦ Method:

- Project scene onto hemi-cube positioned at centroid of patch  $i$
- Count pixel coverage to determine form factors  $F_{i*}$
- Can use hardware z-buffer
- Image precision (aliasing)



## Matrix Solution Methods

### ♦ Invert the Matrix:

- Gaussian elimination
- $O(n^3)$

### ♦ Gauss-Seidel Iteration

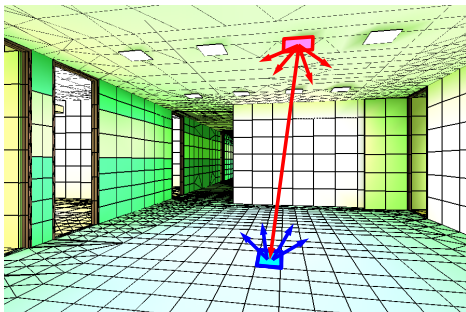
- Iterate gather radiosity to patches
- Relax rows of variables
- $O(n^2)$

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \dots & -\rho_2 F_{2n} \\ -\rho_3 F_{31} & -\rho_3 F_{32} & \dots & -\rho_3 F_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \dots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_n \end{bmatrix}$$

## Matrix Solution Methods

### ♦ Progressive refinement

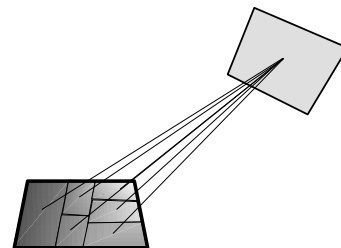
- Iterate shoot radiosity from patches
- Relax columns of variables
- Select patches to shoot in order of unshot radiosity
- $O(n^2)$  computation, but get pretty good solutions more quickly



## Mesh Substructuring

### ♦ Adaptive subdivision

- Partition each patch into multiple “elements” (along gradients of radiosity)
- $O(np)$  if only gather to elements from patches

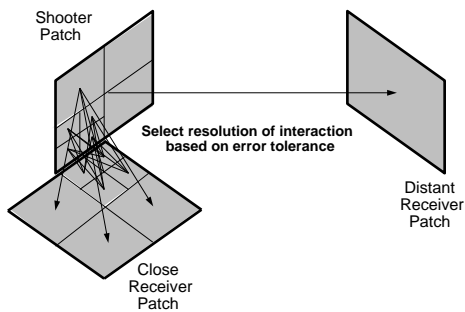


Patch partitioned into 6 elements

## Hierarchical Radiosity

### ♦ Multiresolution Computation

- Substructure patches into quad-tree
- Transfer energy using lower resolution mesh elements if can do so within error tolerance
- $O(n)$  computation



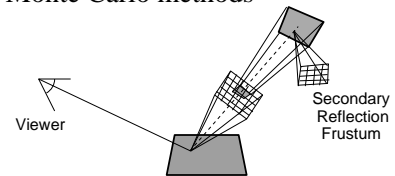
## Ray Tracing and Radiosity

### ♦ Dilemma:

- Radiosity is good at diffuse interobject reflection
- Ray tracing is good at specular interobject reflection

### ♦ Combine them

- Example: compute diffuse interobject reflections in a ray tracer
- Monte Carlo methods



## Summary

- ♦ Radiosity equation
- ♦ Computing form factors
- ♦ Solution methods
- ♦ Combining radiosity and ray tracing